Abstract

Light interacts differently with different materials, depending on the properties of both light and the material. According to which interactions occur in the material, we can roughly differentiate the materials into three groups: transparent, translucent and opaque. Light cannot penetrate the surface of opaque materials, whilst transparent and translucent materials allow the light to pass through.
1 Introduction

Vision is one of the most important senses. Human eyes can detect light in the visible part of the spectrum, and are able to distinguish between different wavelengths to a certain accuracy. Different wavelengths in general correspond to different colors, as seen in Figure 1.

![Figure 1: Different colors corresponding to different wavelengths. (source: wikipedia)](image)

In order for the human eye to see an object, light has to come from the light source, interact with an object and travel to the human eye. Detected colors depend on both the emitted light and the way that light interacts with the given object.

Some materials are proving to cause more difficulties at analyzing texture and form of an object, for instance human skin. Animation studios are investing a lot of time and effort into researching the optical properties of the skin in order to reproduce as life-like characters as possible, as our eyes are very good at discriminating different interactions of the material with light.

In this seminar we will take a short look at different ways light can interact with a material once it penetrates the surface and how that interaction can lead to our eyes detecting different colors.
2 Microscopic Interaction of Material and Light

Optical properties of the material depend on the interaction of the material with light, electromagnetic radiation in visible part of the spectrum. These interactions can lead to various phenomena and are a result of photons interacting with electrons in the material. The electromagnetic wave causes the electrons in the material to move, they vibrate and radiate at the same frequency as the incident electromagnetic field, reducing the phase velocity. The response of the material to the external fields generally depends on the frequency of the field and is proportional to electric susceptibility. The factor by which the speed is reduced is called refractive index $n$ and is connected to the dielectric constant $\varepsilon$ as

$$n = \sqrt{\varepsilon}. \quad (1)$$

As the response of the material arises only after applied field, dielectric constants are treated as complex functions of the angular frequency of light $\omega$. Electrons in different chemical bonds resonate at different frequencies $\omega_0$, which causes different, unique dielectric constants for different materials. In the simplest terms, a typical mode of resonance can be described as damped harmonic motion of electron’s position $x$, driven by the external field $E$,

$$\ddot{x} + 2\beta \dot{x} + \omega_0^2 x = eE/m_e. \quad (2)$$

where $m_e$ represents the mass of the oscillating electron, $e$ the electron charge and $\beta$ the damping coefficient. The electron acts as an oscillating dipole, which gives a resonant contribution

$$\varepsilon \propto \left(\left(|\omega_0|^2 - \omega^2\right) - 2\beta\omega i\right)^{-1} \quad (3)$$

to the dielectric function. The entire response is a superposition of numerous resonances that correspond to bond rotations, vibrations, atomic orbital resonances and other phenomena, giving rise to a unique response for each material.

The refractive index is as well complex and can be written as follows:

$$n = n_0 + i\kappa, \quad (4)$$

where $n_0$ is the refractive index, indicating phase velocity and is connected to scattering, and $\kappa$ is related to absorption in the material.

3 Absorption

The most trivial way light interacts with light is simply to get absorbed by it. Absorption of light is an optical phenomenon in which the photon is absorbed by the matter and its energy transformed into internal energy.

Assume light only travels in direction $x$ and the material is homogeneous. Radiant flux of light close to the light source equals $j_0$ and radiant flux of light at the distance $x$ from the source is $j(x)$. We assume that change in radiant flux in thin layer is proportional to layer thickness $dx$ and incident light flux $j(x)$. From our assumptions we can derive the Beer-Lambert’s law

$$\frac{dj(x)}{dx} = -\mu j(x)$$

$$j(x) = j_0 e^{-\mu(x)x} \quad (5)$$
Attenuation coefficient $\mu$ with unit m$^{-1}$ is in general a function of wavelength $\mu(\lambda)$ and is proportional to $\kappa$ (Equation 5). When the absorption is constant through all the spectrum, the material only dims the light and is perceived as gray.

The phenomenon in which only a part of spectrum is absorbed is called selective absorption. [1] If the selective absorption takes place in the visible spectrum, the material appears colored, depending on which part of the spectrum is transmitted. When color is only a consequence of light absorption of the material and the light source emits white light, then the color perceived is complementary to the color absorbed, as seen in Table 1.

<table>
<thead>
<tr>
<th>Spectral range of absorption [nm]</th>
<th>Light color of the spectral range</th>
<th>Perceived color of the material</th>
</tr>
</thead>
<tbody>
<tr>
<td>380-440</td>
<td>Violet</td>
<td>Yellow-green</td>
</tr>
<tr>
<td>440-490</td>
<td>Blue</td>
<td>Yellow (Orange)</td>
</tr>
<tr>
<td>490-560</td>
<td>Green</td>
<td>Red</td>
</tr>
<tr>
<td>560-580</td>
<td>Yellow-green</td>
<td>Violet</td>
</tr>
<tr>
<td>580-605</td>
<td>Yellow (Orange)</td>
<td>Blue</td>
</tr>
<tr>
<td>605-780</td>
<td>Red</td>
<td>Green</td>
</tr>
</tbody>
</table>

*Table 1: Absorbed and perceived colors for transparent objects on white background.* [2]

Materials that mainly transmit light are called transparent. Colored transparent materials are characterized by wavelength-dependent absorption coefficient [2]. Examples of these materials are most colored liquids, plant leaves, wine etc. The color of transparent materials is only perceived indirectly by how they filter light from objects and light sources behind them. Against a black background, no color can be detected.

Materials that do not transmit light, are called opaque, like metal, plastic and clay. Those materials reflect light at point of illumination and light ideally does not penetrate the surface. Many materials in our everyday life, however, can be classified as translucent, and this will be the topic of a later chapter.

### 3.1 Dichroism

Some colored liquids, however, can exhibit peculiar properties when observed as thick or thin layers. One of those liquids is pumpkin seed oil. Pumpkin seed oil looks reddish when observed in thick layers (inside a container) and greener when in thin layers (on the salad) (Figure 2). This optical property of pumpkin seed oil is called dichroism and can be explained by Beer-Lambert law and the property of the eye.

From the Beer-Lambert's law (Equation 5), we can see absorbance for any wavelength is linearly proportional to the thickness of the absorbing medium $x$. As the material gets thicker, less light gets transmitted. However, the amount of transmitted light does not decrease proportionally, but exponentially. Therefore, the differences between transmission of different wavelengths can be greatly distorted (Figure 2c).

As shown in Figure 2c, pumpkin seed oil has greatest absorption from under 400 to around 450, where there is a great dip in absorbance. From there on absorption does not immediately show any greater dips or peaks. As the layer thickens, the shape of the absorbance graph remains the same as per the Beer-Lambert’s law (Equation 5). However, the shape of the graph showing the amount of the transmitted light (Figure 2d) is visibly distorted as the layer thickens. The transmitted flux of light with wavelengths below 630 nm reduces greatly, while intensity at wavelengths above 630 nm remains high (Figure 2d). The green thin layer of liquid therefore turns red [3].
4 Absorption and Scattering

Light can be attenuated via absorption, scattering or reflection. In materials like milk, wax and human skin, light is not only absorbed, but also scattered, therefore these materials cannot be described as transparent, rather as translucent. Humans are fairly good at differentiating between transparent, translucent and opaque materials, and also between various levels of translucency. Therefore research of translucent materials has become of great interest in the last few decades, especially in the fields of computer graphics, film animation, and medicine.

4.1 Scattering

Scattering can be defined as the redirection of radiation from the original direction of the incident radiation. Light scatters due to interaction of incident photons with charged particles in the material, where light scatters in various directions.

Depending on the size of the scattering particle, we can observe three special cases:

- **Rayleigh scattering**: the incident light wavelength is smaller than the diameter of the scattering particle;
- **Mie scattering**: the incident light wavelength is comparable to the diameter of the scattering particle;
- **Reflection and refraction**: where the wavelength of the incident light is much bigger than the scattering particle.
Rayleigh scattering

Rayleigh scattering is the special case of scattering, when a particle is smaller than the wavelength of light. When such a particle is exposed to the electromagnetic waves, every part of it experiences the electromagnetic field $E_0$ simultaneously. The small particle with size smaller than the wavelength gets polarized in the incident radiation and its dipole moment in the oscillating periodic field is given by

$$ p = p_0 e^{-i(k(r-ct))} $$

(7)

where $r$ is the distance between the particle and the detector and $k$ is the wave vector. If we measured the scattered electric field at a large distance $r$ from the dipole, we would get to

$$ E = \frac{1}{c^2 r} \frac{1}{\partial r} s \sin(\gamma) = -E_0 \frac{e^{-i(k(r-ct))}}{r} k^2 \alpha \sin(\gamma), $$

(8)

where $\alpha$ is the polarizability of the particle and $\gamma$ is the angle between the scattered dipole moment and the direction of the observation. Using that $I = \frac{1}{4\pi} \frac{c}{\partial} |E|^2$ and assuming the incident light beam is not polarized, we get

$$ I = I_0 \frac{\alpha \pi \gamma^2}{2 \pi^2} \frac{1}{n^2} n^2 \frac{1}{n^2+1} (1 + \cos^2 \theta), $$

(9)

where $V$ is the particle volume, $n$ is the ratio between the refraction indices of the particle and the medium, $\theta$ the angle between the incident and the scattered light, and $I_0$ the incident light intensity [5]. It is apparent from the Equation 9 that the amount of light scattered backwards is equal to the amount of light scattered in the direction of the incident light (Figure 3a). Rayleigh scattering is consistent with dipole antenna radiation.

As scattering is inversely proportional to the fourth power of wavelength, we can deduce that smaller wavelengths get scattered more. As a result we get the blue skies during the day and red sunsets in the evening.

Mie scattering

When the wavelength of light is comparable to the particle diameter, the Rayleigh theory is no longer applicable, as the field is no longer uniform over the entire particle volume. Assuming the particles are spherical, each photon only gets scattered once, the refractive index is small (< 2.0) and the diameter of the scattering particle being comparable to the wavelength of the incident light, the scattering in the direction of incident light becomes greater than scattering in the backwards direction (Figure 3b) [6]. Mie scattering does not depend greatly on the wavelength, therefore all wavelengths get scattered at a similar rate, leading to clouds and fog looking white, as water droplets in air are comparably big to the wavelengths of visible light. Similar principle works for latex particles in paint and droplets in emulsion, including milk.

![Figure 3](image)

**Figure 3**: Schematic representation the angular distribution of scattering. (a) Rayleigh scattering for scattering particles smaller than the wavelength of light. Amount of light scattered in the direction of incident ray is similar to the amount of light scattered in the opposite direction. (b) Mie scattering for scattering particles comparable in size to wavelength of incident light. Scattering is greater in the direction of the incident light. [6]

Reflection and refraction

When scattering particles become much larger than the wavelength of incident light, Mie scattering meets the limit of geometrical optics. Depending on the material, incident can reflect and refract at different rates. [6]
4.2 Subsurface Scattering

As an object is illuminated, the rays split with some energy being reflected and other part refracting into the surface. When hitting the surface of an opaque material, the light gets reflected immediately, without penetrating deeper (Figure 4a). Although usually some light gets reflected off the surface even with translucent materials, it also penetrates beneath the surface. Light that enters a translucent material scatters and re-emerges from the surface. This is called subsurface scattering (Figure 4b) and is an example of multiple scattering. Though light can travel through these materials, the silhouette is blurred due to scattering, hence the objects look somewhat softer, as the light spreads beneath the surface and reemerges in a larger region around the point of illumination.

![Figure 4: (a) Surface reflection in opaque materials. (b) Subsurface scattering of translucent materials. [7]](image)

If we look at computer-graphics in older animated films and games, objects looked fairly plastic and not very life-like. Until recent years, computer graphics only accounted for the reflections on the surface (Figure 4a), however, now animation studios are researching the topic of subsurface scattering in order to produce more realistic models (Figure 4b). Human skin, for instance, directly reflects only about 6% of the incident light, the remaining 94% are due to subsurface scattering.

The parameters we need to analyze in order to understand subsurface scattering of a material, are the refractive index of the material, the angular distribution of scattering, and scattering and absorption coefficients. The later two give us the probability that a photon will either get absorbed or scattered when traveling a certain distance inside the material. In most materials that also means they determine the color and the degree of translucency of the material. The higher the degree of translucency, the harder it is to analyze the shape of an object, as most of the lighting comes from subsurface scattering.

As so many everyday objects are somewhat translucent, human eye is very perceptible in detecting different degrees of translucency. Fleming et. al. (August 2004) have published a research in which they tested how human perception of translucency changes with different physical factors, for instance effects of highlights, light-source direction, contrast, color saturation etc. (Figure 5).

Research of translucent materials has produced many different theories and models to describe the phenomenon with different degrees of success and accuracy, one of which is Kubelka-Munk theory, used in determining and classifying different colorants used in the industry.
4.3 Kubelka-Munk Theory

Translucent materials usually get their color from added selectively absorbing colorants. In order to better classify the colors for translucent materials, a theory had to be invented to predict the fraction of light reflected from a surface exposed to illumination from all directions of a hemisphere. Although many theories have been developed in order to cover this problem, the most widespread in the industry is the Kubelka-Munk theory, having been able to provide simple analytical equations that can predict the reflectance of a material with reasonable accuracy.

In order to get to a simple solution, we need to assume the layer has uniform composition and it is extensive enough not to have any losses at the edges. The layer is freely suspended and illuminated from above with diffuse light from above with intensity \( i(x) \) (Figure 6).

Let the layer be infinitely thin with thickness \( dx \). As the light enters this layer, it gets scattered by factor \( s \) and absorbed by factor \( \mu \), therefore it transmits light with intensity lessened by the sum of those two coefficients. The light exiting the layer \( i(x + dx) \) in the direction of the incident light equals to the sum of the transmitted light and the scattered light from the light reflected back from the background, as seen in Equation 10.a. Similarly, we can get to the amount of light exiting the layer in the opposite direction, see Equation 10.b.

\[
\begin{align*}
i(x + dx) &= i(x)(1 - (s + \mu)dx) + j(x + dx)sdx \quad (10.a) \\
j(x) &= j(x + dx)(1 - (s + \mu)dx) + i(x + dx)sdx \quad (10.b)
\end{align*}
\]

By rearranging the equations (10.a) and (10.b), we get the following set of differential equations:

\[
\begin{align*}
di &= -(\mu + s)dx + js \, dx \\
-dj &= -(\mu + s)dx + is \, dx 
\end{align*}
\]

\[ (11) \]
Solving this set of differential equations gives us a very complicated and for industrial purposes impractical solution:

\[
R = \frac{e^{\frac{\mu_s}{2 + \frac{\mu}{E}} S x}}{(e^{\frac{\mu_s}{2 + \frac{\mu}{E}} S x}) - 1}
\]

By assuming the layer is infinitely thick and any background is infinitely far away with reflectance 0, we arrive to the following equation for diffuse reflectance, which is more widely used in the industry:

\[
R_\infty = 1 + \frac{\mu}{S} - \sqrt{\frac{\mu}{S} (2 + \frac{\mu}{S})},
\]

Here \(\mu\) and \(S\) are the absorption and scattering coefficients of the sample at given wavelength, respectively. Reflected light intensity therefore depends on absorption and scattering of the material in question.

If we look at the limit of absorption coefficient being zero, the Kubelka-Munk theory gives us the following equation:

\[
R_{\mu=0} = \frac{S x}{1 + S x}
\]

This shows that if there is no absorption, all light must be scattered until it reappears at the surface again. However, if the scattering coefficient would be set to zero, the limit would be the Beer-Lambert’s Law (Equation 5).

Kubelka-Munk theory is particularly useful in the technology of paints and paper, however neither can be too absorbing in order for the results to be reasonably accurate. For a thinner layer, solution of the same equations tells us how the color of the substrate shows through.

### 4.4 Nanotechnology in Lycurgus Cup

The model of a single electron or a chemical bond oscillating works well most dielectric materials, but in metals, collective motions of conduction electrons play an important role. Surface plasmon resonance (SPR) occurs at the frequency at which conduction electrons oscillate in response to alternating electric field of the incident electromagnetic radiation [9]. For nanometre-sized particles of metal, their size strongly affects the frequency of SPR. This is why bulk gold looks yellow in reflected light, however already in thin layer we can observe blue in transmitted light. When particle size gets reduced even more, to about 3 nm, the color steadily changes through different shades of orange, purple and red.

Lycurgus cup is a Roman glass cage cup, known for the change of color it undergoes when the lighting conditions change. It is an outstanding piece of art and technology as it represents use of nanotechnology in as early as fourth century A.D. The color changing phenomenon is the result of the metal nanoparticles in the glass, namely gold and silver colloids [10], that are contained in less than 1% of the cup. These nanoparticles can both scatter and transmit light, so the empty cup appears green when front lit and red when back lit (Figure 7).

The change in color with change in illumination is due to small amounts of gold nanoparticles in the glass, which had been discovered using a transmission electron microscope. Due to the effect of SPR, electrons start oscillating at a specific frequency. Light at wavelengths corresponding to those of the oscillating electrons, is able to travel through the glass, as the electrons are unable to screen it. Light with different frequencies, however, is shaded by the oscillating electrons and therefore scattered back. The end effect in transmitted light is therefore somehow similar to that of absorption.
5 Conclusion

Intricate details of internal structure of materials mean virtually every material has its own unique way of interacting with light. We presented the simplified model of absorption and scattering, which is adequate for understanding most everyday materials and can still be solved analytically. More advanced models must be used for describing phenomena such as material anisotropy, polarization, optical activity, fluorescence, and interference effects.

Development of better models for transparent and especially translucent materials is still very much ongoing, both for academic purposes in material science, graphical engineering, and in efforts for realistic film animation. The new theories are trying to patch the limitations of Kubelka-Munk, such as analyzing reflectivity on heavily inked paper and darker paint, and there is practically a new way of rendering translucent materials with every new animated film. More advanced models are usually tailor-made to a specific material, and face challenges in practicality and suitability for commercial adoption.

6 References