Abstract

In this seminar I present string breaking, a phenomenon of transition from a quark-antiquark pair to a meson-antimeson pair, which is the underlying mechanism responsible for quark confinement. It is described in the framework of quantum chromodynamics (QCD). Since string break occurs in nonrelativistic regime, where the interaction strength of the strong force is large, nonperturbative approach is required. Therefore, explicit calculations are done with approximate theory of QCD - lattice QCD.
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Introduction

The main purpose of particle physics is to list all of the fundamental particles in Nature and describe the inter-
actions between them on the most fundamental level. The idea of the existence of irreducible building blocks
leads way back in history, but it was only in the beginning of the previous century, that physicists started to get
close to the picture of the world that we have now.

Based on the experiments of Rutherford and Chadwick up to 1932, the nucleus was known to be composed of
protons and neutrons. It introduced the idea of nuclear force, since the electromagnetic and gravitational, which
were the only two known at that time, were insufficient to bind the protons and neutrons into such a small
region. In years that followed a lot of experiments had been done by colliding them together and observing the
products. If the protons and neutrons would be fundamental, one would expect the outcome being the same as
the income, except with possible changes in trajectories. But the result was quite different. The final state was
composed of several copies of the initial particles and/or even new particles that were never observed before.

In 60s the idea of quarks was proposed by Murray Gell-Mann and George Zweig. With only three of them one
could construct patterns which matched the observed set of particles. The problem was that one should propose
very weak interactions between them when being close together, but that it is impossible to isolate them. Despite
physicists trying to find unbound quarks, they would only find them as a part of a barion or a meson, which
are known to be composed from three quarks and a quark-antiquark pair respectively. So from experiments The
Principle of Confinement was proposed.

But more unexpected experimental data was concerned with quarks. In order to explain the observed spectrum
of particles, they ought to have fractional charges of \( \frac{2}{3} \) and \( -\frac{1}{3} \) that of a proton. But not only that, while being
fermions with spin \( \frac{1}{2} \), they supposed to have symmetric wave functions, which was a clear contradiction. Expect-
ing interesting results, a series of experiments was done from 1967 to 1973 on the Stanford Linear Accelerator
Center (SLAC) by scattering highly energetic electrons of protons and neutrons. They discovered that indeed
there are quark-like particles composing nucleons and when they suffer a large momentum transfer, behave as
being free.

Theoretical approach to strong interaction was also developing parallel to experimental observations. Break-
through was made in 1973 by David Gross, Frank Wilczek and independently that same year, David Politzer.
They found a particular theory among many models, which was suited to describe the phenomena of the strong
interaction. It proposed the existence of three kinds of charges, with complete symmetry among them (idea of
three-valued charge arose from experiments even before 1973). The term charge was replaced by the colour
and consequently the theory is called quantum chromodynamics (QCD). (Wilczek, 2004)

First part of this seminar is considering our underlying theory of quantum chromodynamics. I briefly summarize
the concept of local gauge invariance and how it leads to interactions between quarks and gluons. I finish this
part with describing two phenomena, typical for non-Abelian gauge theories such as QCD, that is asimptotic

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freedom and colour confinement.
Second part is concerned with lattice QCD (LQCD), a nonperturbative approach to solving the theory of strong interactions. We discuss the transition from a smooth space-time theory of QCD to discrete lattice QCD. We also discuss the path integral approach, which is used in the latter theory.
The last and the longest part contains the calculation of potential between a quark-antiquark pair and the analysis of string breaking.

1 Quantum Chromodynamics

1.1 Fundamentals
Quantum chromodynamics is a field theory describing quarks, gluons and interactions between them. As such it is an indispensable part of the Standard Model and vital for understanding the world on the smallest scales we know of.
To obtain the dynamics of a strongly interacting system, we proceed by means of local gauge invariance. Since the strong interaction does not change the quark type (that is u, d, ...), we can describe only one type without the loss of generality. The starting point is the Lagrangian, which for a free quark field can be written as

\[ \mathcal{L}_f = \overline{\psi} (i \gamma^\mu \partial_\mu - m) \psi, \]  

where \( \psi \) is a bispinor, \( \gamma^\mu \) are so-called gamma matrices satisfying anticommutation relation \( \{ \gamma^\mu, \gamma^\nu \} = 2g^{\mu\nu} \) and \( m \) is the mass of a quark. We should note that a complete quark field \( \psi \) determines spatial wave function, bispinorial state and also its colour state. Therefore we can write it with explicit indices as \( \psi_{\alpha c}(x) \) (\( \alpha \) being Dirac and \( c \) being colour index). The latter is determined with a unit vector in a complex three-dimensional space, colour space, with the basis of red, green and blue state. We immediately see that (1) is invariant under global transformations \( U \in SU(3) \) (space-time independent), that act on a colour state of a quark:

\[ \psi \rightarrow U \psi, \quad \overline{\psi} \rightarrow \overline{\psi} U^+, \]

since the identity \( U^+ U = 1 \) holds. Colour charges are postulated to be completely symmetrical amongst themselves, therefore we require our physical Lagrangian to be invariant under local transformations \( U(x) \in SU(3) \) (space-time dependent). While the mass term suffices this property, the derivative term does not. We introduce the covariant derivative \( D_\mu = \partial_\mu + igA_\mu \), where \( g \) represents strong coupling constant and \( A_\mu \) gluon field or gluon potential matrix with the following transformation property under local transformations \( U(x) \):

\[ A_\mu \rightarrow U(A_\mu + \frac{i}{g} \partial_\mu)U^+. \]  

New Lagrangian of the form

\[ \mathcal{L}_q = \overline{\psi} (i \gamma^\mu D_\mu - m) \psi \]  

is now invariant under local transformations

\[ \psi \rightarrow U(x) \psi, \quad \overline{\psi} \rightarrow \overline{\psi} U^+(x) \]

and simultaneous so called local gauge transformations (2). As expected local gauge invariance introduced interaction term between quarks and gluons through new gauge potentials,

\[ \mathcal{L}_{\text{int}} = -g \overline{\psi} \gamma^\mu A_\mu \psi. \]  

Since the gluons are physical objects which carry energy and momentum, we have to write down additional term(s) to complete QCD Lagrangian in order to describe dynamics of their own. (Introduction to QCD and the Standard Model, 2009) In analogy with the electromagnetic field we form (gauge invariant) antisymmetric field strength tensor

\[ F^{\mu\nu} = -\frac{i}{g} [D^\mu, D^\nu] = \partial^\mu A^\nu - \partial^\nu A^\mu - g^2 [A^\mu, A^\nu]. \]  

2
The purely gluonic term is then
\[ \mathcal{L}_g = -\frac{1}{4} \text{Tr}(F^\mu_\nu F^\nu_\mu). \] (6)

New features arise from the fact of noncommutativity of \( A^\mu \) and \( A^\nu \), which are not familiar from the theory of quantum electrodynamics (QED), that is gluon self-interactions. Since in the interaction term the gluon fields appear in products of three and four, possible processes are those schematically represented with Feynman diagrams in the picture below (Figure 1). The very different nature of strong force leads to phenomena (among others) called asymptotic freedom and colour confinement, which we briefly discuss.

![Figure 1: Two Feynman diagrams representing possible purely gluonic interactions.](image)

### 1.2 New Features

Asymptotic freedom was first explained in 1973, when Gross, Wilczek and Politzer showed on theoretical grounds of QCD that the interaction strength \( \alpha_s = \frac{g^2}{4\pi} \) between quarks weakens as the quarks approach one another. This is quite the contrary to QED where we know that due to the polarization of vacuum, schematically shown on the diagram below, the coupling constant \( \alpha_c = \frac{e^2}{4\pi} \) becomes much larger at small distances. The main difference between the two behaviours of the coupling constants is due to the second diagram (Figure 2, right) which contributes only in QCD, while the first one (Figure 2, left) contributes to both. This phenomenon introduces an intrinsic QCD energy scale \( \Lambda_{QCD} \sim 250 \text{ MeV} \), which determines the energy at which the coupling strength \( \alpha_s \) becomes large (Introduction to QCD and the Standard Model, 2009). This is precisely the fact that makes the theory of QCD highly challenging for theoretical predictions, since for the processes that take place at low energies, the coupling strength approaches or exceeds 1 and the theory becomes nonperturbative.

![Figure 2: First diagram causes screening effect, which we already know from theory of QED. The second diagram is a special feature of QCD, which is possible due to the three-gluon interaction, and leads to antiscreening effect.](image)

While describing processes above the \( \Lambda_{QCD} \) in the last paragraph, we now take a look on the other side at low-energy domain, where \( \alpha_s \) becomes large and colour confinement occurs. A rough statement is that any strongly interacting system must be a colour singlet at a distance scale larger than \( 1/\Lambda_{QCD} \). (Introduction to QCD and the Standard Model) By colour singlet we mean that colours of individual constituents are coupled to zero. It immediately follows that isolated quarks cannot exist in nature, which is sometimes also called quark confinement. A hand-waving explanation of this phenomenon goes in the following way. As a quark-antiquark pair in a colour singlet gets separated, the interaction between them gets stronger as the distance between them
gets larger. When at a certain critical distance there is enough energy accumulated in the gluon field, a new quark-antiquark pair arises from the vacuum and two mesons form, which are again - colour singlets. The so called string breaking occurred. As pointed out the above discussion was not in any way precise. To really describe the phenomenon of string breaking, we must invoke nonperturbative calculational methods as the theory itself is nonperturbative in this energy domain. The only one we know today is lattice quantum chromodynamics (LQCD), the topic of the following chapter.

2 Lattice QCD

2.1 Construction

Formulation of lattice QCD starts with replacing space-time with a finite four dimensional Euclidean lattice $\Lambda$:

$$\Lambda = \{ n = (n_1, n_2, n_3, n_4) \mid n_i = 0, 1, \ldots N - 1, \; i = 1, 2, 3; \; n_4 = 0, 1, \ldots N_T - 1 \},$$

where arrays $n$ represent points of discrete space-time separated by a lattice constant $a$. Introduction of such a lattice into the theory serves two purposes. First, the discrete space-time acts as a regulator, since the finite value of lattice constant provides an ultraviolet cutoff at $\pi/a$. The second purpose is that LQCD can be simulated on a computer using methods similar to those used in statistical mechanics, as we will show later, that allow for calculations of various quantities in hadronic systems. (Gupta, 2008)

By turning from continuous to discrete, we must also construct our theory in a compatible way. First, we have to limit our quark fields to have values only at the lattice points. Quark degrees of freedom are then finite and are

$$\psi(n), \; \bar{\psi}(n) \; n \in \Lambda,$$

where Dirac and colour indices are suppressed. For reasons that will be apparent later we make a transformation of time coordinate $it_M = t$, so that with respect to $t$ metric of space-time becomes Euclidean. $t_M$ is just the ordinary Minkowski time. With this novelty the free quark part of the QCD Lagrangian becomes a discrete version of (1), written as (Gattringer and Lang, 2010)

$$L_q = \bar{\psi}(n) \left( \sum_{\mu=1}^{4} \gamma_\mu \frac{\psi(n + \hat{\mu}) - \psi(n - \hat{\mu})}{2a} + m\psi(n) \right),$$

where we approximated the derivative term with the central difference. To recover gauge fields, we again require invariance of the Lagrangian under local rotation of colour indices of the quark fields. This time though we cannot have a smooth $SU(3)$ matrix field, we can attach an arbitrary transformation $U(n) \in SU(3)$ to each lattice point. In other words we impose the following transformations on quark fields,

$$\psi(n) \rightarrow U(n)\psi, \; \bar{\psi}(n) \rightarrow \bar{\psi}(n)U^+(n).$$

We note that to the first order in $a$, $U_\mu$ and $A_\mu$ are connected via the following relation (Gattringer and Lang, 2010):

$$U_\mu(n) = \exp(iA_\mu(n)) = 1 + iaA_\mu(n).$$

Similarly to original derivation of gauge fields, the mass term suffices the above requirement, while the derivative term, where products of the form $\bar{\psi}(n)\psi(n+\hat{\mu})$ appear, does not. If we introduce a field $U_\mu(n)$ with the directional index $\mu$ and with the following property under gauge transformations $U(n)$,

$$U_\mu(n) \rightarrow U(n)U_\mu(n)U^+(n + \hat{\mu}),$$

then terms of the form

$$\bar{\psi}'(n)U_\mu'(n)\psi'(n + \hat{\mu}) = \bar{\psi}(n)U_\mu(n)\psi(n + \hat{\mu})$$

(13)
are obviously gauge invariant. Thus if we want to modify Lagrangian (9) so that it will be invariant under (10), we can write it in the following way (Gattringer and Lang, 2010):

\[
\mathcal{L}_f = \bar{\psi}(n) \left( \sum_{\mu=1}^{4} \gamma_\mu U_\mu(n) \psi(n + \hat{\mu}) - U_{-\mu}(n) \psi(n - \hat{\mu}) \right) \frac{1}{2a} + m\psi(n),
\]  

(14)

where we have introduced \( U_{-\mu}(n) \equiv U^+_{\mu}(n - \hat{\mu}) \) for notational convenience. Our gauge fields are now \( U_\mu \), which are elements of the group \( SU(3) \) and are sometimes referred to as link variables (Figure 3). We note that the naive discretization in the form (14) has the so-called fermion doubling problem, which we will not discuss here. The problem is solved by adding the so-called Wilson term into the Lagrangian (14). (Gattringer and Lang, 2010)

Again to complete the description of the whole system we must add a term which in the continuum limit as \( a \to 0 \) reduces to (6).

\[
\mathcal{L}_g = \frac{2}{g^2} \sum_{\mu < \nu} \text{Re} \ tr[1 - U_{\mu\nu}(n)],
\]

(15)

where \( U_{\mu\nu}(n) \) is the so-called plaquette defined with the product

\[
U_{\mu\nu}(n) \equiv U_\mu(n)U_\nu(n + \hat{\mu})U_{-\mu}(n + \hat{\mu} + \hat{\nu})U_{-\nu}(n + \hat{\nu}),
\]

which can be visualised as shown in the picture (Figure 3) (Gattringer and Lang, 2010).

### 2.2 Euclidean Correlator and Path Integral

To proceed we turn our attention to one of the main objects, that allows us to interpret lattice field theory, the Euclidean correlator, which we express in two different ways. It is defined as (Gattringer and Lang, 2010)

\[
\langle O_2(t)O_1(0) \rangle_T = \frac{1}{Z_T} \text{tr} \left[ e^{-(T-t)\hat{H}} \hat{O}_2 e^{-t\hat{H}} \hat{O}_1 \right],
\]

(16)

where the normalization factor \( Z_T \) is

\[
Z_T = \text{tr} \left[ e^{-T\hat{H}} \right].
\]

(17)

Taking the limit \( T \to \infty \), we obtain (assuming that the vacuum is unique)

\[
\lim_{T \to \infty} \langle O_2(t)O_1(0) \rangle_T = \sum_n \langle 0 | O_2 | n \rangle \langle n | O_1 | 0 \rangle e^{-tE_n}.
\]

(18)

The above equation can be used, for example, in the following way: Let \( O_1 \) be an operator that creates from the vacuum a state with the quantum numbers of a pion and \( O_2 \) that annihilates the latter. Only nonzero matrix elements in (18) are those where the states \( | n \rangle \) match the pion’s quantum numbers. Since the pion has excited states, there will be several such contributions. But assuming that one can evaluate (18), and we will show how it is done, the ground state can be extracted from long-time limit due to different exponential damping of the individual terms. (Gattringer and Lang, 2010)

By showing the usefulness of the above expression we briefly sketch how to actually calculate it.

Our starting point is the path integral, which was introduced by Feynman. To partially justify the upcoming formula, we discuss this idea in the framework of usual nonrelativistic quantum mechanics.

Consider a general QM system, described with an arbitrary set of coordinates \( q^i \), conjugate momenta \( p^i \) and...
Hamiltonian $H(q,p)$. A simple transition amplitude of a particle from point $q_a$ at time 0 to point $q_b$ at time $T$ is given by

$$U(q_a, q_b; T) = \langle q_a | e^{-iT\hat{H}} | q_b \rangle. \quad (19)$$

It can be shown that for the usual Hamiltonian (19) can be rewritten as

$$U(q_a, q_b; T) = \int Dq(t) e^{iS[q(t)]}, \quad (20)$$

where $S = \int dt L = \int d^4x L$ is of course the action (Peskin and Schroeder, 1995). Equation (20) directly resembles superposition principle of QM, since we are adding up contributions from all the possible paths, constrained only by the boundary points. Important thing to note is that while the starting expression for transition amplitude contains operators, the right-hand side of (20) contains only numbers.

Similarly, we can write an analog of path integral (20) for Euclidean correlator in LQCD (Gattringer and Lang, 2010),

$$\langle O_2(t)|O_1 \rangle = \frac{1}{Z} \int D[\psi, \overline{\psi}] \ D[U] \ e^{-S[\psi, \overline{\psi}, U]} O_2[\psi, \overline{\psi}, U] O_1[\psi, \overline{\psi}, U] \quad (21)$$

where $Z$ is

$$Z = \int D[\psi, \overline{\psi}] \ D[U] \ e^{-S[\psi, \overline{\psi}, U]}. \quad (22)$$

Taking into account that degrees of freedom are now $\psi$, $\overline{\psi}$ and $U$ field, the structure of (21) is precisely that of (20). The only difference is in the minus sign in the exponential of (21), which appeared due to the time transformation made above. Since we are on the lattice, the integration measure of (21) and (22) is (Gattringer and Lang, 2010)

$$D[\psi, \overline{\psi}] = \prod_{n \in \Lambda} \prod_{a,c} d\psi_{ac}(n) d\overline{\psi}_{ac}(n), \quad D[U] = \prod_{n \in \Lambda} \prod_{\mu=1}^4 dU_\mu(n). \quad (23)$$

The number of degrees of freedom on a finite lattice is finite, which allows for implementation of calculations on a computer.

### 3 Confinement and Potential between quark-antiquark Pair

With a review of QCD and in particular lattice QCD in the last two sections, we now describe string breaking. First we show the application to spatial dependence of potential between static quark-antiquark pair, the initial state of string breaking process.

We start with a gauge invariant object called *Wilson loop*, which will allow for the extraction of the potential. It is defined as

$$W_C[U] = \text{tr} \left( \prod_{(n,\mu) \in C} U_\mu(n) \right), \quad (24)$$

where $C$ denotes a closed loop in (discrete) space-time, made out of four straight lines, where two are so called *Wilson lines* $S(n, m, n_t)$, which connect two different space-time points $n$ and $m$ at the same time slice $n_t$, and the other two are *temporal transporters* $T(n, n_t)$, which connect two different space-times points at the same spatial position. If we, in addition, fix a temporal gauge where $U_4(n) = 1$ for every $n$, then (24) can be written as

$$W_C[U] = \text{tr} [S(m, n, n_t) S(m, n, 0)^+] \quad (25)$$

Writing down Euclidean correlator of (25) we get, according to (18)

$$\langle W_C \rangle = \sum_k \langle 0 | S(m, n)_{ab} | k \rangle \langle k | S(m, n)_{ba}^+ | 0 \rangle \ e^{-tE_k} \approx Ae^{-V(r)t} \text{ at large } t. \quad (26)$$
We argue, based on the same transformation properties under gauge group as $\psi(m)\bar{\psi}(n)$, that operator $S^+(m,n)$ creates a static quark-antiquark pair located at spatial positions $m$ and $n$ and similarly $S$ annihilates such a state. Therefore the lowest lying state should be the one describing static quark-antiquark pair, where excited states could contain for example another such a pair. The ground energy is thus identified with the static potential $V(r)$ at spatial quark-antiquark separation $r = a|m - n|$.

First we investigate the strong coupling limit - large $g$. The vacuum expectation value is

$$\langle W_C \rangle = \frac{1}{Z} \int D[U] \exp \left( -\frac{2}{g^2} \sum_P \text{Re} \text{tr}[1 - U_P] \right) \text{tr} \left[ \prod_{l \in \mathcal{C}} U_l \right],$$

where $P$ denotes plaquettes, which are summed only over one orientation, and $C$ determines a Wilson loop. Using group integration measure it can be shown that to lowest order the following relation holds (Gattringer and Lang, 2010):

$$\langle W_C \rangle = 3 \exp \left( n_r n_t \ln \left( \frac{3}{g^2} \right) \right) \left( 1 + \mathcal{O}(\frac{1}{g^2}) \right).$$

On the other hand, the asymptotic form is

$$\langle W_C \rangle \approx \exp(-a n r V(r)).$$

From (28) and (29) we conclude that potential at large distances is of the linear form (see Figure 4)

$$V(r) = \alpha r,$$

where string tension $\alpha$ is

$$\alpha = -\frac{1}{a^2} \ln \left( \frac{3}{g^2} \right) \left( 1 + \mathcal{O}(\frac{1}{g^2}) \right).$$

Potential at short ranges where $g$ is small, is precisely of the Coulomb form (Figure 4), which can be seen from (5) where QCD effectively reduces to Abelian gauge theory, that is, has the same structure as quantum electrodynamics (Gattringer and Lang, 2010).

4 String Breaking

In the previous section we have introduced a field line called string, representing colour field between two static colour sources. String breaks, when the distance between a quark and antiquark reaches some critical value $r_c$ and a transition to a meson-antimeson pair occurs, in particular heavy-light meson-antimeson pair. We briefly discuss the procedure and results of a calculation, describing the above process.

The initial state is a quark-antiquark pair and at small separations $r$ between the two it is also a physical eigenstate of a system. On the other hand, the final state is a meson-antimeson pair, which is at large separations $r$ between the two also a physical eigenstate of a system. Only in the vicinity of the location of QCD string breaking the initial and final state interfere to form the physical eigenstate. The latter is referred to as mixing. Denoting our initial and final state with $|Q\rangle$ and $|B\rangle$ respectively and the two physical states with $|1\rangle$ and $|2\rangle$, we can write down mixing in the following way (Bali, et. al., 2005a):

$$|1\rangle = \cos(\theta) |Q\rangle + \sin(\theta) |B\rangle,$$

$$|2\rangle = -\sin(\theta) |Q\rangle + \cos(\theta) |B\rangle.$$
where we have parametrized unitary transformation between the two sets of states with a mixing angle $\theta$, which depends on the distance $r$ between the static quark-antiquark (meson-antimeson) pair. From our previous discussion we already expect it to take a value of $\theta = 0$ at small separations $r$, where the main contribution goes to $|Q\rangle$, and $\theta = \frac{\pi}{2}$ at large separations $r$, where the main contribution goes to $|B\rangle$, when considering for example $|1\rangle$. If the energy of the first physical eigenstate is $E_1(r)$ and of the second is $E_2(r)$, then we identify string breaking at a distance $r_c$, where the difference

$$\Delta E(r) = E_2(r) - E_1(r)$$

is minimal. (Bali, et. al., 2005a) To obtain the energies and the mixing angle dependence, we write down the main object of study, the correlation matrix,

$$C(t) = \begin{pmatrix} C_{QQ}(t) & C_{QB}(t) \\ C_{BQ}(t) & C_{BB}(t) \end{pmatrix}$$

(35)

where $C_{AB}(t)$ is the Euclidean correlator for operators $A$ and $B$. We note that the operator $Q$ creates a static quark-antiquark pair and $B$ a heavy-light meson-antimeson pair. Inverting the above transformation (32) and (33), we get

$$|Q\rangle = \cos(\theta) |1\rangle - \sin(\theta) |2\rangle,$$

(36)

$$|B\rangle = \sin(\theta) |1\rangle + \cos(\theta) |2\rangle.$$  

(37)

With that in hand, we can write down matrix elements simply as

$$C_{QQ}(t) = \cos^2(\theta) e^{-E_1 t} + \sin^2(\theta) e^{-E_2 t},$$

(38)

$$C_{BB}(t) = \sin^2(\theta) e^{-E_1 t} + \cos^2(\theta) e^{-E_2 t} \quad \text{and}$$

(39)

$$C_{QB}(t) = \sin(\theta) \cos(\theta) [e^{-E_1 t} - e^{-E_2 t}].$$

(40)

On the other hand we can evaluate coefficients as described in section 3.3 considering path integral. Fitting the results to the above set of equations (38)-(40) we can extract wanted quantities - $E_1(r)$, $E_2(r)$ and $\theta$, where the first two are graphically shown in the picture below for some particular calculation. In this case we can see, that string breaking occurs around $r_c \approx 15a \approx 1.25 \text{ fm}$. (Bali, et. al., 2005b)

![Figure 5: Diagram is representing numerical results for energies of physical eigenstates $|1\rangle$ and $|2\rangle$. Energies are normalized so that the mass of two static (heavy) quarks is zero. We can see that the difference of the two energies is minimized around $\frac{r}{a} \approx 15$. That is where string breaking is identified. (Bali, et. al., 2005b)](image_url)
5 Conclusion

We have discussed the theory of quantum chromodynamics and its peculiarities. The phenomenon of anti-screening makes it highly challenging to make predictions, since the usual perturbative approach fails for strong processes that occur below some energy parameter. The only (approximate) approach to solving equations in the latter energy domain known today is lattice quantum chromodynamics. It allowed for possibility to calculate the observed mass spectrum of hadrons, which agreed with experiments to very high accuracy. We, here, have touched upon a different problem, that of quark confinement, and provided a mechanism responsible for it - string breaking. We showed the results in the last part, where it is seen that a typical critical distance is around 1 \text{fm}.

References


