The strong CP problem and axions
Seminar I

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Ljubljana, 2017

Abstract

We investigate the possibility of CP symmetry violation in strong interactions, also called the strong CP problem. We show how the problem arises in the theory of quantum chromodynamics and its potential resolution involving a new light scalar particle, the axion. We present some of the experiments performed in search of the axion and the constraints the results place on the axion mass and on the coupling of the axion to two photons.
1 Introduction

The theory that is used to describe the dynamics of strong interactions is called quantum chromodynamics or QCD. One of its few deep problems was discovered in 1970s, when it was realized that the strong interaction could theoretically violate the symmetry to the combined transformation of charge conjugation and space inversion, otherwise known as CP symmetry. At the same time the experimental results clearly showed CP to be respected by strong interactions to a very high precision. The question why this is so constitutes the strong CP problem. The problem has remained unsolved to this day. In what follows, we present the basics of how CP violating terms arise in the theory of QCD and move on to show the main ideas involved in suppressing them, leading to the prediction of a new light scalar particle, the axion. Finally, we also present some experimental results of the search for axions.

2 QCD Lagrangian

Strong interaction is responsible for binding of quarks into heavier composite particles called hadrons, an example of which are the nucleons. Analogously to the electromagnetic interaction which arises between charged particles as an exchange of virtual photons, strong interaction takes place between quarks exchanging massless gluons. The charge, associated with strong interaction is called color charge. This analogy between quantum electrodynamics (QED) and quantum chromodynamics remains relevant at the fundamental level, which is why it is often instructive to first investigate the more familiar case of QED.

The quantity from which we derive the equations of motion in any quantum field theory is called the Lagrangian. For QED, it is given by

$$L_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi,$$

where the first term features the photon field $A_\mu(x)$
forming the electromagnetic field strength tensor

\[ F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}, \]

(2)

while the second term contains the fermion field $\psi(x)$ of mass $m$. There is also the covariant derivative

\[ D_{\mu} = \partial_{\mu} + ieQA_{\mu}, \]

(3)

where $e$ is the coupling constant of QED, and $Q$ is a number which is characteristic of the field $\psi$ and may therefore vary depending on the type of field involved.

Taking example in QED, we may now construct the Lagrangian for QCD. Strong interaction takes place between quark fields $q(x)$ and gluons $G_{\mu}^{a}(x)$. As already mentioned, quark fields carry color charges red, green and blue. We therefore write quark fields as

\[ q = \begin{pmatrix} q_{R} \\ q_{G} \\ q_{B} \end{pmatrix}, \]

(4)

where each of the entries on the right-hand side is a Dirac spinor of color determined by its index. The quarks thus form the fundamental representation of the group SU(3) of color. We use gluon fields along with the QCD coupling constant $g$ and the SU(3) structure constants $f_{abc}$ to define the gauge field strength tensor

\[ G_{\mu}^{a} = \partial_{\mu} G_{\nu}^{a} - \partial_{\nu} G_{\mu}^{a} - g f_{abc} G_{\mu}^{b} G_{\nu}^{c}. \]

(5)

The notable difference between QCD and QED field strength tensors is in the extra term, appearing in this equation. Another important aspect of QCD is the fact that instead of a single photon, there are eight gluons which is why there is an extra index on each gluon field. With these definitions, we write the Lagrangian

\[ \mathcal{L}_{QCD} = -\frac{1}{4} G_{\mu\nu}^{a} G_{\mu\nu}^{a} + \bar{q}_{j} (i \gamma^{\mu} D_{\mu} - m_{j}) q_{j}, \]

(6)

where the index $j$ runs over all quark flavors and where $D_{\mu}$ is the covariant derivative, defined using SU(3) generators in the fundamental representation $T_{a}$ as

\[ D_{\mu} = \partial_{\mu} + ig T_{a} G_{\mu}^{a}. \]

(7)

Usually we define the generators as $T_{a} = \frac{1}{2} \lambda_{a}$, where $\lambda_{a}$ are the Gell-Mann matrices.

For a general gauge field theory, the first term in (6), also called pure gauge term, should be written as $-\frac{1}{4} G_{\mu\nu}^{a\dagger} G_{\mu\nu}^{a}$, but we may justify the absence of hermitian conjugation by taking $G_{\mu}^{a}$ to be real valued fields. We should also note that the SU(3) structure constants $f_{abc}$ must be real if the relevant generators are hermitian, which certainly holds for our choice of the generators. Therefore the gauge field strength tensor must be hermitian as well.

### 3 P, C and T symmetries

In what follows, we will be focusing on the properties of the QCD Lagrangian under various discrete transformations.

#### 3.1 Parity

The first discrete operation that we will study is called space inversion or parity, denoted by $\mathcal{P}$. By definition, it changes the sign of all spatial coordinates while leaving the time component unaffected, thus transforming a vector $x^{\mu} = (t, \mathbf{x})$ to

\[ \tilde{x}^{\mu} = (t, -\mathbf{x}). \]

(8)

To prove parity invariance of a Lagrangian, we need to find transformations that satisfy the relation

\[ \mathcal{P} \mathcal{L}(x) \mathcal{P}^{-1} = \mathcal{L}(\tilde{x}), \]

(9)

where the fact that the fields are functions of space-time coordinates is made explicit. In other words, we are looking for parity transformations which keep the form of the Lagrangian in the new coordinates the same as it is before the transformation in the original coordinates. If we can find such transformations, then the Lagrangian is invariant under parity. This is indeed true for QCD, where the transformation rules for quark and gluon fields are

\[ \mathcal{P} q(x) \mathcal{P}^{-1} = \eta_{P} \gamma_{0} q(\tilde{x}) \]

\[ \mathcal{P} \bar{q}(x) \mathcal{P}^{-1} = \eta_{P} \bar{q}(\tilde{x}) \gamma_{0} \]

\[ \mathcal{P} G_{\mu}(x) \mathcal{P}^{-1} = G_{\mu}(\tilde{x}) \]

\[ \mathcal{P} G_{i}(x) \mathcal{P}^{-1} = -G_{i}(\tilde{x}), \]

(10)

where $\eta_{P}$ is a parameter that may take values $\eta_{P} = \pm 1$. To prove that the Lagrangian conserves parity,
we could take the transformed fields from (10) and use them to replace all original fields in (6). It would then become apparent that the result is the same as the one obtained by simply replacing all instances of $\tilde{x}$ by $\tilde{x}$ and appropriately adjusting the signs in spatial derivative terms.

The reason for the presence of parameter $\eta_P$ is that fermion fields come in pairs in all terms of the Lagrangian so the only constraint on the parameter is $\eta_P^2 = 1$, which is easily seen in the mass term. On the other hand, the interaction term $-g\tilde{q}\gamma^\mu T_\alpha q G_\mu$ fixes any such multiplicative parameter for all components of the gluon field.

### 3.2 Charge conjugation

The next discrete symmetry of interest is charge conjugation $C$, which transforms a particle into its antiparticle. Charge conjugation does not change any space-time coordinates, which implies that if the Lagrangian is invariant, its form will remain unaffected by the operation. The only change can therefore be induced on the fields themselves. It can be shown that the QCD Lagrangian is invariant to C symmetry if the quark bilinears along with gluon fields undergo the transformations

$$Cq\tilde{q}^{-1} = \tilde{q}$$
$$C\gamma_\mu \frac{\lambda_a}{2} q^{-1} = -\eta(a)\tilde{q}\gamma_\mu \frac{\lambda_a}{2} q$$
$$CG_\mu^{-1} = -\eta(a)G_\mu$$

where

$$\eta(a) = \begin{cases} +1 & \text{if } a = 1, 3, 4, 6, 8 \\ -1 & \text{if } a = 2, 5, 7. \end{cases}$$

The coefficient $\eta(a)$ is a consequence of the SU(3) structure of QCD. To obtain transformation rules for QED, we would simply omit all occurrences of $\eta(a)$ and SU(3) generators $\frac{1}{2}\lambda_a$.

### 3.3 Time reversal

The last of the discrete symmetries that we study is time reversal $T$. Its effect on a vector in space-time can be defined in an analogous manner to that of parity. It changes the sign of the temporal component and leaves the spatial ones unaffected, thus transforming a vector $x_\mu = (t, \mathbf{x})$ to

$$\tilde{x}_\mu = (-t, \mathbf{x}).$$

As in the case of parity, the Lagrangian is invariant to time reversal if there exists such a transformation of fields that

$$T\mathcal{L}(x)T^{-1} = \mathcal{L}(-\tilde{x}).$$

A similar procedure as in the case of parity can be employed to find the transformations for quark bilinears and gluon fields,

$$T\tilde{q}(x)q(x)T^{-1} = \tilde{q}(-\tilde{x})q(-\tilde{x})$$
$$T\tilde{q}(x)\gamma_0 \frac{\lambda_a}{2} q(x)T^{-1} = \tilde{q}(-\tilde{x})\gamma_0 \frac{\lambda_a}{2} q(-\tilde{x})$$
$$T\tilde{q}(x)\gamma_i \frac{\lambda_a}{2} q(x)T^{-1} = -\tilde{q}(-\tilde{x})\gamma_i \frac{\lambda_a}{2} q(-\tilde{x})$$
$$T\tilde{G}_0(x)T^{-1} = \tilde{G}_0(-\tilde{x})$$
$$T\tilde{G}_i(x)T^{-1} = -\tilde{G}_i(-\tilde{x}).$$

With such transformations of fields, QCD Lagrangian is invariant to time reversal.

Throughout this section, it was implicitly assumed that all the components of the color triplet behave in the same way under any of the discrete symmetries.

### 4 CP violating effects

#### 4.1 CP violating term in QED

We have seen that the Lagrangian of QCD is invariant separately under C, P and T transformations. By extension, it is also invariant under the CP transformation, which is the combination of C and P, as the name suggests. The same conclusions then clearly hold for the QED Lagrangian as well, since all of the terms it contains can also be found in the QCD Lagrangian. Because QED is the somewhat simpler theory, it is instructive to first discuss potential CP violating effects in electrodynamics before returning to QCD.

Aside from what was written as the QED Lagrangian in (1), there is another type of gauge invariant object that we could consider including in the theory. More specifically, we could contemplate the term

$$\tilde{\mathcal{L}}_{QED} = \alpha F_{\mu\nu}\tilde{F}^{\mu\nu},$$

where $\alpha$
where we have introduced the dual of the electromagnetic field strength tensor, defined by

\[ \tilde{F}_{\mu \nu} = \frac{1}{2} \varepsilon_{\mu \nu \lambda \rho} F^{\lambda \rho} \]  

(17)

using the completely antisymmetric Levi-Civita symbol \( \varepsilon_{\mu \nu \lambda \rho} \) and a numeric constant \( \alpha \). To determine the properties of the new term under the discrete symmetries, we expand the field strength tensors in terms of photon fields and use explicit space-time values for Lorentz indices to obtain

\[ F_{\mu \nu} \tilde{F}^{\mu \nu} = 2 \varepsilon^{0ijk} \left( (\partial_0 A_i)(\partial_j A_k) - (\partial_i A_0)(\partial_j A_k) + (\partial_i A_j)(\partial_0 A_k) - (\partial_i A_j)(\partial_k A_0) \right). \]  

(18)

Now let us look at what happens when we perform parity transformation on this object. On the one hand, using parity induces the change in the coordinates \( x \rightarrow \bar{x} \). This changes the signs of all spatial derivatives and at the same time induces the transformation of the photon fields \( A_\mu(x) \rightarrow A_\mu(\bar{x}) \). This means that signs are changed for the first and the third term within the parentheses. On the other hand, parity transformation can be performed by transforming the photon fields directly, using the gluon transformation rules from (10). It is easy to see that in this case, only the second and the fourth term would flip signs. Therefore, the \( F \tilde{F} \) term violates P symmetry. By following similar steps, we could show that T is also violated while C remains a symmetry of the Lagrangian even with the new term included.

It is useful to note that Levi-Civita symbol interchanges space-space components with time-space components. Using the definitions of electric and magnetic field strengths \( E^i = F^{0i} \), \( B_i = -\frac{1}{2} \varepsilon_{ijk} F^{jk} \), we can rewrite the term as

\[ F \tilde{F} = 4E \cdot B. \]  

(19)

To examine this new contribution to the Lagrangian in greater detail, we rewrite it in terms of photon fields by expanding the field strength tensor and its dual according to equations (2) and (17). By taking into account the antisymmetry of \( \varepsilon_{\mu \nu \lambda \rho} \), the \( F \tilde{F} \) term can be put into the form of a total derivative

\[ \alpha F_{\mu \nu} \tilde{F}^{\mu \nu} = \partial_\mu \left( \alpha \varepsilon^{\mu \nu \lambda \rho} A_\nu F_{\lambda \rho} \right). \]  

(20)

In most cases, such four-divergence terms do not contribute anything significant to the theory. The reason for this becomes apparent when we calculate the action from these contributions. We can use the Gauss theorem to rewrite the action integral as

\[ \tilde{S} = \int dS_\mu \alpha \varepsilon^{\mu \nu \lambda \rho} A_\nu F_{\lambda \rho}, \]  

(21)

where the integration is now performed over the three-dimensional boundary of space-time. In order for the energy functional of any quantum field theory to give finite values of energy, all terms in the relevant Lagrangian must vanish at infinity. Looking at (1), we conclude that the fermion field and photon field strength tensor must vanish at space-time infinity. Since there is a factor of \( F_{\lambda \rho} \) in the last equation, the action integral vanishes. The \( F \tilde{F} \) term therefore has no physical consequence in QED.

### 4.2 Contribution from strong interaction

We now perform a similar procedure for QCD. Once again, we define the new term as

\[ \tilde{\mathcal{L}}_{QCD} = \alpha G^a_{\mu \nu} \tilde{G}^{a \mu \nu}, \]  

(22)

where we have used the dual of the gluon field strength tensor

\[ \tilde{G}^{a \mu \nu} = \frac{1}{2} \varepsilon^{\mu \nu \lambda \rho} G^a_{\lambda \rho}. \]  

(23)

This may again be rewritten as a total derivative

\[ \alpha G^a_{\mu \nu} \tilde{G}^{a \mu \nu} = \partial_\mu \left( \alpha \varepsilon^{\mu \nu \lambda \rho} G^a_\nu (G^a_{\lambda \rho} + \frac{1}{3} g f_{b c a} g^b_\lambda g^c_\rho) \right), \]  

(24)

whose contribution to the QCD action is

\[ \tilde{S} = \int dS_\mu \alpha \varepsilon^{\mu \nu \lambda \rho} c_\nu \left( G^a_{\lambda \rho} + \frac{1}{3} g f_{b c a} g^b_\lambda g^c_\rho \right). \]  

(25)

The important thing to notice here is the second term in the parentheses which was not present in QED. It contains SU(3) structure constants, from which it is clear that there cannot be any such terms in QED. They can only arise in non-abelian gauge theories. The first term in (25) again vanishes, since finiteness of energy dictates the quark fields and gluon field strength tensors to vanish at space-time infinity. There is no requirement for the gluon fields themselves to vanish, so second term remains in the contribution...
to QCD action. We summarize this by writing the full action of QCD as
\[
S = \int d^4x \, L_{QCD} + \frac{\alpha g}{3} f_{abc} \varepsilon^{\mu \nu \lambda \rho} \int dS_\mu \, G^a_\mu G^a_\nu G^b_\rho .
\] (26)

We now investigate the properties of the $G\tilde{G}$ term under discrete symmetries. To show that P and T symmetries are violated, let us use equations (5) and (23) to expand expression (22) and focus on the term that is cubic in gluon fields. Finally, we replace Lorentz indices by the ones denoting specific space-time components, and we get
\[
G\tilde{G}_{\text{cubic}} = -2g \varepsilon^{0jk} f_{abc} (\partial_0 G^a_i \partial_j G^a_k - (\partial_0 G^a_i) \partial_j G^a_k - (\partial_0 G^a_i) \partial_j G^a_k) .
\] (27)

Directly using parity transformation on this expression, all fields would undergo $G^a_i(x) \rightarrow G^a_i(\bar{x})$. Also, the latter three terms would change signs because of the spatial derivatives. The gluon field transformation rules from (10) clearly do not produce the same effect since this would change the signs of all four terms.

We can conclude that parity is violated. We can use the same kind of argument to show that time reversal is violated as well. Charge conjugation remains a symmetry of the Lagrangian, which can be verified using the transformation rules (11). We would find that any terms that change signs under the effect of charge conjugation are multiplied by vanishing structure constants. Therefore, since parity is violated and charge conjugation conserved, the combined CP symmetry is violated.

Finally, we redefine the constant $\alpha$ and use it to rewrite the equation (22) as
\[
\bar{L}_{QCD} = \frac{\theta_{QCD} g^2}{32\pi^2} G^a_\mu \tilde{G}^\mu_\nu .
\] (28)
where we have introduced the parameter $\theta_{QCD}$. The normalization is chosen so that performing the boundary integral in (26) would give the result $n\theta_{QCD}$, where $n$ is the winding number of the gluon field, which is a topological parameter.

4.3 Contribution from the electroweak interaction

There is another important point to make regarding the term $G\tilde{G}$. According to the discussion so far, it is an additional possible contribution to QCD Lagrangian. But there is another source of exactly this type of contribution. It can be shown that performing the axial transformation
\[
q \rightarrow \exp (i\beta \gamma_5) q
\] (29)
on all quark flavors for some value of the parameter $\beta$, the Lagrangian obtains an extra term
\[
\frac{g^3 \beta}{16\pi^2} G^a_\mu \tilde{G}^\mu_\nu ,
\] (30)
which is of the same form as what we have already encountered in (28). The important feature of this contribution is that it arises entirely out of quantum loop corrections. The Noether current, associated with the transformation (29), calculated for the QCD Lagrangian, only contains contributions that vanish in the limit where the quarks are massless.

This means that in the massless limit, the transformation (29) is a symmetry of the Lagrangian on the classical level, but is broken when quantum effects in the form of loop diagrams are taken into consideration. Such contributions are called anomalies. Since the transformations from the equation (29) are axial rotations forming a group U(1), it is said that QCD has a U(1)$_A$ anomaly. Such transformations are exactly what we need in electroweak theory when we calculate quark masses from the mass term
\[
\mathcal{L}_{\text{mass}} = \bar{u}_{Li} M_{ij}^{(u)} u_{Rj} + \bar{u}_{Rj} M_{ij}^{(u)\dagger} u_{Lj} + \bar{d}_{Li} M_{ij}^{(d)} d_{Rj} + \bar{d}_{Rj} M_{ij}^{(d)\dagger} d_{Lj} .
\] (31)
In this equation, $u$ and $d$ represent up and down-type quarks, and $i$ and $j$ are generation indices, which have been made explicit. $L$ and $R$ are used to denote chirality. The mass matrices $M$ therefore in general mix quarks of different generations, so in order to obtain mass terms in which both quark fields belong to the same family, mass matrices need to be diagonalized. In such a procedure, transformations of the type (29) are used, and as a consequence, a term of the form (30) is added to the QCD Lagrangian. The amount of rotation is given as $\beta = \arg \det (M^{(u)} M^{(d)})$, which is the complex phase of the determinant of the mass matrices. Since this electroweak effect is formally equivalent to the QCD contribution, it can be
added to the already known parameter $\theta_{\text{QCD}}$. Thus we obtain the effective $\theta$ parameter

$$\theta_{\text{eff}} = \theta_{\text{QCD}} + \arg \det \left( M^{(u)} M^{(d)} \right), \quad (32)$$

which is the value upon which all experimental results will depend.

### 4.4 Physical effects of $\theta_{\text{eff}}$

We have seen that strong interaction may violate CP symmetry through the term in the Lagrangian containing the parameter $\theta_{\text{eff}}$. This term violates symmetries P and T while allowing C to remain a symmetry of the Lagrangian. To find the value of $\theta_{\text{eff}}$, measurements must be done on physical quantities that have the same properties under these transformations. One such quantity is the electric dipole moment (EDM) of a particle. To see this, let us imagine a neutron with an internal charge distribution that creates a non-zero EDM. The orientation of the neutron is determined by its spin. By performing a parity transformation on the neutron, we reverse the charge distribution and with it the direction of the dipole moment while spin remains unchanged. Time reversal leaves the charge distribution unaffected but it reverses the spin of the neutron. Both P and T are thus violated. The combined transformation CPT is an exact symmetry of nature at the fundamental level and so any T violation also implies an opposite CP violation.

We could prove that EDM violates CP in a more systematic manner by discussing matrix elements associated with the electric dipole of a fermion. It turns out that such matrix elements contain combinations of $\gamma_5$ in fermion bilinears, which is what signals CP violation since the transformation properties of such bilinears differ from those already shown in (10) and (15).

Let us now look at the interaction that gives rise to neutron EDM through the CP-violating $\theta$ term. It turns out that the leading contribution can be expressed as an effective interaction Lagrangian in the form [9]

$$\mathcal{L}_{\text{CPV}}^{\text{str}} = -\theta_{\text{eff}} m q_i \hat{\Phi}_{ij} f_\pi q_j, \quad (33)$$

where we have introduced the octet matrix of pseudoscalar mesons $\hat{\Phi}_{ij} = \sum_{i=1}^{8} \Phi_a (\lambda_a)_{ij}$ which operates in flavor space. We also have the pion decay constant $f_\pi$ and the mass coefficient

$$m = \frac{m_u m_d m_s}{m_u m_d + m_u m_s + m_d m_s}. \quad (34)$$

It is clear that if any of the quarks were massless, the whole term (33) would vanish. The diagram that contributes to the strong CP-violating part of the neutron EDM is shown below. Scalar loop in this diagram consists of pions or kaons.

![Figure 1: The strong CP-violating contribution to neutron EDM. The black circle represents the interaction from (33).](image)

High precision measurements of the neutron electric dipole moment $d_N$ have been performed. An estimate for its magnitude is expressed as [6]

$$d_N \sim \frac{e \theta_{\text{eff}} m_\pi^2}{m_N^3}, \quad (35)$$

where $e$ is the electromagnetic coupling constant, $m_\pi$ is the pion mass and $m_N$ is the neutron mass. Experimental results put the approximate upper limit on the value of $\theta_{\text{eff}}$ at

$$|\theta_{\text{eff}}| \leq \mathcal{O}(10^{-10}), \quad (36)$$

which is an extremely small value, since there is no reason not to expect anything other than $\theta_{\text{eff}} \sim \mathcal{O}(1)$. We know that quark mass matrices must have a complex phase, which gives rise to CP violation in weak interaction. Therefore, the experimental result would imply that the smallness of $\theta_{\text{eff}}$ is due to a delicate cancellation of the two terms in (32), which is extremely unlikely, judging by the fact that both of them originate in completely different sectors of SM. This is what constitutes the strong CP problem. One possible solution of this problem would present itself if any
of the quark masses were zero, since in this case we
could perform a chiral rotation of the quark fields for
an amount that would cancel the $\theta$ term. However,
this solution is excluded by lattice QCD simulations
which show quarks to have non-zero masses [5].

5 Axions

There is another type of mechanism that allows us
to preserve CP symmetry in strong interaction. We
can achieve this by imposing an additional spontane-
ously broken global chiral symmetry on the full SM
Lagrangian, which implies the existence of a new scalar
particle called axion. By appropriately expanding
the QCD Lagrangian we determine the form of the
term involving the axion $a$ to be of the form

$$\mathcal{L}_a = \frac{\xi a}{f_a} \frac{g^2}{32\pi^2} G^a_{\mu \nu} \tilde{G}^{\mu \nu} ,$$

(37)

where $\xi$ is a model-dependent coefficient and $f_a$
is the axion decay constant. The similarity between this
term and the $\mathcal{G}\tilde{G}$ term from (28) is immediately ap-
parent. We can join both of them together and see that
they form an effective potential for the axion field,

$$\mathcal{L}_{\mathcal{G}\tilde{G}} = \left( \theta_{\text{eff}} + \frac{\xi a}{f_a} \right) \frac{g^2}{32\pi^2} G^a_{\mu \nu} \tilde{G}^{\mu \nu} .$$

(38)

This potential is minimized at the vacuum expecta-
tion value of the axion,

$$\langle a \rangle = -\frac{\theta_{\text{eff}} f_a}{\xi} ,$$

(39)

for which the whole term vanishes. By expanding the
physical axion as

$$a_{\text{phys}} = a - \langle a \rangle ,$$

(40)

we rewrite the $\mathcal{G}\tilde{G}$ term as

$$\mathcal{L}_{\mathcal{G}\tilde{G}} = \frac{\xi a_{\text{phys}}}{f_a} \frac{g^2}{32\pi^2} G^a_{\mu \nu} \tilde{G}^{\mu \nu} .$$

(41)

Thus the parameter $\theta_{\text{eff}}$ is gone from the Lagrangian
and CP is once again conserved in strong interactions.
In a sense, this solution promotes the static parameter
$\theta_{\text{eff}}$ to a dynamical field whose excitations around
zero are associated with the axion.

6 The search for axions

At present, the axion remains elusive despite great
experimental efforts. Nevertheless, much of the possi-
ble interval for its mass has been constrained. In
the next section, we shall see some of the experi-
mental methods and observations through which this has
been achieved.

6.1 The interaction Lagrangian

The mass of the axion is one of the few free para-
eters in an axion model. It is related to the axion
decay constant by

$$m_a = \frac{\sqrt{z} f_{\pi} m_{\pi}}{1 + \frac{z}{f_a} (f_a/10^{10}) \text{GeV}} = 0.60 \text{meV},$$

(42)

where $m_{\pi}$ is the pion mass and $f_{\pi}$ is the pion de-
cay constant. The values of constants are $z = 0.56$,
$f_{\pi} = 92 \text{MeV}$ and $m_{\pi} = 135 \text{MeV}$. This contribu-
tion to axion mass exists because axions undergo mixing
with neutral pions. Consequently, the properties of
axions and pions are closely related. Generally, axion
interactions differ from the corresponding pion cou-
plings only by a constant coefficient.

Before moving on to experimental techniques, it
is useful to see in more detail how axions couple to
other fields. The effective Lagrangian for the inte-
raction with nucleons, electrons and photons is given
by

$$\mathcal{L}_{\text{eff}} = i g_{aNN} \frac{1}{2m_N} \partial_\mu a (\bar{N} \gamma^\mu \gamma_5 N)$$

$$+ i g_{aee} \frac{1}{2m_e} \partial_\mu a (\bar{e} \gamma^\mu \gamma_5 e) + g_{a\gamma\gamma} a E \cdot B .$$

(43)

In the following sections, we will focus on the term
involving photons since most experiments probe this
part of the interaction Lagrangian.

The axion directly couples to fermions only, so
its interaction with photons must contain more than
one vertex. The diagram of this type of interactions
is shown here.
The explicit form of the coupling constant for this process is

\[ g_{a\gamma\gamma} = \frac{\alpha}{2\pi} \left( \frac{E}{N} - \frac{2}{3} \frac{4 + z}{1 + z} \right) \frac{1 + z}{\sqrt{z}} \frac{m_a}{m_{\pi} f_{\pi}} , \]  

where the ratio \( E/N \) depends on the choice of the axion model and \( \alpha \) is the fine structure constant. The coefficients are related to electromagnetic (\( E \)) and color (\( N \)) effects. The coupling \( g_{a\gamma\gamma} \) becomes minimal when \( E/N \sim 1.95 \), which is what the second term within the parentheses amounts to for \( z = 0.56 \). In general, \( E/N \) can take a wide range of values which may be positive or negative.

Another notable thing regarding the coupling constant \( g_{a\gamma\gamma} \) is its dependence on \( m_a \). In fact, all of the coupling constants in (43) are proportional to \( m_a \), which means that should the axion have a tiny mass, its interactions will be extremely weak.

Turning back to the photon coupling constant, we can use it to calculate the axion decay width for the process \( a \rightarrow \gamma\gamma \). By setting \( E/N = 0 \) and \( z = 0.56 \), it reads

\[ \Gamma_{a\rightarrow\gamma\gamma} = \frac{g_{a\gamma\gamma}^2 m_a^3}{64\pi} = 1.1 \times 10^{-24} \text{ s}^{-1} \left( \frac{m_a}{\text{eV}} \right)^5 . \]  

In order for the axion lifetime to be greater than the age of the universe, its mass should be \( m_a \lesssim 20 \text{ eV} \). Therefore, if the axion mass is sufficiently small, they may yet be present in the universe today. This opens up the possibility of axions being a component of dark matter.

6.2 Photon regeneration experiments

The process, shown in Fig. 1, has been used in experiments involving the conversion of an axion into a photon [15].

In this type of experiment, an intense laser beam \( E \) travels through a region of length \( L \) of a transverse magnetic field \( B \). From the interaction term \( g_{a\gamma\gamma} a \mathbf{E} \cdot \mathbf{B} \), it is clear that the magnetic field must be parallel to the polarization of the laser beam for the greatest probability of conversion. The axions are then produced in the direction of the beam. The momentum transfer in these interactions must be small, so that they are coherent and so they may be seen as photon-axion oscillations. After traveling the distance \( L \), the laser beam is blocked. The emergent axions then travel through the barrier and into a second magnetic field, which is identical to the first one. In the secondary field, the oscillations may continue and thus some of the axions are converted back to photons, which are finally detected by a photomultiplier. The probability for the conversion of a photon into an axion at the exchanged momentum \( q = |k_\gamma - k_a| \) is [3]

\[ P_{\gamma\rightarrow a} = \frac{1}{4} (g_{a\gamma\gamma} BL)^2 \left( \frac{2}{qL} \sin \frac{qL}{2} \right)^2 . \]  

The probability for the conversion of the axion back into a photon is the same, which gives the combined probability

\[ P_{\gamma\rightarrow a \rightarrow \gamma} = P_{\gamma\rightarrow a}^2 . \]  

The schematic for the experiment OSQAR, performed at CERN, is shown in the figure below. The results from this experiment showed

\[ |g_{a\gamma\gamma}| < 5.7 \times 10^{-8} \text{ GeV}^{-1} \]  

in the limit where the axion is massless.

Figure 3: The OSQAR photon regeneration experiment consists of two 14.3 m long LHC dipole magnets, operating at 9 T. The laser beam provides photons at the wavelength 532 nm of power 15 W. Resulting axions travel through the wall into the second identical dipole region, where a lens and a CCD detector detect any regenerated photons [3].
6.3 Photon polarization

Another type of experiment, involving the same coupling has also been performed [16]. Similarly to the photon regeneration experiments, light passes through a transverse magnetic field. This time, the beam is polarized at an angle of $45^\circ$ relative to the magnetic field. Since only the component $E_{||}$ is used to produce axions, it becomes depleted and thus slightly rotates the polarization of light. This effect is called dichroism. For axion masses

$$\frac{m_{a}^{2}L}{2\omega} \ll 2\pi$$  \hspace{1cm} (49)$$

the magnitude of dichroism is unrelated to $m_{a}$, but for $m_{a} > \omega$, the effect vanishes. There is another type of polarization that arises in a similar manner. It occurs as a quantum loop correction in the form of virtual axions in the $E_{||}$ component, but not in $E_{\perp}$. In this way, linearly polarized light develops an elliptical polarization. This effect is called birefringence. It is present irrespective of the axion mass. The experiments limited the coupling constant at $m_{a} \lesssim 7 \times 10^{-4}$ eV to

$$|g_{a\gamma\gamma}| < 2.5 \times 10^{-6} \text{ eV}. \hspace{1cm} (50)$$

6.4 Astrophysical observations

Light, weakly interacting particles are produced in hot plasmas within astrophysical objects, such as stars. Such particles have the potential to quickly and efficiently transport the energy out of the stars and thus shorten their lifetimes. We may use the observations of stellar energy loss rates and lifetimes to determine the coupling strength of such particles with ordinary matter.

First let us look at possible axion production in our Sun, where the majority of axions are produced via Primakoff process, $\gamma + Ze \rightarrow Ze + a$. Here, $Ze$ denotes ions and electrons that produce magnetic fields to which axions may couple. The effective Feynman diagram for such a process is depicted here.

By applying a solar model and performing the energy integration, we obtain the axion luminosity [1]

$$L_{a} = g_{10}^{2} 1.85 \times 10^{-3} L_{\odot}, \hspace{1cm} (51)$$

where $g_{10} = |g_{a\gamma\gamma}| \times 10^{10}$ GeV. From this we can calculate the number flux at Earth as

$$j_{a} = g_{10}^{2} \times 3.75 \times 10^{11} \text{ cm}^{-2} \text{ s}^{-1}. \hspace{1cm} (52)$$

Including the axion in the description of solar dynamics would mean that we need to also take into account the extra nuclear energy that is lost due to axion radiation. The increased number of nuclear reactions in turn increases the neutrino luminosity. Measurements of solar neutrino fluxes used alongside a standard solar model set an upper limit of the axion luminosity to

$$L_{a} \lesssim 0.1 L_{\odot}, \hspace{1cm} (53)$$

from which it directly follows that $g_{10} < 7$.

There exists a more stringent limit for the value of $g_{10}$, which is determined by observations of globular clusters. The stars on the horizontal branch of the HR diagram are in the helium-burning stage, which is accelerated due to axion emission via Primakoff effect. On the other hand, red giants that are present in the same clusters remain unaffected by this effect, since they only fuse hydrogen in the shell while the pressure in their core is insufficient to start helium fusion, so it remains inert. It is possible to determine $g_{a\gamma\gamma}$ by comparing the number of the stars on the horizontal branch to the number of red giants. In this way, the upper bound for the coupling constant was found to be [2]

$$|g_{a\gamma\gamma}| < 6.6 \times 10^{-11} \text{ GeV}^{-1}, \hspace{1cm} (54)$$

which holds for a wide range of axion masses.
We can also try to directly detect solar axions instead of using energy losses to estimate rates of emissions. The devices, used for this end, are called axion helioscopes. This experimental method is based on the idea that we can detect the solar axions which were created by the Primakoff process, by having them undergo the reverse process on Earth under the influence of a magnetic field. We can use a pressurized gas to give the emergent photons an effective mass \( m_{\gamma} = \omega_{\text{plas}} \) which can be adjusted so that it is equal to the mass of the axion. This ensures coherence throughout the detector. In this way we may extend the range of detection towards larger axion masses.

### 6.5 Cosmological implications

In cosmology, one of the most important features of the axion is the possibility that it constitutes the cold dark matter, or CDM [13]. One of the production mechanisms of axions is called re-alignment. The main idea is that in the very early universe with a high enough temperature, axion field could take any value. As the universe cooled, QCD effects became significant and caused the effective potential for the axion, shown in (38). Oscillations of the axion field around the minimum of this potential thus result in a condensate of axion population that forms the CDM.

Telescope searches for axion decays into photons have been performed by attempting to detect nearly monochromatic emissions from galaxies and galaxy clusters. Low mass axions in dwarf galaxies emit a weak radio line, which is very narrow, since the axion velocities in the small gravitational potential of such galaxies should also be small due to virial theorem. However, such searches have yet to exclude any plausible axion models [4].

Another possibility for detecting axions in the galactic halo is a resonant electromagnetic cavity in a strong static magnetic field [17], also called haloscope. The cavity is designed so that it allows for axion conversion into a photon of its resonant frequency. By delicately tuning the frequency of the cavity, we may detect axion signals. Since galactic halo axions travel at speeds \( \beta \sim 10^{-3} \), the frequency of detection should be

\[
\nu = m_a \left( 1 + \mathcal{O}(10^{-6}) \right) .
\]

(55)

All of the experimental results regarding the value of \( g_{a\gamma\gamma} \) and \( m_a \) that we discussed in this section are shown in the next figure.

![Figure 5: Experimental boundaries on the values of axion mass in relation to the coupling constant with two photons are represented by the colored areas. Predictions of the axion models lie within the yellow area. Different lines in this area denote models with different values of \( E/N \) [14].](image)

### 7 Conclusion

It is obvious that even if current experiments have not been able to directly detect the axion, there are many orders of magnitude of mass yet to scan. Axion remains a highly compelling idea as an extension of the standard model. Aside from providing a natural solution to the strong CP problem, it is also one of the contenders for the identity of dark matter, the exploration of which is at the forefront of modern physics. It is therefore not unreasonable to expect the axion to play a role in the shaping of theories yet to come.
References


