Hail detection using single-polarization radar

Abstract

Hail related to summertime thunderstorms is a small-scale phenomenon, and it often has a short time duration. Because of the high spatial and temporal variability of hail, the proper detection of hail occurrences is almost impossible using ground station reports alone. An alternate approach uses information from weather radars. Several algorithms that use single-polarisation radar data have been developed for hail detection. The criteria consider different levels or thresholds of radar reflectivity, some of them complemented by estimates of the 0°C level or cloud top temperature. In the seminar three independent studies are presented, where they tested and optimized hail detection algorithms for specific geographic areas in Germany, Czech Republic and Netherlands.
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1 Introduction

Hail has been a subject of scientific interest for many decades because of the severe damage it causes to agriculture, buildings and cars. For a specific location hail is a low probability high impact weather event. Hail is characterized by a strong local-scale variability of the occurrence and intensity, and the small extent of the affected areas referred to as hailsteaks. As a consequence, point observations of hail are not representative for larger areas. Weather radar-based hail detection algorithms can be useful for investigating hail frequency over larger areas and in regions where long observation time series do not exist. In the fifties of the previous century, first studies were conducted to investigate the presence of hail in thunderstorms. Since then a number of hail detection algorithms has been developed, and trying to catch the best correlation between hail and radar reflectivity data [1].

2 Hail

Theoretical studies and observations have shown that hail can grow at many different locations and altitudes within thunderstorms. Thus it appears that there are several distinct classes of hailstones caused by the different interaction of microphysics and storm dynamics [2]. Hail growth begins at altitudes between 5 and 10 km in a cloud of supercooled droplets (temperature < 0°C), and a broad region of moderate updraft is necessary to suspend hail aloft in the prime growth layer. The major ice particles involved in the growth of hailstones are graupel, small hail and ice pellets. Graupel consists of a central ice crystal that has accreted supercooled droplets that freeze after impact. The graupel often has a conical shape, a diameter of about 5 mm, and a density of 800 kg/m$^3$. Because graupel is composed of a myriad small frozen particles it appears white. Small hail represents an intermediate stage between graupel and a hailstone. Small hail forms from graupel by intake of liquid water into the air capillaries of its ice structure wherein liquid water is produced either by accretion of warm cloud droplets or by partial melting of the graupel. The size can be similar to graupel but the density would be larger because of water accretion. Hailstones are lumps of ice or ice and water, with air
inclusions and diameters > 5 mm. First measurements of hailstone size distributions were made by Douglas in 1964 who collected samples in wire-mesh baskets. He found an exponential form of size distributions. For hailstone samples collected on the ground from seven storms in Alberta, Cheng and English (1983) to propose a single parameter exponential size distribution for hailstones:

\[ n = n_0 \exp(-\Lambda d), \quad \text{with } n_0 = 115\Lambda^{3.36}, \quad (\text{in m}^{-3}\text{mm}^{-1} \text{ and } \Lambda \text{ in mm}^{-1}). \]  

This affords in the convenience of using a single parameter to describe hailfall rate. The terminal fall velocity of a hydrometeor is the speed at which its downward acceleration due to gravity balances the upward acceleration due to the drag of the air and is, relative to the air itself. Maximum terminal velocity as a function of its diameter is shown in table 2.

<table>
<thead>
<tr>
<th>Kind</th>
<th>Diameter [cm]</th>
<th>Velocity [m/s]</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>&lt; 0.5</td>
<td></td>
<td>Grain</td>
</tr>
<tr>
<td></td>
<td>0.5-1.0</td>
<td></td>
<td>Pea</td>
</tr>
<tr>
<td></td>
<td>1.0-1.5</td>
<td>19</td>
<td>Mothball; small marble</td>
</tr>
<tr>
<td></td>
<td>1.5-2.0</td>
<td></td>
<td>Cherry; marble</td>
</tr>
<tr>
<td>Large</td>
<td>2.0-2.5</td>
<td>24</td>
<td>Large marble</td>
</tr>
<tr>
<td></td>
<td>2.5-3.0</td>
<td>28</td>
<td>Walnut</td>
</tr>
<tr>
<td></td>
<td>3.0-4.0</td>
<td>31</td>
<td>Golfball</td>
</tr>
<tr>
<td></td>
<td>4.0-5.0</td>
<td>34</td>
<td>Small egg</td>
</tr>
<tr>
<td>Giant</td>
<td>5.0-6.2</td>
<td>38</td>
<td>Egg</td>
</tr>
<tr>
<td></td>
<td>6.2-7.5</td>
<td></td>
<td>Tennis ball</td>
</tr>
</tbody>
</table>

Table 1: Hail diameter, its maximum terminal velocity as a function of its diameter and related descriptions [3].

3 Convection

The term convection in meteorology is used to refer to heat transport by the vertical component of the flow associated with buoyancy. Hazardous weather events (large hail, damaging wind gusts, tornadoes, and heavy rainfall) are generally the result of the energy released by phase changes of water [4]. A circular convective cloud with a 5 km radius and 10 km deep contains, about \(8 \times 10^8\) kg of condensed water, assuming an average condensed water content of \(1 g/m^3\). During the condensation of that water, roughly \(10^{14}\) J of latent heat energy is released over a time scale of roughly 25 minutes. Most of the energy is expended against gravity, but some portion also may create hazardous weather. The released heat contributes to buoyancy, \(B\), an essential aspect of convective storms. Buoyancy is defined most simply by

\[ B = g \left( \frac{T - T'}{T} \right) \]  

where \(g\) is the acceleration due to gravity, \(T\) is the temperature of a parcel, and \(T'\) is the temperature of the surrounding environment. Buoyancy can be either negative or positive. If \(B\) is integrated from the level of free convection (LFC) to the equilibrium level (EL) above the LFC, the result is convective available potential energy (CAPE), as illustrated in Fig. 1. It is this energy that is responsible for the convective updraft and for many of the hazards produced by the convection.
Deep moist convection (DMC) is the result of instability. The vertical momentum is described by the equation:

\[
\frac{dw}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g,
\]

where \( w \equiv \frac{dz}{dt} \) is the vertical component of the flow, \( z \) is geometric height, \( \rho \) is the density, and \( p \) is the pressure. The vertical pressure gradient force is the first term on the right hand side. Since the vertical acceleration is zero in a hydrostatic atmosphere, buoyancy is associated with an unbalanced pressure gradient force, caused by density perturbations. Equation 3 can be transformed to:

\[
\frac{dw}{dt} = \frac{d^2z}{d^2t} = B = -\frac{g}{T} (\Gamma - \gamma) z
\]

where \( \Gamma \) is the parcel lapse rate \((-dT/\partial z)\), and \( \gamma \) is the environmental lapse rate \((-dT'/\partial z)\). When the coefficients are constant, equation is a simple, second-order differential equation that has a simple solution:

\[
z(t) = z_0 \exp(iNt),
\]

where \( z_0 \) is the initial height of the parcel and \( N \) is the so-called Brunt-Väisälä, or buoyancy frequency

\[
N^2 = \frac{g}{T} (\Gamma - \gamma) z
\]

The solution of equation 5 implies an instability whenever the square root of \( N^2 \) is imaginary. Since \( \gamma \) is normally not greater than the dry adiabatic lapse rate, the context clearly is associated with conditional instability (i.e., \( \gamma > \Gamma_m \), where \( \Gamma_m \) is the moist adiabatic lapse rate). Actual parcel instability leading to DMC is primarily associated with finite vertical displacements; hence, the key to the possibility for growth of convective storms is the presence of CAPE, not the environmental lapse rates alone. In most cases, energy must be supplied to lift the parcel through its condensation level to its LFC. From the LFC to the EL, the parcel accelerates vertically, drawing the energy for this acceleration from the CAPE.
The origin of buoyant instability is heat, both latent and sensible, that is produced at low levels in the atmosphere as a result of solar heating and evapotranspiration of water vapor (also due to solar heating) into the lower troposphere. The process of convection alleviates the instability created by the accumulation of heat at low levels. DMC takes the excess sensible heat and water vapor from low levels and expels it into the upper troposphere, and transports potentially cold, dry air downward, thereby alleviating the instability.

Our perceptions of convective structure are highly dependent on the observing system being employed. Recent decades meteorological radar has become established to observe development and consequences of deep moist convection.

4 Meteorological radar

Conventional radar (Radio Detection and Ranging) transmits powerful, very brief pulses of electromagnetic energy with high frequency into the atmosphere at equally spaced intervals of time. The energy is concentrated in a beam with small solid angle by a directive antenna. All types of targets present in the beam intercept some of the incident energy, which they absorb and radiate in different directions. The fraction returned to the radar is the useful signal. For hydrometeor observations the most common frequencies lie between 3 and 35 (GHz) and therefore belong to the microwave region. The corresponding wavelengths \( \lambda = c/f_0 \) range form 10 cm to 0.8 cm [5].

<table>
<thead>
<tr>
<th>Bands</th>
<th>Wavelength [cm]</th>
<th>Usage</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>15-30</td>
<td>Long range; for studying turbulence at high altitude</td>
</tr>
<tr>
<td>S</td>
<td>8-15</td>
<td>Range up to 250 km</td>
</tr>
<tr>
<td>C</td>
<td>4-8</td>
<td>Shorter range;</td>
</tr>
<tr>
<td>X</td>
<td>2.5-4</td>
<td>For studies of cloud dynamics</td>
</tr>
<tr>
<td>K</td>
<td>&lt; 2.5</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Doppler radar band [7].

The wavelengths are approximately ten times the diameter of the droplets or ice particles of interest, which causes Rayleigh scattering. This means that part of the energy of each pulse will backscatter on these small particles in the direction of the radar station. The result of radar measurements is the ratio between the emitted and the received radiation power. Hydrometeors will radiate as dipoles, if the emitted wavelength of the radar beam is much larger then the diameter of droplets. Their backscattering cross section is:

\[
\sigma_b = \frac{\pi^5}{\lambda^4} K^2 D^6
\]

\[
|K|^2 = \frac{(m^2 - 1)}{(m^2 + 2)}
\]

where \( D \) is diameter of droplets, \( \lambda \) is the wavelength and \( m = n - j\kappa \) is the complex refractive index of water. The refractive index is \( n \), \( \kappa \) is the attenuation index. \( K \) is a coefficient related to the dielectric constant of water or ice. \( |K|^2 \) for water varies between 0.91 and 0.93 for wavelengths between 0.01 and 0.10 m and is practically independent of temperature. Ice spheres have \( |K|^2 \) of about 0.18, a value independent of temperature as well as wavelength in the microwave region. The result of radar measurements is the ratio of emitted and received power of radiation. Received power as a result of reflections from targets at a distance \( r \) from the radar describes a generalized radar equation:
\[ P_r = C|K|^2 \frac{Z}{r^2} \]  \hspace{1cm} (8)

where \( C \) is radar constant and \( Z \) is the radar reflectivity. Defined as:

\[ Z = \int_0^\infty D^6 \frac{\partial N}{\partial D} dD \]  \hspace{1cm} (9)

element \( \frac{\partial N}{\partial D} \) describes the distribution of precipitation droplets by diameter \( D \). The value of \( Z \) is expressed in mm\(^6\)m\(^{-3}\). Because \( Z \) commonly encountered in weather observations span many orders of magnitude, radar meteorologists use a logarithmic scale that indicates the ratio of a physical quantity relative to a specified or implied reference level \( Z(\text{dBZ}) = 10 \log[Z \text{ (mm}^6 \text{ m}^{-3})] \). The backscattering cross section of hydrometeors are proportional to \( D^6 \). Therefore the contribution to the echo of the small hydrometeors is negligible compared to that of the big ones. For distribution of diameter droplets we take the exponential distribution:

\[ N(D) = N_0 e^{-\Lambda D} \]  \hspace{1cm} (10)

The relation between radar reflectivity and the rainfall intensity [mm h\(^{-1}\)] is defined by the following equation:

\[ Z = a R^b \]  \hspace{1cm} (11)

where \( a \) and \( b \) are empirically determined coefficients. Just like the microstructure of clouds and precipitation on which they depend, these coefficients vary in space and time. We therefore can consider instantaneous or local values and average values. The meteorological radar Lisca uses the value of \( a = 200 \) and \( b = 1.5 \). In figure 2, horizontal and vertical cross-sections of hailstorm and a general scheme of the precipitation zone are shown.

Figure 2: Horizontal and vertical cross sections of hailstorm and general scheme of precipitation zone: H is hail, H/R is hail-rain mixture, Rh is heavy rain, Rm is moderate rain, Rl is light rain [6].

5 Radar-based hail detection algorithm

5.1 Zmax

Zmax method uses the maximum radar reflectivity in a vertical column (\( Z_{\text{max}} \)). Zmax method as a hail warning product is present in the Rainbow processing software of Gematronic radars. The Mason criterion is one of the simplest approaches. It predicts hail on the ground if \( Z \geq 55 \text{ dBZ} \). Because the radar reflectivity increases substantially with the diameter of the hydrometeors (\( Z \sim D^6 \)), larger hail produce reflectivity values that cannot be reached by large raindrops. For a successful hail detection the most important fact is that the size of droplets has a limit, because the larger droplets split when falling into smaller raindrops. Radar reflectivity above 55 dBZ can therefore only be caused
by precipitation elements that are larger than the maximum possible raindrops - these are wetted ice grains [7].

5.2 VIL

The use of the entity "Vertically Integrated Liquid water" (VIL) as a new analysis tool has been introduced by Greene and Clark in 1972, and they anticipated that this technique would be useful for both severe storm and hydrological applications. Empirically VIL is a nonlinear function of reflectivity, and converts radar reflectivity data into an equivalent liquid water content value based on studies of a drop-size distribution and empirical studies of a reflectivity factor and liquid water content. The first step in the calculation of VIL is to convert all reflectivities to liquid water content (M) using the semi-empirical relation between M in g/m$^3$ and Z in mm$^6$/m$^3$:

$$M = 3.44 \times 10^{-3} Z^{7/4}$$  \hspace{1cm} (12)

Subsequently, the obtained liquid water content at each location is integrated vertically:

$$\text{VIL} \equiv \int_0^{H_{top}} M \cdot dh = 3.44 \times 10^{-3} \int_0^{H_{top}} Z^{7/4} dh$$  \hspace{1cm} (13)

where the VIL is expressed in kg/m$^2$ or in mm of "potential rainfall" and the height in km. The three-dimensional radar data is thus converted to a plan-position indicator of the amount of liquid water present in a vertical column above a certain position. Because the rate of precipitation formation is roughly proportional to the updraft velocity, VIL is a function of both updraft and cloud depth. A height value of VIL correlates well with the occurrence of severe thunderstorms and hail. In stratiform situations VIL rarely exceeds a value of 10 kg/m$^2$, in thunderstorms, however, VIL is usually much higher (values of $\geq 25$ kg/m$^2$). There is, however, no agreement in literature on the best warning threshold for the detection of hail with the VIL method [3].

5.3 VIL-density

In an attempt to eliminate the problems with thresholds for VIL-based hail warnings, Amburn and Wolf have proposed to "normalize" the VIL value using echotop heights of a certain reflectivity threshold, for instance 7 dBZ [11]. This would capture the observation that some high-topped thunderstorms do not produce hail and some low-topped thunderstorms with low VIL values do produce hail. The "VIL-density" is defined as follows:

$$\text{VIL density} = \frac{\text{VIL}}{H_{top}}$$

where the VIL-density will be in g/m$^3$ when VIL is given in kg/m$^2$ and $H_{top}$ in km. Amburn and Wolf have proposed a universal VIL-density threshold of 3.5 g/m$^3$ for issuing hail warnings. Unfortunately, the advantage of the use of this "universal" VIL-density threshold over just VIL is disputed. Edvards and Thompson (1998) note that the use of a warning threshold of 38 g/m$^2$ for VIL on the data of Amburn and Wolf would result in the same performance as the use of the VIL-density threshold. In addition, they note that for VIL values exceeding 43 kg/m$^2$ hail is always observed independent of echotop height. Currently, both the severe hail index (SHI) and the VIL are used by forecasters in the United States to detect (severe) hail [3].

5.4 Waldvogel

Waldvogel investigated and verified radar-based hail detection criteria over Switzerland. They presented a methodology, which considers the vertical distance between the melting level $H_{0gC}$ and a
certain upper-level reflectivity, originally the 45 dBZ reflectivity $H_{45 \text{dBZ}}$. If the vertical distance is greater or equal to 1.4 km, then the presence of hail is likely:

$$\Delta H_{45} = H_{45 \text{dBZ}} - H_{0^\circ C} \geq 1.4 \text{ km.}$$

In this approach, $\Delta H_{45}$ serves as a proxy for the vertical extent of the zone where hail may grow in cumulus clouds by riming, even if the most significant growth zone is between $-5$ and $-15 \, ^\circ C$ [8].

The Waldvogel technique was originally examined with data from X-band radar. For the use of C-band radar data later some authors varied the two thresholds of 45 dBZ and $\Delta H_{45}$ and evaluated the results with the verifications data. Witt et al. [10] developed a Hail Detection Algorithm (HDA) for the network of WSR-88D radars in the United States. The HDA contains two separate components, one for detecting hail of any size and one for detecting severe hail. The first part of the HDA is based on the Waldvogel criteria. The maximum height of the 45 dBZ reflectivity above the zero isotherm level is converted into a probability of hail of any size. Height differences of 1.6 km and 5.5 km correspond to probabilities of 0% and 100%, respectively. The HDA is used to determine the probability of a hail occurrence. The zero isotherm height is identified from a nearby aerological sounding. Figure 3 shows the probability curve of hail at the ground as a function of height difference between $0^\circ C$ isotherm and top of the $>= 45 \text{ dBZ}$ reflectivity.

![Figure 3: Probability of hail at the ground as a function of ($H_{45} - H_0$). Here $H_{45}$ is the height of the 45-dBZ echo level and $H_0$ is the height of the melting level [10].](image)

### 5.5 POSH

The Probability of Severe Hail (POSH) according to Witt et al. [10] is estimated from the energy flux of hail kinetic energy $\dot{E}$ defined by:

$$\dot{E} = 5 \cdot 10^{-6} \cdot 10^{0.084 Z(dBZ^{-1})} \cdot W_1(Z)$$

(14)

here $Z$ is reflectivity in dBZ, $\dot{E}$ in Jm$^{-2}$s. The weighting function $W_1(Z)$ depends on the levels of the 40 and 50 dBZ reflectivity and it can be used to define a transition zone between rain and hail reflectivities. Default values of levels cannot be treated as fixed. $\dot{E}$ is closely related to the potential damage of hail at the ground. Vertical integration of the cell’s reflectivity profile from the melting level $H_0 = 0^\circ C$ to the echo top height according to equation 14 yields a parameter referred to as Severe Hail Index (SHI):
\[ SHI = 0.1 \int_{H_0}^{H_{\text{top}}} W_2(H) \dot{E} dH, \quad (15) \]

where \( H_{\text{top}} \) is the height of the top of the storm cell. In this formulation, a second weighted function \( W_2(T) \) is considered, which is a function of the actual height \( H \) relative to the 0 and -20 °C levels. During the initially testing of SHI, they found that the SHI values are close to 300 Jm\(^{-1}\)s\(^{-1}\) for hail with diameters of 19 mm and greater. Finally, POSH is estimated from \( H_0 \) and SHI:

\[ POSH = 29 \ln \left( \frac{SHI}{57.5 \cdot H_0 - 121} \right) + 50 \quad (16) \]

This empirical equation estimates the probability of hail in percent. Negative values are set to zero, values in excess of 100% are truncated to 100. From the evaluation of the case studies, Kunz et. al. [8] found the best prediction skill for a threshold of 80%, which is further used in the study. While a lower threshold unrealistically increases the area where hail is predicted higher values underestimate the hail probability.

### 6 Verification scores

In the verification process (the comparison between the outcomes of the different hail detections methods and the verification data), the hail events will be classified using a 2-by-2 contingency table. Hail detected by a radar-based method which is confirmed by the verification data will be classified as a hit (H), hail detected by a radar-based method which is not confirmed by verification data as a false alarm (F), hail observations or reports in the verification data that are not detected by the radar-based method as a miss (M), and no event at all as a non-event (N). These four classes can be shown schematically in the 2-by-2 contingency table:

<table>
<thead>
<tr>
<th></th>
<th>Hail</th>
<th>No hail</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radar detection</td>
<td>H</td>
<td>F</td>
</tr>
<tr>
<td>No radar detection</td>
<td>M</td>
<td>N</td>
</tr>
</tbody>
</table>

Table 3: 2-by-2 contingency table for verification radar detection algorithm scores.

In the case of rare events, like summer hail, the number of "non-events" N will be the largest by far. The dominance of the none events in the case of rare events can be circumvented by the use of verification scores that do not include N. The Probability Of Detection (POD), the False Alarm Ratio (FAR) and the Critical Success Index (CSI) are defined as:

\[ POD = \frac{H}{H + M} \quad (17) \]
\[ FAR = \frac{F}{H + F} \quad (18) \]
\[ CSI = \frac{H}{H + M + F} = \left[ \frac{1}{POD} + \frac{1}{1 - FAR} - 1 \right]^{-1} \quad (19) \]

The probability of detection and the false alarm ratio always have to be used together to characterize
the result of a verification, where the method with a high POD and a low FAR is preferred, while the critical success index characterizes the verification result in a single number, where the method with the highest CSI is preferred [3].

On the basis of these indexes we can evaluate and compare the effectiveness of radar based hail detection algorithms with each other and optimize empirical factors of each method.

6.1 Overview: the results of detection algorithm

This section presents the results of the verifications of detection algorithms which are conducted by three different studies in Europe. Each distinct study dealt with a variety of detection methods, for all were common methods Waldvogel, Zmax and POSH:

- **Germany**: Study, where tested five different hail detection criteria that are applied to 2D and 3D reflectivity from C-band radar over southwest Germany between 1997 and 2011. Verification based on loss data provided by a building insurance company. Best results are obtained for the Hail Detection Algorithm (HDA) and POSH, where index CSI achieved values around 0.3 and 0.25 [8].

  ![Figure 4](image)

  Figure 4: Different skill scores obtained from the different hail criteria. The Waldvogel criterion is based on three different thresholds: $\Delta H_{45 \text{ dBZ}} > 4.5 \text{ km}$ (WV 45 dBZ), $\Delta H_{50 \text{ dBZ}} > 3.0 \text{ km}$ (WV 50 dBZ) and $\Delta H_{55 \text{ dBZ}} > 2.0 \text{ km}$ (WV 55 dBZ) [8].

- **Czech Republic**: The study aimed to find the optimal threshold values for the applications of these techniques over the Czech territory and for evaluating the climatology of hail events. Seven algorithms were tested on well documented recent hail events from 2002 to 2011. The results showed that the optimized Waldvogel with 56 dBZ threshold and the NEXRAD severe hail algorithm (SHI, POSH) were the most accurate methods for hail detection over the area of interest [9].
Figure 5: The scores of POD, FAR and CSI for the tested hail detection methods as a function of the corresponding threshold value [9].

- **Netherlands**: The results of the comparison and verification of the five different hail detection methods. For this, the radar data and verification data of 15 selected days with thunderstorms during the summer of 1999 are used. For this period the Waldvogel and the severe hail algorithm SHI were the most accurate hail detection algorithms. Values of CSI index for these two algorithms exceeds 0.4 [3].

Figure 6: Detection methods as a function of the warning threshold. The scoring parameters are deduced from the comparison of the methods with the verification data [3].
7 Conclusion

Over past decades, several methods have been developed to estimate hail from single-polarization weather radars. Because each of the different hail criteria has its own features and limitations, it is unclear which one provides the most reliable and most robust results over a long-term period. The seminar presented three independent studies, where they tested five different hail detection criteria that are applied to 2D and 3D reflectivity from C-band radar for different geographical areas and different time periods. The studies aimed to find the optimal threshold values for specific geographic areas. In two of them the Waldvogel and POSH have the best verification score, but the difference are small. Verification score parameters are strongly dependent on the verification data (ground based observations and insurance loss data). Based on verification results the climatology maps of the hail frequency are made. Results show a high spatial variability of hail events.

References


