Magneto-optic effects

Seminar I

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Abstract
This seminar discusses magneto-optic effects, phenomena in which optical properties of a medium and therefore its interaction with light are dependent on a magnetic field. First, we will explain the theoretical background of magneto-optic effects from both macroscopic and microscopic points of view. We will then describe the most common magneto-optic effects, such as the Faraday and the magneto-optic Kerr effect, and briefly review others. Finally, we will describe some applicative uses of magneto-optic effects in measurement devices, in wave modulators, and in memory devices.
1 Introduction
Magneto-optic effects represent phenomena in which light interacts with the medium that has been changed by the quasistatic magnetic field, which can be either intrinsic (like in ferromagnets) or external. Because of that, the material response to light changes and we can obtain changes in intensity and polarization for both transmitted and reflected light, as well as double refraction. Similar effects can be produced by electro-optic effects, in which we modulate material response to light with the quasistatic electric field, but their main disadvantage in contrast to magneto-optic effects is that much larger fields are needed for them to become considerable (Size order of 100V/m compared to size order of 0.1T).

While discovered long ago (first magneto-optic effect, the Faraday effect, was discovered by Faraday in 1845), they are still relevant and used in many devices. More recent technologies, such as laser, allow us to explore some other aspects of magneto-optics like nonlinear magneto-optic effects, while modern techniques allow us to create new magneto-optic structures with improved optical characteristics. Therefore, magneto-optic effects remain an exciting area of research in the modern day.

2 Theoretical description
Magneto-optic effects can be described by either macroscopic or microscopic theory. Macroscopic theory describes the interaction of light with the magneto-optic medium with the wave equation, derived from the Maxwell equations in matter, and relations that describe material response (which is in our case dependent on the magnetic field) to an electromagnetic field. It is usually used for the quantitative calculation of magneto-optic effects.

Microscopic theory describes the interaction of light with such medium through the response of its electrons to an electromagnetic wave in the presence of a magnetic field. Use of quantum mechanics is needed to obtain correct results. Microscopic theory is used if we want to fully understand the origin for magneto-optic effects or in cases where considered medium is very small (like a magnetized monolayer in multilayer) and we can’t use the wave equation, since we obtain it from the Maxwell equations in matter that can be only used for macroscopically sized bodies.

2.1 Macroscopic theory
Let us start with relations that describe the response of a material to an electromagnetic field. In most cases, this response can be approximated to linear and these relations can be written as follows:

$$ D = \varepsilon_0 \varepsilon \mathbf{E}, $$  

$$ B = \mu_0 \mu \mathbf{H}. $$

Here, $\varepsilon_0$ and $\mu_0$ are permittivity and permeability of vacuum, while $\varepsilon$ and $\mu$ are relative permittivity and relative permeability of the considered medium and are second-order tensors. Both $\varepsilon$ and $\mu$ are for magneto-optic materials dependent on a magnetic field, but for a majority of magneto-optic materials $\mu$ is approximately constant and equal to 1. There are some cases where $\mu$ dependence on a magnetic field is not negligible, but since dependence on a magnetic field isn’t much different...
between \( \tilde{\varepsilon} \) and \( \tilde{\mu} \), we don’t obtain any new magneto-optic effects, only additional terms in magneto-optic effects already derived from \( \tilde{\varepsilon} \). Therefore, we will assume in our seminar that \( \tilde{\mu} \) is equal to 1.

We will also assume that our light is a monochromatic normal mode \( (E = E_0e^{i(kr - \omega t)}, \text{where } \omega \text{ is the wave frequency and } k \text{ is the wave vector}) \). Consequently, our wave equation in the medium for such case is

\[
k^2E - k(k \cdot E) = \frac{\omega^2}{c_0^2} \tilde{\varepsilon}E,
\]

where \( c_0 \) is the speed of light. Equation (3) has nontrivial solutions when the following equation is valid:

\[
det \left[ k^2 \delta_{ik} - k_k \frac{\omega^2}{c_0^2} \varepsilon_{ik} \right] = 0.
\]

Here, \( \delta_{ik} \) is the Kronecker delta function (equal to 1 if \( i = k \) otherwise 0), while \( \varepsilon_{ik} \) are the components of the relative permittivity tensor \( \tilde{\varepsilon} \). From obtained wave vectors, we can calculate the corresponding refractive indexes with the equation \( n = \frac{\omega}{c_0} |k| \). To fully calculate magneto-optic effects, we also need relation that describes reflection and transmission of the light on the surface of the medium. It is derived from the Maxwell equations in the matter and is in the case when the light falls perpendicularly on the surface equal to

\[
E_{ot} = E_{oi} - E_{or} = \frac{2n_1}{n_1 + n_2} E_{oi}.
\]

where \( E_{oi}, E_{or}, \) and \( E_{ot} \) are the electrical amplitudes of the incident, transmitted and reflected light, \( n_1 \) is the refractive index of the medium that contains the incident and the reflected light, and \( n_2 \) is the refractive index of the medium that contains the transmitted light.

Since we now have all equations required for the calculation of light interaction with the medium, let us now observe the shape of the relative permittivity tensor for magneto-optic media. First, let us consider the simplest case of a magneto-optic material that is magnetically unordered and optically isotropic in the absence of magnetic field. Let the applied magnetic field \( (B) \) be directed along the z-axis of the coordinate system. Then the relative permittivity tensor \( [11] \) is

\[
\tilde{\varepsilon} = \begin{bmatrix} \varepsilon_i & 0 & 0 \\ 0 & \varepsilon_i & 0 \\ 0 & 0 & \varepsilon_i \end{bmatrix} + \begin{bmatrix} b(B) & ig_z & 0 \\ -ig_z & b(B) & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \varepsilon_i & ig_z & 0 \\ -ig_z & \varepsilon_i & 0 \\ 0 & 0 & \varepsilon_i \end{bmatrix}
\]

and we obtain the following relation between \( D \) and \( E \):

\[
D = \varepsilon_0\tilde{\varepsilon}E = \varepsilon_0\left( \varepsilon_i E + iE \times g(B) + b(B)(E_{ux}, E_{uy}, 0) \right).
\]

Here, \( \varepsilon_i \) is the initial relative permittivity in the absence of the applied magnetic field, \( b(B) \) is equal to \( \varepsilon_i - \varepsilon_i \), and \( g_z \) is the z component of the gyration axial vector\(^2\) \( g(B) = (g_x, g_y, g_z) \) (both x and y component are in our case equal to zero). \( b(B) \) is usually small and can in many cases neglected, while the gyration axial vector is usually proportional to the applied magnetic field and therefore equal to

\[
g(B) = aB,
\]

where \( a \) is a constant that is related to the linear magnetic susceptibility of medium. The gyration vector is not always proportional to the applied magnetic field. It can contain higher orders of the applied magnetic field or even have imaginary terms, which are related to nonlinear magnetic susceptibilities of medium. If \( g, b, \) and \( \varepsilon_i \) from (6) are real, there is no absorption in such medium. Equations (7) and (8) can also be used for magnetically ordered materials, such as ferromagnets that are optically isotropic in the absence of magnetization, if the applied magnetic field \( B \) in these equations is replaced with the magnetization \( M \) [4].

Equations (3) and (4) can be used to obtain the eigenmodes of light in the medium with the relative permittivity tensor from (6). For the practically important case where the wave vector of light is parallel to the applied magnetic field, we obtain following wavevectors \( (k_\pm) \) and their corresponding refractive indexes \( (n_\pm) \) by inserting (6) into equation (4):

\[
k_\pm = \sqrt{\frac{\omega^2}{c_0^2} \varepsilon_i \left( 1 \pm \frac{g_z}{\varepsilon_i} \right)} = \frac{\omega}{c_0} n_\pm.
\]

\(^2\)An axial vector is a quantity that in contrast to a vector doesn’t reverse sign when the coordinate axes are reversed.
Here, we assumed that \( g_z \) is much smaller compared to \( \varepsilon_z \). When we insert obtained wavevectors into equation (3), we obtain the following corresponding solutions for the electric field:

\[
E_\pm = \frac{E_0}{2} e^{i(\varepsilon_z z - \omega t)} \left( \hat{e}_x \pm \hat{e}_y e^{i(\frac{\pi}{2})} \right).
\]

Here, we utilized the fact that the electric field is for our case perpendicular to the wave vector, as can be seen from the Maxwell equation \( \nabla \times \mathbf{D} = \nabla \left( \varepsilon_0 \varepsilon_z \mathbf{E} \right) = i \mathbf{k} (\varepsilon_0 \varepsilon_z \mathbf{E}) = 0 \). As we can see, we obtained two circularly polarized waves that rotate in the left circular \( (\mathbf{E}_-) \) or the right circular \( (\mathbf{E}_+) \) direction and propagate with different phase velocities \( (c_\pm = c_0/n_\pm) \) for our medium. The reason for magneto-optic effects in our case is the fact that \( g_z \) and therefore \( n_\pm \) from (10) that affect the propagation of light are dependent on the applied magnetic field (or magnetization). This leads to the Faraday effect and the polar Kerr effect, as we will see in chapter 3.

Let us now write the most general form of relative permittivity tensor for magneto-optic effects [1, 4]. It is equal to

\[
\varepsilon = \begin{bmatrix}
\varepsilon_{xx} + b_x & \varepsilon_{xy} + i g_x & \varepsilon_{xz} - i g_y \\
\varepsilon_{yx} - i g_x & \varepsilon_{yy} + b_y & \varepsilon_{yz} + i g_x \\
\varepsilon_{zx} + i g_y & \varepsilon_{zy} - i g_x & \varepsilon_{zz} + b_z
\end{bmatrix}
\]

where \( \varepsilon_{ik} \) represent components of the relative permittivity tensor in the absence of magnetic field, \( \mathbf{b} = (b_x, b_y, b_z) \), which is dependent on magnetic field, is again usually small and can be neglected, while \( \mathbf{g} = (g_x, g_y, g_z) \) represents the gyration axial vector for which the equation (8) is usually valid.

It is worth noting that magneto-optic effects are not constricted to the visible light wavelength region and its proximity. For example, there exist useful magneto-optic effects in the x-ray region. In the cases of stronger electromagnetic waves (like in lasers), \( \mathbf{D} \) isn’t proportional\(^3\) to \( \mathbf{E} \), but is equal to

\[
\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_0 \mathbf{E} + \varepsilon_0 \left( x^{(1)} \mathbf{E} + x^{(2)} \mathbf{E}^2 + x^{(3)} \mathbf{E}^3 + \ldots \right).
\]

Here, \( x^{(n)} \) are nth-order susceptibilities of the medium. Since nonlinear susceptibilities (from \( x^{(2)} \) onward) are also dependent on a magnetic field, we obtain nonlinear magneto-optic effects.

### 2.2 Microscopic theory

To understand microscopic origins for magneto-optic effects, we first need to explain some of the microscopic properties of the light. Light consists of photons, which can have two polarization states: The right circular and the left circular polarization. Arbitrary polarization of light is produced by the superposition of the mentioned polarizations. Photon also contains spin, which is directed along the direction of the movement of the photon and has two possible values: \( h \) for the left circular and \( -h \) for the right circular polarization.

Magneto-optic effects manifest themselves as a difference in absorption and a difference in phase velocities for these polarizations. To calculate these differences, we would need to use quantum mechanics and calculate electron response to the right and left circularly polarized light in the quasistatic magnetic field with the use of perturbation theory. From there, we could calculate the absorption rate and phase velocities for both polarizations. The quasistatic magnetic field isn’t necessarily external, since for example the magneto-optic effects in ferromagnets are produced by the intrinsic magnetic field that gives rise to spin-orbital coupling [3].

However, the causes of magneto-optic effects can be roughly explained with the following example [6]. Let the light propagate in an isotropic medium in the external magnetic field that is parallel to the wave vector of light. Let the intrinsic magnetic field of the medium be negligible. As we know, arbitrary polarization of light is a sum of the right and the left circular polarization, therefore it is enough to observe the response of the medium for these polarizations. Atoms of medium consist of relatively static nucleons and electron cloud. Right and left circular polarization induce left and right circular movement on the electron cloud. If we now consider the influence of magnetic field, which acts with the force \( -e (\mathbf{v} \times \mathbf{B}) \), we see that this force acts towards the nucleon for the right circular and away from the nucleon for the left circular movement. This means that effective radius between the nucleon and the electron cloud is reduced for the left circular and augmented for the right circular polarization. This difference in radii gives us different refractive indexes and therefore

\(^3\)There also exists nonlinear dependence between \( \mathbf{M} \) and \( \mathbf{B} \), but is much weaker and can be neglected.
different phase velocities for the left and the right circular polarization. This effect is magneto-optic since it is induced by the magnetic field that produced the difference in radii.

On the same example, a difference in absorption can also be roughly explained [4]. Let the light in the above-mentioned medium be at such wavelength that the energy of its photons is roughly equal to the energy difference between the s and p shells of the atoms in the medium, so that electrons can be excited from s to p shells. To simplify matters, we will neglect the spin of electrons to reduce the number of transitions we have to consider. Because of the external magnetic field, we obtain a split in energies for the different projections of orbital angular momentum \( m_L \) in the direction of the magnetic field (Zeeman effect, seen on figure 1), which is also the direction of light propagation. Light consists of photons with right and left circular polarization, which can excite electrons from s to p shell. For the left circular polarization (its spin is equal to \( -\hbar \)), the transition of \( \Delta m_L = 1 \) is allowed, while for the right circular polarization (its spin is equal to \( -\hbar \)) the transition of \( \Delta m_L = -1 \) is allowed. From figure 1, we can see that there exists an energy difference between the s and p shells for transitions with \( \Delta m_L = -1 \) and transitions with \( \Delta m_L = 1 \). Of these transitions, the transition with the energy difference closer to the energy of photons is more likely to happen. Since each of these two transitions can only occur with only one type of circular polarization, we obtain a difference in absorption rates for the left and the right circular polarization. This effect is magneto-optic since it is induced by the magnetic field that produced energy splitting of states. In similar way, we can also roughly explain the origin for magneto-optic effects in ferromagnets where the main cause is the energy splitting of states due to spin-orbital coupling.

![Figure 1: Energy splitting due to the Zeeman effect. Adapted from [4].](image)

The example above may give us the basic idea behind magneto-optic effects, but is flawed, since for example, it doesn’t explain how is relative permeability \( \mu \) dependent on a magnetic field. Therefore, we see that we need to use quantum mechanics to obtain correct results.

3 Magneto-optic effects

As mentioned before, we can obtain intensity and polarization changes with magneto-optical effects for both transmitted and reflected light. We can also obtain double refraction in previously isotropic material. In this chapter, we will describe the Faraday effect for transmitted light, the Kerr effect for reflected light and briefly review some other effects.

3.1 Faraday effect

Faraday effect is the rotation of polarization for linearly polarized light in the medium in the applied magnetic field that is parallel to the wave propagation direction, as seen in figure 2. In many cases, it can be described with a simple equation:

\[
\beta = v Bd,
\]

where \( \beta \) is the angle of rotation, \( B \) is the size of the applied magnetic field in the direction of the wave vector of light, \( d \) is the length of an object, and \( v \) is the Verdet constant of the material (usually in units of radians per tesla per meter). The Verdet constant can be finite in all materials, however for most materials it is very small (size orders of \( 10^{-4} \) rad/T·m are common). It is usually the largest for paramagnetic materials such as terbium gallium garnet\(^4\), where it is equal to \(-134\ \text{rad}/(\text{T} \cdot \text{m})\) for the wavelength of light at 632nm [3].

![Figure 2: Visual representation of the Faraday effect. Taken from [3].](image)

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\(^4\) Synthetic garnet with the chemical composition \( \text{Tb}_3\text{Ga}_5\text{O}_{12} \).
Equation (14) can be obtained for the medium with the relative permittivity tensor from (6), where we assume that \(\varepsilon_0, \varepsilon_1\) and \(g_z\) are real so there is no absorption in the medium. Let the incident wave propagate in the z-direction and be polarized in the x-direction outside the medium. As we know from the equations (10) and (11), transmitted light that enters the medium splits into the right and the left circularly polarized waves \(E_x\), each of them with the different corresponding refractive indexes \(n_x\). Incident light is written as a sum of the right and the left circularly polarized waves, where the right (left) circularly polarized wave outside the medium transform into the right (left) circularly polarized wave inside the medium. This can be written with equations

\[
E_1 = E_0 \hat{e}_x e^{i(kz - \omega t)} + E_0 \hat{e}_y e^{i(kz - \omega t)},
\]

\[
E_2 = \frac{2n_1}{n_1 + n_2} E_x + \frac{2n_1}{n_1 + n_2} E_y = \frac{2}{2 + \frac{2n_1}{n_1 + n_2}} (E_x + E_y) = C(E_x + E_y).
\]

Here, the factors before \(E_x\) in the equation (16) are the transmission rates obtained from the equation (5), \(n_1 \approx 1\) is the refractive index outside the medium and \(k\) is equal to \(\frac{\omega}{c_0}\). As we see, transmission rates for the left and the right circularly polarized light are different because of the different corresponding values of \(n_x\). However, we made the approximation that transmission rates are equal since \(n_+ \approx n_- \approx \frac{n_1 + n_2}{2}\), which can be justified with the fact that \(\frac{\omega}{c_0}\) in the equation (10) is much smaller than \(\sqrt{\varepsilon_1}\). Since the right and the left circular polarizations in (16) have different sizes of the corresponding wave vectors and therefore propagate with the different phase velocities \((c_+ = c_0 n_+ c_0 n_-)\), we obtain a phase shift between them, which is (after both waves complete a path \(d\), which is equal to the length of material) equal to

\[
\Delta f = k_x d - k_y d = \frac{\omega}{c_0} (n_+ - n_-) d = \frac{\omega}{c_0} \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}. (17)
\]

Because of the large size of the \(\frac{\omega}{c_0}\) for the visible light (size order of \(10^7\) per meter), the phase shift is not negligible even though \(n_+ \approx n_-\). Therefore, \(E(d)\) is (we assume that the light entered the medium at \(z = 0\) equal to

\[
E(d) = C(E_x(d) + E_y(d)) = C \frac{E_0}{2} \left( \hat{e}_x (e^{i(\omega_0 n_+ n_2 d - \omega t)} + e^{i(\omega_0 n_- n_2 d - \omega t)}) + \hat{e}_y (e^{i(\omega_0 n_+ n_2 d - \omega t)} + e^{i(\omega_0 n_- n_2 d - \omega t)}) \right) =
\]

\[
C \frac{E_0}{2} e^{-i\alpha t} e^{i(\omega_0 n_+ n_2 d)} \left( \hat{e}_x (e^{i(\omega_0 n_+ n_2 d)} + e^{i(\omega_0 n_- n_2 d)}) + \hat{e}_y (e^{i(\omega_0 n_+ n_2 d)} + e^{i(\omega_0 n_- n_2 d)}) \right) =
\]

\[
C \frac{E_0}{2} e^{-i\alpha t} e^{i(\omega_0 n_+ n_2 d)} \left( \hat{e}_x 2 \cos(\frac{\Delta f}{2}) + \hat{e}_y 2 \sin(\Delta f/2) \right) (18)
\]

At \(z = d\), some of the light exits the medium, as can be seen from (5), and propagates again as a linearly polarized light:

\[
E_x(z > d) = \frac{2 n_+ + n_-}{n_1 + n_2} E(d) e^{i(kd)} =
\]

\[
2 \frac{n_+ + n_-}{n_1 + n_2} \frac{E_0}{c_0} e^{i(\omega_0 n_+ n_2 d)} \left( \hat{e}_x 2 \cos(\Delta f/2) + \hat{e}_y 2 \sin(\Delta f/2) \right) e^{i(kd - \omega t)}. (19)
\]

Here, we again made the approximation that the transmission rates of the right and the left circularly polarized waves in the medium are equal since \(n_+ \approx n_- \approx \frac{n_1 + n_2}{2}\). From equation (19), we can see that light now contains a component in the y-direction and we have therefore obtained a shift in the angle of polarization, which is equal to

\[
\tan(\beta) = \frac{E_y}{E_x} = \frac{2 \sin(\Delta f/2)}{2 \cos(\Delta f/2)} = \tan(\Delta f/2) (20)
\]

\[
\beta = \frac{\Delta f}{2} = \frac{\omega}{2c_0} (n_+ - n_-) d = \frac{\omega}{2c_0 \sqrt{\varepsilon_1}} d. (21)
\]

If \(g_z\) is proportional to the applied magnetic field, then the same holds true for the angle of rotation \(\beta\), and this is in accordance with the equation (14). In the derivation, we neglected the electromagnetic waves that repeatedly reflect from the inner surface of the medium before exiting it.

The Faraday effect is a nonreciprocal effect, since if a ray of light is passed through material and reflected back through it, the rotation of polarization doesn’t go back to zero but doubles because the size of the magnetic field in the direction of the
wave vector of the light changes sign in the equation (14). This is useful in the construction of nonreciprocal devices, as can be seen in chapter 4.2 [4].

In cases where \( g_s \) also contains an imaginary component, we obtain (in addition to absorption of light) a phase shift between \( x \) and \( y \) component of the electric field and the light becomes elliptically polarized. This is named magnetic circular dichroism\(^5\) (MCD), since we have different absorption rates for the right and the left circularly polarized light [4].

### 3.2 Kerr effect

The magneto-optic Kerr effect (MOKE) manifests itself as a change of intensity and polarization for linearly polarized light when it is reflected from the magnetized surface. This effect is usually linear in magnetization (or in the applied magnetic field if we are observing magnetically unordered material), which means that the size of the mentioned effects is proportional to the magnetization.

We have three different types of MOKE: The polar effect, where the magnetization is perpendicular to the surface, the longitudinal effect, where the magnetization is parallel to the surface and the parallel component of the wave vector of the light, and the transversal effect, where the magnetization is parallel to the surface but perpendicular to the wave vector of the light [7]. Arbitrary magnetization can be written as a sum of mentioned magnetizations. For the polar and longitudinal Kerr effect, we can obtain a change in the angle of polarization for reflected light and appearance of elliptical polarization, while for the transversal Kerr effect, which only occurs in materials that absorb light, we obtain changes in the intensity of reflected light.

![Figure 3: Three types of the magneto-optic Kerr effect. Taken from [7].](image)

Let us derive the polar Kerr effect for the case in which the light falls perpendicularly on the surface of the material with the relative permittivity tensor from (6) and therefore refractive indexes from (10) to show the main ideas behind the calculation of the Kerr effect [4]. Let the incident light be oriented along the \( z \)-axis of the coordinate system and be polarized along the \( x \)-axis of the coordinate system. As we know, some of the light is reflected from the surface. The incident and the reflected light are decomposed into the right and left circular polarization since these polarizations interact differently with the surface of the material because they have different refractive indexes \((n_\pm)\) inside the material, as we know from equations (10) and (11). Therefore, the electric field of the incident and reflected light can be written as

\[
E_i = E_{i0} e^{i(kz - \omega t)} = \frac{E_{i0}}{2} e^{i(kz - \omega t)} \left( \hat{e}_x + \hat{e}_y e^{-i\frac{\pi}{2}} \right) + \frac{E_{i0}}{2} e^{i(kz - \omega t)} \left( \hat{e}_x + \hat{e}_y e^{i\frac{\pi}{2}} \right),
\]

\[
E_r = \frac{E_{i0}}{2} e^{-i(kz - \omega t)} \left( \hat{e}_x + \hat{e}_y e^{i\frac{\pi}{2}} \right) + \frac{E_{i0}}{2} e^{i(kz - \omega t)} \left( \hat{e}_x + \hat{e}_y e^{-i\frac{\pi}{2}} \right) = \frac{\hat{e}_x e^{i(-kz - \omega t)}}{2} (E_{0r+} + E_{0r-}) + \frac{\hat{e}_y e^{i(-kz - \omega t)}}{2} (e^{-i\pi/2} E_{0r+} + e^{i\pi/2} E_{0r-}) \approx \frac{e^{i(-kz - \omega t)}}{2} \left( \frac{n_+ - n_-}{n_+ + n_-} \right) \hat{e}_x + e^{i\pi/2} \frac{\delta_0}{\sqrt{\varepsilon_1(n_1 + \sqrt{\varepsilon_1})}} \hat{e}_y.
\]

Here, \( k \) is equal to \((\omega/c_0)\), and \( E_{0r\pm} \) is equal to \( E_{i0} \frac{n_\pm - n_0}{n_1 + n_\pm} \), as can be obtained from the equation (5), where \( n_1 \) is the refractive index outside the medium. As we can see, we obtained a polarization in the \( y \)-direction for the reflected light since \( E_{0r+} \) is different from \( E_{0r-} \). The size of the polar Kerr effect is usually proportional to the magnetization (or to the applied magnetic field for the magnetically unordered materials), since \( \delta_0 \) is usually proportional to the magnetization (or to the applied magnetic field for the magnetically unordered materials), as we know from the equation (9).

The magneto-optic effect is usually relatively weak (rotations of polarization at around 0.1 degrees are typical [4]), but still strong enough for utilization in different types of devices.

\(^5\) Dichroism is a difference in absorption rates for differently polarized lights.
### 3.3 Other effects

Another effect that we can obtain for reflected light from some of the magnetized media is the orientational magneto-optic effect (OME) [4]. When the orientation of magnetization changes from transversal to longitudinal, intensity of reflected light variates quadratically. This effect can be described with the formula

$$\delta_{OME} = \frac{I_1 - I_2}{I_0},$$

where $I_1$ and $I_2$ are the intensities of reflected light for the transverse and longitudinal magnetization and $I_0$ is the intensity of reflected light from the unmagnetized surface. Typical values of $\delta_{OME}$ are at around $0.0005 - 0.001$. This variation of intensity is produced by those components of the permittivity tensor that contain quadratic dependence on the magnetization; therefore it cannot be seen in all materials since not all of them contain such components.

For transmitted light, we can obtain double refraction in an initially isotropic medium. If the light is very strong (like in laser), we obtain nonlinear magneto-optic effects. Most important of these is the nonlinear Kerr effect, where we obtain the reflected light with doubled frequency compared to the incident light. This can be useful in surface probing of magnetized materials. There also exist magneto-optic effects for the light in the x-ray region. For example, magnetic X-ray circular dichroism (MXCD) that manifests itself as a difference of absorption for the right and the left circularly polarized light can be useful in measuring of the magnetic properties of atoms, such as their spin and orbital magnetic moment [4].

### 4 Applications

Magneto-optic effects can be used in many different ways. Typically, these effects are used in devices that measure magnetic field, in devices that measure magnetization and in wave modulators [4,6]. Historically, magneto-optic effects were also used for data storage on computer disks and tapes, but they are currently rarely used. However, recent progress shows us that there exists potential for the renewed use of magneto-optic effects in memory devices [8].

#### 4.1 Applications in measurement devices

Let us start with the magneto-optic sensor of magnetic field [4]. Magneto-optic magnetic field sensors are based on the Faraday effect. Their advantages are low weight, small size and long-distance signal transmission for remote operation. In those types of sensors, we measure the rotation of polarization, which is the consequence of the Faraday effect, of the linearly polarized light that passed through our magneto-optic medium. The Faraday effect is induced by the magnetic field that we are measuring. Larger the magnetic field, larger the Faraday rotation and this can be used for the measurement of magnetic field. Usually, materials with strong Faraday rotations are used to obtain high sensitivity (up to around 0.1mT).

Magneto-optic effects can also be used for the measurement of the magnetization of media. Here, we can use both transmitted and reflected waves. Since the magnetization influences the interaction of the light with a medium, we can deduce its size from the changes that happened to light. Because we usually measure the magnetization of ferromagnets that strongly absorb light, we normally use reflected waves and the magneto-optic Kerr effect for the measurement of magnetization. However, the problem with this approach is that we can only measure the magnetization on the surface.

Let us present one of the simplest devices for the measurement of the magnetization on the surface that is based on the Kerr effect and is shown in figure 4 [6]. We will use s and p polarizations to describe the polarization of light. The s polarization is parallel to the surface of material (and perpendicular to the wave vector), while the p polarization is perpendicular to the s polarization and the wave vector. We have laser light that is p-polarized after it went through the first polarizer\(^6\). When such light is reflected from a non-magnetized surface, it remains p-polarized. Since the sample is magnetized, the reflected light contains a small s component and the dominant p component, with $E_s/E_p = \phi' + i\phi''$, where $\phi'$ is the Kerr rotation which tells us for how much was the p-polarized light rotated when reflected from the surface, while $\phi''$ is the Kerr ellipticity that describes the phase shift between the s and p component of light. Both the Kerr rotation and the Kerr ellipticity are dependent on the magnetization of the surface. We could now directly measure the intensity of the s polarized light (and from it deduce the magnetization of the sample) by placing the second polarizer in such position that would block the p polarization of the reflected light (therefore we would have only the s component of the light on a photodiode\(^7\)), but because it is hard to measure the intensity of the s polarized light on a photodiode because of its small intensity, we use an alternative approach, in which the polarizer between the photodiode and the sample is rotated by a small angle $\delta$ from the p axis and therefore some amount of the p polarized light passes through the polarizer. Consequently, the intensity of the light on the photodiode is

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\(^6\)Polarizer is an optical filter that lets the light of specific polarization pass through while blocking light of other polarizations.

\(^7\)Photodiode is a semiconductor device that measures intensity of light by converting it into an electrical current.
where $I_0$ is equal to $|E_p|^2 \delta^2$, $\delta$ is small, $\phi'$ is small compared to $\delta$ and $\phi''$ is negligible. Since the Kerr rotation $\phi'$ is usually proportional to the magnetization, we obtain a linear dependence between the intensity on the photodiode and the magnetization. This can be used for the measurement of magnetization. To avoid measuring the absolute value of the intensity of light, we measure the relative change of the intensity between the non-magnetized and magnetized surface and use external magnetic field to change magnetization.

Other effects can also be used for the measurement of magnetization. For example, we can use the orientational magneto-optic effect or the nonlinear Kerr effect for the measurement of magnetization from reflected light. From transmitted light, we can measure the magnetic properties of the atoms, such as their spin and orbital magnetic moment, with the Magnetic X-ray circular dichroism, as we already mentioned.

4.2 Wave modulators
Magnetooptic effects can also be used for modulation of intensity or polarization of the light. For example, we can create non-reciprocal devices that suppress reflected light [4]. This is particularly useful in optical fibers, which are used for the transmission of the light between the two ends of the fiber and consequently for the transfer of information. However, reflected light, mostly produced on the contact of the optical fibers and the optical elements, has a negative impact on the performance of the optical fibers. Therefore, we use non-reciprocal devices that can be explained with following scheme (as seen on figure 5): We have two polarizers with their transmission axes rotated for 45 degrees between each other and a magneto-optical Faraday rotator that rotates the polarization of the linearly polarized light for 45 degrees. Component of the light with polarization parallel to the transmission axis of the first polarizer passes such system undisturbed since the polarization of such light coincides with the transmission axis of the second polarizer because of the Faraday rotation. However, the reflected light is not transmitted through such system since its polarization is perpendicular to the transmission axis of the first polarizer because the Faraday rotation doubles to 90 degrees after the light goes through the Faraday rotator for the second time (non-reciprocal effect). In this scheme, we neglected that the Faraday rotation is dependent on the wavelength of light and the temperature of the Faraday rotator. Realistic non-reciprocal devices, while usually still containing magneto-optic elements, are therefore much more complicated, since the temperature and wavelength dispersion needs to be as small as possible, and can be only used within the limited temperature and wavelength ranges.

Magneto-optic effects can also be used in other ways, such as for conversion between different modes in optical fibers, for optical spatial filtering, for optical switches, for bistable optical devices, for optical circulators (all are described in [4]) etc.

If we neglect absorption and reflection of the polarizers and the Faraday rotator.
4.3 Magneto-optic memory devices

Historically, magneto-optic effects were used in memory disks and tapes for computers. Nowadays, they are rarely used (instead we use USB drives), but new advances in the modulation of matter magnetization open up the possibility for the renewed use of magneto-optic memory devices [8].

As we know, the memory for computers is stored in bits that can have two possible values: 0 or 1. In magneto-optic disks (similar procedure is also used for magneto-optic tapes), this is realized in the following way: Magneto-optic disks (which usually look same as the usual computer disks), created from a ferromagnetic material, consists of magnetic domains which can have the magnetization oriented in two possible orientations: up and down. Each magnetic domain represents one bit.

When we project the laser light on such domain, some of the laser light is reflected, and because of the magneto-optic polar Kerr effect, we obtain two possible rotations of polarizations and therefore two possible polarization states for the reflected light. The reading device above the disk that accepts the reflected laser light is then able to differentiate between both polarization states and translate them into the language of the bits (zero and one). After this is done, we move to the next magnetic domain.

![Figure 6: The basic idea behind magneto-optic memory devices. Taken from [9].](image)

Magneto-optic memory devices have become obsolete because of their relatively low storage capabilities (few gigabytes at best since domains were too big) and their slow writing of information on the device, which has been done by applying the external magnetic field on particular domain to change its magnetization (after the domain was heated with laser), which is relatively slow process (at around 100 ps). However, recent progress can be used to eliminate the mentioned disadvantages. Instead of using the external magnetic field to change the magnetization of the domain, we can use sub picosecond laser pulses for it [8]. This can improve the writing data speed on each domain up to around 10/f’s. This optical manipulation may also allow us to control magnetization on smaller dimensions and therefore improve storage capacity because of smaller domains. With that, magneto-optic memory devices may become widely used again.

5 Conclusion

In this seminar, we briefly described magneto-optic effects and their applications. Even if most theoretical aspects of them were described long ago, magneto-optic effects remain an exciting area of research (especially in the applicative sense), with new technologies opening up new possibilities. To conclude, magneto-optic effects are and will remain useful in many different ways.

References


