Lateral boundary errors in regional numerical weather prediction models

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Abstract

Regional models are used in many national weather services for short-range forecasts. These models are covering smaller areas with higher resolution with respect to global models. Thus, they are expected to be more accurate than global models. Unlike global models, they require boundary conditions at the lateral borders of the horizontal domain. For that purpose regional models are nested into coarser models. However, there are few problems associated with nesting. Some of them are the subject of this seminar.
1 Introduction

Equations for numerical weather prediction (NWP) are mathematical representation of physical processes in the atmosphere. They include equations for the conservation of momentum, mass and energy and equation of state for perfect gases [1]:

1. Conservation of momentum or equation of motion

\[
\frac{d\vec{v}}{dt} = -\frac{1}{\rho} \nabla p + \vec{g} - 2\vec{\Omega} \times \vec{v} + \vec{F}
\]

2. Conservation of mass or continuity equation

\[
\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{v})
\]

3. Conservation of energy or thermodynamic equation

\[
Q = C_p \frac{dT}{dt} - \frac{1}{\rho} \frac{dp}{dt}
\]

4. Conservation of water mass

\[
\frac{dq}{dt} = E + P
\]

5. Equation of state for a perfect gas

\[
p = \rho RT
\]
These are strongly nonlinear equations and can be solved only numerically. Given an initial state of wind velocity $\vec{v}$, density $\rho$, pressure $p$, temperature $T$ and water vapor mixing ratio $q$, they describe evolution of atmospheric-weather forecast. Every NWP model solves the same system of equations (1-5) but uses different methods for their numerical discretization. Furthermore, equations 1-5 describe dynamical processes. Terms $\vec{F}$, $Q$, $P$ and $E$ include physical processes, many of which have to be parametrized. The terms $\vec{g}$, $\vec{\Omega}$, $\vec{F}$, $Q$, $C_p$, $E$ and $P$, $R$ represent gravity, angular velocity of earth, frictional force, heating (per unit mass), specific heat at constant pressure, evaporation and condensation (precipitation) and gas constant for air, respectively.

Many important physical processes that cannot be explicitly resolved by model so they are parameterized. These are processes that occur at a molecular scale, i.e., condensation, evaporation, friction, radiation and processes from a few centimeters to the size of the model grid, i.e., turbulent motions. Because they affect the large-scale processes, they are parameterized - their effect is formulated in terms of the resolved fields. Model’s equations are applied on a specific domain, discretized over a grid and numerically solved. Initial and boundary conditions are needed for integration. The solutions are prognostic fields of variables such as pressure, wind, temperature and moisture. Other useful meteorological quantities are derived from these prognostic variables. For NWP, two types of models are in use: global models covering the whole earth and regional models (limited area models) covering smaller areas.

Global NWP models are used for large-scale (synoptic) motions with horizontal length scale of the order of $10^3$ km (planetary waves, cyclons and anticyclons). If we are interested in weather forecast for a smaller area and small-scale processes, resolution of a large-scale model is not adequate. Since refining resolution of global models is computationally very demanding, new solutions have to be found. In 1971 the first limited area model was introduced [2]. Limited area models (LAMs) are nested within host models, which are often global, hemispheric or just larger-domain models. For that reason LAMs are also called nested models. Figure 1a shows general example of such nesting and Fig. 1b shows the domain of Slovenian numerical weather prediction model. Unlike large-scale models, LAMs have finer grid with typical resolution of 1-10 km. The need to have a fine grid in a limited area model in numerical forecast is very important since some significant weather phenomena occur on meso- and small-scales. Local phenomena are produced by the fine structured topography, the land-sea interaction and land-surface characteristics. LAMS, thus, represent fine scale physical and dynamical processes thanks to their high resolution.

Beside initial, top and bottom boundary conditions LAMs need one more thing - lateral boundary conditions (LBCs). LBCs for LAMs are obtained from
larger-domain models and provide regional models with information from outside of their domains (it is very important for LAM to know if some significant atmospheric structure is approaching, e.g., a cyclone). The problem is, how to combine the solutions from various grids. This is known as coupling. There are number of ways to formulate LBCs for LAMs, one of them will be presented in section 2.1.1. None of these formulations is perfect, so we are dealing with error structures that propagate from the boundary inward and spurious wave reflections which are traveling inside interior grid and may influence LAM's forecast. It is important to formulate boundary conditions in the best possible way and to understand these errors that occur.

2 Boundary error generation

When we define LAM's domain, we actually extract sub-volume of the global atmosphere. It is expected that flow into and out of the LAM's domain passes freely without being changed. However, lateral boundaries of domain are numerical and cannot be perfectly determined. Consequently, flow through these boundaries is changed. On the other hand, boundaries should be able to assure free propagation of outgoing waves and fine-scale meteorological flow that are generated inside the LAM. Otherwise waves will reflect back into the domain and make noise in a forecast. Boundary errors have two sources: the boundary conditions formulation and inaccurate data specification at the boundary which are discussed in sections 2.1 and 2.2, respectively.

Beside boundary errors there are other sources of forecast error, i.e., initial conditions, numerical algorithms, surface forcing and physical-process parameteri-
zation [5]. In general, the boundary error is much smaller than the total forecast error [6].

2.1 Formulation of lateral boundary conditions

If the interaction between the nested and host model is unidirectional, i.e., from host to nested model, that type of interaction is called one-way nesting. Host and nested models are calculating partial differential equations independently on their grids over a period of time. Boundary conditions for inner domain are taken from the solution (forecast) of outer domain. The problem occurs on mutual grid points, where the solution is not unique (ill-posed problem). Over-specification results in unstable integration and false nonphysical reflections into the inner domain. These errors start to grow at the boundary and propagate towards the interior. Boundary problem deals with formulating boundaries so that errors are minimized.

There are two main types of schemes with which we are trying to avoid over-specification - those which are operating only with the boundary grid points and those which are applied within a boundary zone (which consists of the boundary grid points and next few neighboring points).

The first method is pseudo-radiation boundary scheme shown in Fig. 2a. Large-scale values (red points) from the host model are only specified on the boundary grid points if the flow is directed into the inner domain. If it moves out, boundary values are taken from the nested model. The idea is to allow interior disturbances to pass through the boundary, if directed outward, and thus not to allow reflection at the interface. These scheme causes minimal over-specification which together with errors in the estimated phase speed of the flow contribute to partial reflection of outward propagating waves [7].

The second approach are schemes where boundary zone inside the inner domain is created as shown in Fig. 2b. Boundary zone is used to couple the prognostic values from nested model (blue points) with the large-scale values from host model (red points). Coupled values (green points) are interpolated between the values on both edges of the boundary zone. All variables are specified at all boundary points as they are taken from the solution of the larger-scale model. These values are horizontaly interpolated on the remaining boundary points of the nested model (black circles between red points). Nested model is receiving information from the host model every few hours and that values are than also interpolated in time. Over-specification in this case is treated by effectively damping these errors (noise) in the boundary zone. Coupling is applied on the boundary zone every time step just after the forecast calculation in the inner
domain is finished.
There are few types of coupling that are used: diffusive damping scheme, tendency modification scheme and flow relaxation scheme. The latter one is the most widely used so we will describe this scheme.

2.1.1 Flow relaxation scheme

This subsection follows [1]. The behavior of this scheme will be shown for a simple flow system. Motion of the one-dimensional non-rotating, inviscid and shallow fluid:

\[
\frac{\partial u}{\partial t} + c \frac{\partial u}{\partial x} = 0.
\]  

Equation (1) describes 1D linear advection, where \( u = u(x,t) \) is a zonal velocity component, \( c \) is a nonzero constant speed of passive advection of \( u \). Equation (1) is modified by adding a relaxation term over a boundary zone:

\[
\frac{\partial u}{\partial t} + c \frac{\partial u}{\partial x} = -K(u - \overline{u})_{BZ},
\]  

where \( K = K(x) \) is the relaxation coefficient that is positive and nonzero only in the boundary zone (subscript BZ) and \( \overline{u} \) is the externally specified field (from host model). Relaxation term continuously reduces the difference between the nested and the host model’s values in the boundary zone by adding a damping. Since the host model follows the wave equation:

\[
\frac{\partial \overline{u}}{\partial t} + c \frac{\partial \overline{u}}{\partial x} = 0,
\]  

the "error" equation (the difference between the nested value of \( u \) and the host model value of \( \overline{u} \)) takes the form:
\[
\frac{\partial u'}{\partial t} + c \frac{\partial u'}{\partial x} = -Ku'.
\] (4)

where \( u' \) represents error. According to Eq.(4), the error is advected from or toward the boundary and damped when it reaches a boundary zone as determined by the value of \( K \). If \( K \) increases abruptly, spurious reflections of outgoing waves are produced. Thus, this coefficient has to be a smoothly varying function.

When Eq.(2) is discretized and arranged, the updated relaxed solution for nested model (green points in Fig.2b) in any time step \( n + 1 \) is obtained:

\[
u^{n+1} = (1 - \alpha)u_i^{n+1} + \alpha\bar{u}_{n+1}.
\] (5)

In Eq.(5) \( \bar{u}^{n+1} \) represents the solution of the host model (red points in Fig.2b) and \( u_i^{n+1} \) represents the solution of the nested model (blue points in Fig.2b) before it was relaxed toward the host solution. \( \alpha = 2K\Delta t \) is a weighting coefficient whose value is one on the boundary and zero for all points beyond the boundary zone. Thus, boundary zone is an area where prognostic values of host model and nested model are combined (green points in Fig.2b) according to \( \alpha \). Some of \( \alpha \) functions that are used are shown in Fig.3.

This method does not prevent spurious reflections at the boundary. However, with careful choice of the boundary zone width and the relaxation coefficient this unwanted effects can be reduced. Thus, if \( K \) increases abruptly, it will induce reflection. On the other hand, too small value of \( K \) will not damp the outgoing waves adequately and a reflection from the boundary interface can be expected [7].

**Figure 3:** Examples of functions that are used for relaxation. Here the five-points width boundary zone is used. Index \( j \) represents boundary zone points.
2.2 Inaccurate data specification at the boundary

Factors that influence the quality of the lateral boundary conditions imposed by the host model [5]:

- LBC values are obtained from models with the coarser horizontal, vertical and temporal resolution. Thus, the information has to be interpolated temporally and spatially. This was explained in 2.1.
- The coarser forecast that provides boundary values may be wrong.
- Nested models are usually using more advanced physical-process parameterizations.
- Fast inertia-gravity modes may occur that are numerically and not physically based. They could make nested model forecast difficult for interpretation.

3 Examples of the LBC error

In this section two examples of influence of LBCs will be presented. In the first example the resolution of the host model which provides LBCs will be changed and in the second example size of the nested domain will be changed. These two cases are taken from [6]. The nested model that was used had size of 90° longitude by 75° latitude domain centered over central USA. Nested model horizontal resolution was 2.5°. The boundary information was provided by 5° resolution host (hemispheric) model.

3.1 Changing the host model resolution

To isolate the first error source, they used identical model configuration for host and nested model. In first case 5° resolution was used for global model which was in second case improved to 2.5° resolution. Figure 4 shows time evolution of errors (differences between host and nested model forecast) of the 6 km pressure using 5° and 2.5° host resolution.
Figure 4: Time evolution of the 6 km pressure errors, caused by changing resolution in host model, for 12h, 24h, 36h and 48h (from left to right). Differences between forecasts of 5° host model and 2.5° nested model (top row) and between forecasts of 2.5° host model and 2.5° nested model (bottom row). Contour interval is 1 hPa, negative values are dashed, [6].

There are certain similarities in the location and the shape of errors in both cases. Large pressure errors with magnitudes of 5-10 hPa propagate rapidly, especially from the western and northern boundaries. Magnitudes of these errors are smaller in improved version of the host model (in most observed times up to 50%). Errors in both cases are growing rapidly during first day. During the second day they are being reduced and stabilized. This behavior is most likely caused by the boundary formulation, when spurious transients occur early in the forecast which are damped slowly as the forecast progresses. Another thing they have in common is inward propagation of these error structures which contaminate central area (especially in 5° case). Large error is produced near the western boundary which grows up to 7 hPa in magnitude and is propagated inward at the rate of 20° – 30° per day. In general, the stronger synoptical activities (more active boundary) are, the higher the magnitude of error is. It is not shown here, but during the host forecast on the west boundary very strong synoptic systems were entering inner domain.

3.2 Changing the boundary location

As mentioned in subsection 3.1, boundary errors are propagating inward and eventually they cover entire domain. Lateral boundaries should be far away so that our domain of interest is protected from boundary errors for short-range forecasts. 5° hemispheric and 2.5° nested model were used which were identical
except for the lateral boundaries. Initially defined inner domain was expanded for 20° of longitude in the west and east, the southern boundary was moved northwards by 10° of latitude and the northern boundary remained the same. Time evolution of the 6 km pressure differences between forecasts of host and nested model (errors) of the expanded domain are shown in Fig. 5.

![Figure 5: Time evolution of the 6km pressure errors for the expanded domain. Contour interval is 1 hPa and negative values are dashed, [6].](image)

If we compare the original domain in Fig. 5 and top row of Fig. 4, we can see that the biggest change is on the east and west boundaries, which are also the most active. Errors in the early forecast of the original domain are now reduced by the factor of 4. The west boundary error even changed the sign (due to strong influence of boundary specification). There is no error on the east boundary of the original domain during the first day of forecast (in Fig. 4 it reaches 6 hPa). The northern boundary was held fixed, so this error is similar in both cases during the first day of the forecast. Errors are growing during the forecast in both domains contaminating them. In the expanded domain it takes longer time for errors to expand over the center. Propagation speed of error in this case is similar to that from the previous case.

Expanding the inner domain can significantly reduce the error. Domain must be large enough to protect the main region of interest from boundary effects. The boundaries are set with respect to typical error propagation speed and the forecast duration [6].
4 Conclusion

Quality of the weather forecast depends on the numerical prognostic model that is used (model numerics, dynamics and physics) and quality of the lateral boundary conditions that limited area models require. The boundary conditions should be specified so that rapid amplification of fast spurious waves near the boundary is prevented. These waves are propagating inward and could ruin the calculation for the whole area. When designing a boundary strategy, it is essential to appropriately choose three factors discussed in this seminar; Firstly, boundary formulation should be well tested and known, so that it does not generate fast propagating waves with significant amplitude. An appropriate boundary formulation can also overcome overspecification problem. Secondly, domain size needs to be long enough to ‘protect’ the area of interest. Error structures that boundary zone did not damp need some time to propagate inward and to contaminate the area of interest. Finally, host and nested model similarities would do well (here we discussed a host model resolution, but this is also valid for other factors).

References


