Abstract

We present how the fact of particles having masses works with other theoretical assumptions of particle physics. As it turns out, some experimental observations concerning the weak interaction come into conflict with relativistic way of how to theoretically give masses to particles. In order to properly understand this conflict, we present distinction between left- and right-handed fermions, while studying Lorenz covariance. Striving towards the Standard model we present the three interactions included in it through gauge theory with particular emphasis on the weak interaction. Seeing that mass cannot arise in way previously thought, we conclude that a new mechanism is required. We resolve the mass problem with inclusion of a scalar particle, and present some consequences that this inclusion has. This approach is now called the Higgs mechanism. Having all ingredients of the Standard model, we introduce the Standard model as a holistic theory of particle physics. We conclude the text with experiments that take the presented mass generation mechanism under test.
1 Introduction

The aim of this seminar is to present the Standard model of particle physics, and to understand why it is
the way it is. The answer to this question has a lot to do with the masses of particles. Mass is an everyday
property of some particles, and experimentally well established. But studying phenomena associated with weak
interaction, the experiments showed some logical inconsistencies in the theory [1]. It was realized that, if we
were to include all of this experimental facts, the mass of particles cannot arise in a way previously thought [2].
We will present the properties of the Standard model step by step, and its logical justifications. We will avoid
historical point of view and will rather try to understand the structure of the Standard model in a modern
way. We intend to present the logic behind the Standard model and to qualitatively assess the properties that
follow. We will present each assumption of the Standard model separately and than state it as a whole. The
behavior of the Standard model that we wish to present can be understood on semi-classical level i.e. in the
language of first quantization. This means that this text is about classical field theory and not quantum field
theory. When the problem with mass terms will arise, we will resolve it using, what is now known as, the
Higgs mechanism. We will study the impact of this mechanism to the Standard model and show that masses
of fermions and weak bosons can be a result of interaction with the scalar field that the Higgs mechanism
postulates. The experiments that address the correctness of this approach of mass generation will be briefly
presented with the relevant findings.

1.1 Notation

We use, the so called, natural units ℏ = c = 1. Coordinates are labeled x₀ = t, x¹ = x, x² = y and x³ = z as
it is customary in relativistic theories. So xµ is a space-time coordinate, where µ = 0, 1, 2, 3. We use Einstein
summation convention a♭b♭ = ∑♭a♭b♭ because it reduces the number of factors and terms in our expressions.
We also use compact notation for partial derivatives ∂µ = ∂/∂xµ and ∂µ = ∂/∂xµ, or more expressively
represented as matrices

\[ \partial_\mu = \begin{bmatrix} \frac{\partial}{\partial t}, & \frac{\partial}{\partial x^1}, & \frac{\partial}{\partial y}, & \frac{\partial}{\partial z} \end{bmatrix}^T \]

\[ \partial^\mu = \begin{bmatrix} \frac{\partial}{\partial t}, & -\frac{\partial}{\partial x^1}, & -\frac{\partial}{\partial y}, & -\frac{\partial}{\partial z} \end{bmatrix}. \]

Here we hinted another way of looking at Einstein summation convention where we can, for example, look at
∂µ∂₀ = ∂²/∂t² − ∇² as row and column multiplication, and so we get a distinct interpretation what subscripted
and superscripted indices mean. We also use another abbreviation that reduces the number of terms. The
(h.c.) in our expressions means Hermitian conjugate (-)† of all preceding terms. This is also a convenient way
to ensure hermiticity, since (A + A†) is always Hermitian. For example

\[ \mathcal{L} = \psi^\dagger i\partial_\mu \psi + (h.c.) = \psi^\dagger i\partial_\mu \psi + ((\psi^\dagger)(i\partial_\mu \psi))^\dagger \]

\[ = \psi^\dagger i\partial_\mu \psi + (i\partial_\mu \psi)(\psi^\dagger)^\dagger = \psi^\dagger i\partial_\mu \psi - \psi i\partial_\mu \psi^\dagger. \]

2 Lorentz covariance

One basic property that we require of our model of particle physics is Lorentz covariance. That means that if
we have a solution of our equations and we Lorentz transform that solution we get another legitimate solution
of our equations. So we can rotate a solution or perform a boost on that solution and the transformed solution
is still a solution of our equations. This property makes our model relativistic, which is crucial in high energy
physics.
It is an experimental fact that particles have spin, and this property has profound consequences, so spin should not be ignored. So the state of a particle, and so a solution of equations of our model, is defined as spinor valued wave function \( \psi(x^\mu) \). In case of a particle with spin \( s = 1/2 \) this means that \( \psi : x^\mu \rightarrow s_1(x^\mu)|\uparrow\rangle + s_2(x^\mu)|\downarrow\rangle \). We will now consider a possible Lorentz covariant model for spin \( s = 1/2 \) particle. Massive particles with spin \( s = 1/2 \) are in nature probably the most noticeable, so we start with these.

In case of \( s = 1/2 \) the spinor space is two dimensional. Lorentz transformations (SO(1,3)) fundamentally act on four dimensional vectors like \( \partial_\mu \) and we know, from special relativity, how to transform four-vectors. But now we have two dimensional \( s = 1/2 \) spinor space and we want to know how Lorentz transformations transform \( 1/2 \)-spinors so that we can construct relativistic equations for such spinors. Lorentz transformation of four-vector \( \partial_\mu \rightarrow \partial'_\mu \) is unique, but two possibilities exists how Lorentz transformations can consistently act on two dimensional spinor space. We call one of these possibilities left-handed and the other right-handed [3].

Let us take a look how Lorentz transformations of left-handed and right-handed spinors differ. If \( J_i \) are the generators of rotations and \( K_i \) the generators of boosts, the two possible \( 1/2 \)-spinors (left- and right-handed) transform as

\[
J_i \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}_{L,R} = \sigma^i \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}_{L,R} \quad K_i \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}_{L} = -i\sigma^i \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}_{L} \quad K_i \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}_{R} = i\sigma^i \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}_{R},
\]

where \( \sigma^{i=1,2,3} \) are Pauli matrices and subscripts \( L \) and \( R \) label left- and right-handed spinor. If there are two ways how to Lorentz transform a \( 1/2 \)-spinor, we must include both types of spinors into our model, left- and right-handed. So forcing our model for \( 1/2 \)-spinor to be Lorentz covariant results to two types of \( 1/2 \)-spinors in our model.

So a general \( 1/2 \)-spinor\(^1\) is a sum of its left-handed and right-handed components \( \psi = \psi_L + \psi_R \) and each component transforms differently under Lorentz transformations. With this in mind, it is possible to derive a first order relativistic equation, called Dirac equation, for \( 1/2 \)-spinor function \( \psi \).

\[
\begin{align*}
  \sigma^\mu \partial_\mu \psi_L &= m\psi_L & \mu = 0, 1, 2, 3 \text{ and } \sigma^0 = I_{2 \times 2} \\
  \sigma^\mu \partial_\mu \psi_R &= m\psi_R \\

\end{align*}
\]

This equation is Lorentz covariant by construction,\(^2\) so Lorentz transformed solution is also a solution. The \( \psi \) represents a wave function of noninteracting relativistic fermion with spin \( s = 1/2 \) and mass \( m \). This is the only first order linear equation for \( 1/2 \)-spinor that is Lorentz covariant [3].

Because we intend to add new features to this model (like interactions), it is wise to use Lagrange formalism, so that new features can be presented as new terms in Lagrange density \( \mathcal{L} \). The above Dirac equation and its Hermite conjugated version is equivalent to Euler-Lagrange equations

\[
\begin{align*}
  \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \psi_R} &= \frac{\partial \mathcal{L}}{\partial \psi_R} \\
  \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \psi_L} &= \frac{\partial \mathcal{L}}{\partial \psi_L} \\

\end{align*}
\]

where we treat \( \psi \) and \( \psi^\dagger \) as independent using the following Lagrange density

\[
\mathcal{L} = \psi^\dagger \sigma^\mu \partial_\mu \psi_L + \psi^\dagger \sigma^\mu \partial_\mu \psi_R - m\psi^\dagger \psi_R - m\psi^\dagger \psi_L + (\text{h.c.})
\]

The first two terms in \( \mathcal{L} \) are dynamical terms and they allow propagation of the particle. The other two terms are mass terms that makes a particle massive. Now we have a basic \( \mathcal{L} \) for massive spin \( s = 1/2 \) relativistic particle. The idea of the following chapters is to add other terms to the \( \mathcal{L} \) so that solutions match the observations in nature.

### 3 Gauge covariance

The most obvious deficiency of \( \mathcal{L} \) from previous chapter is lack of interactions. In nature we can observe some forces between particles and we would like to include these observed interactions as new terms in our \( \mathcal{L} \). It is possible to present interactions into our model as a side effect of some geometrical considerations. Gauge theory is the framework that achieves this [3].

Problem with mass is associated with parity violation in weak interaction processes [2]. In order to properly understand this, it is necessary to present weak gauge theory. We will present the notions of gauge theory on the example of strong interaction. This is because we want to present the idea of gauge theory before we continue to some unusual behavior concerning the weak interaction. Learning gauge theory on an example is pedagogical and it helps us to acquaint ourselves with all interactions contained in the Standard model.

\(^1\)A bispinor \( \psi = \psi_L \oplus \psi_R = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}_{L,R} \oplus \begin{bmatrix} s_3 \\ s_4 \end{bmatrix}_{L,R} \) with four components

\(^2\)The \( \partial_\mu \) transforms under \( (1/2, 1/2) \) representation of the Lorentz group and \( \psi_L \) transforms under \( (1/2, 0) \) representation so the components of \( \partial_\mu \psi_L \) transforms under \( (1/2, 1/2) \oplus (1/2, 0) = (0, 1/2) \oplus \ldots \) and this contains the representation \( (0, 1/2) \) which is how \( \psi_R \) transforms under Lorentz transformations, so the Pauli matrices in Dirac equation are just Clebsch-Gordan coefficients of this decomposition of representations [3]
3.1 Gauge theory and strong interaction

Experiments show that strong force has three independent charges [4]. We usually call them red, blue and green charge. In order to attribute some charge to a particle, it is prudent to introduce some internal charge space. Let us say we have a particle that has a charge red = 2, blue = −1 and green = 1/3. Now we can construct a three dimensional vector out of this and say that the charge of that particle is \([2 \quad -1 \quad 1/3]^T\). So the strong charge of a particle is a vector in vector space spanned by the following three dimensional basis

- red = \([1 \quad 0 \quad 0]^T\)
- blue = \([0 \quad 1 \quad 0]^T\)
- green = \([0 \quad 0 \quad 1]^T\).

This is an internal vector space and has nothing to do with physical space-time. Each point of space-time has its own charge space that specifies the charge in that region. Direction of a single vector in a charge space has no particular meaning. What matters is where one vector is pointing with respect to the others. So, in principle, there is no preferred way where for instance red is pointing at. So the choice of this basis is completely arbitrary. This observation is of fundamental importance in gauge theory.

Particle in quantum mechanics is just a field excitation. Its emergence is local. We might have two particles (and so two areas of space-time) and each of them has its own internal charge space with its own basis since the choice of basis is arbitrary. If we want to have some interaction between them as a result of their charges, we must have a way to compare these two charge spaces. We need to glue these two charge spaces together in order to compare charge in one region with the charge in the other region of space-time. One way is to smoothly change the basis of the charge space from point to point, from one region to the other, and so connect in every point of space-time, than the transformed solution is also a legitimate solution. We call this property local gauge covariance and it has strong consequences if we apply it to our model.

How do these smooth changes of basis affect our future model equations? Well, they should not. If charge space basis is pointwise arbitrary, than our model should be covariant to the smooth change of basis of the charge space. That means that if we have solution and we smoothly change the charge space basis differently in every point of space-time, than the transformed solution is also a legitimate solution. We call this property local gauge covariance and it has strong consequences if we apply it to our model.

We introduce a matrix valued function \(g(x^\mu)\) that smoothly changes the basis of the charge space. If the former considerations are valid, we can smoothly change the basis of the charge space using this arbitrary \(g(x^\mu)\) and our model equations should not change. Particles that carry a charge are now represented by spinor and vector valued wave functions\(^{iii}\) where the value of the vector specifies the charge of the particle. So we can have a wave function

\[
\psi = \varphi(x^\mu) \begin{pmatrix} \text{charge}_1(x^\mu) \\ \text{charge}_2(x^\mu) \\ \text{charge}_3(x^\mu) \end{pmatrix}
\]

and this kind of smooth basis transformations, and so transformations of \(\psi\), should not change our model equations. This means that if \(\psi\) is a solution the \(g \cdot \psi\) is also a solution.

Matrices \(g(x^\mu)\) should preserve the values of scalar products \(\langle \psi_1 | \psi_2 \rangle\), and thus probability amplitudes, so they have to be unitary \(g^\dagger \cdot g = I_{3 \times 3}\). We would also like \(g\) to be orientation preserving and so \(\det(g) = 1\). The group of all matrices with such properties is in mathematics called SU(3). The S stands for special \((\det(g) = 1)\), the U stands for unitary \((g^\dagger \cdot g = I)\) and 3 is the dimension of vector space on which \(g\) acts.

We will now force \(L\) from the previous chapter into being gauge covariant. The mass terms are already gauge covariant

\[
-m(\psi^\dagger)^\prime \psi = -m(g \cdot \psi^\dagger)^\prime (g \cdot \psi) = -m(\psi^\dagger \cdot g^\dagger)g \cdot \psi = -m \psi^\dagger (g^\dagger \cdot g) \psi = -m \psi^\dagger \psi,
\]

\(^{iii}\) \(\psi = \varphi(x^\mu) \begin{pmatrix} \text{spin}_1 \\ \text{spin}_2 \end{pmatrix} (x^\mu) \otimes \begin{pmatrix} \text{charge}_1 \\ \text{charge}_2 \\ \text{charge}_3 \end{pmatrix} \) with \(2 \times 3 = 6\) components
but dynamical terms $\psi^i i\sigma^\mu \partial_\mu \psi$ are not because $\partial_\mu (g \cdot \psi) \neq g \cdot \partial_\mu \psi$ since $g = g(x^\mu)$. So terms like $\psi^i \partial_\mu \psi$ cannot be gauge covariant because of the derivative. The covariance can be achieved by refining the derivative $\partial_\mu \rightarrow \partial_\mu + A_\mu$, so that the term that breaks the gauge covariance can be absorbed by $A_\mu$.

$$g \cdot \partial_\mu \psi = \partial_\mu (g \cdot \psi) - (\partial_\mu g) \cdot g^I \cdot (g \cdot \psi) = (\partial_\mu - (\partial_\mu g) \cdot g^I) \psi'$$

not covariant

$$g \cdot (\partial_\mu + A_\mu) \psi = \partial_\mu (g \cdot \psi) - (\partial_\mu g) \cdot g^I \cdot (g \cdot \psi) + g \cdot A_\mu \cdot g^I \cdot (g \cdot \psi)$$

covariant

$$= (\partial_\mu + g \cdot A_\mu \cdot g^I - (\partial_\mu g) \cdot g^I) \psi'$$

$$A'_\mu = g \cdot A_\mu \cdot g^I - (\partial_\mu g) \cdot g^I$$

So if we force $A_\mu$ to transform under gauge transformation as we have written in the last equation, our $\mathcal{L}$ can be made gauge covariant by replacing $\partial_\mu \rightarrow \partial_\mu + A_\mu$. So by forcing our $\mathcal{L}$ to be gauge covariant, we introduced a new object $A_\mu$ to our model. $A_\mu$ can be viewed as a wave function that must transform under Lorentz transformations in the same way $\partial_\mu$ does, which means that $A_\mu$ has spin $s = 1$ and so represents a boson particle.

We can look at $A_\mu$ inclusion in different ways. One way is that in order to obtain gauge covariance we need to add some object that would absorb the anomaly left behind by $\partial_\mu$ during the gauge transformation. So we add $\psi^i i\sigma^\mu A_\mu \psi$ term in $\mathcal{L}$ along with transformation rule $A'_\mu = g \cdot A_\mu \cdot g^I - (\partial_\mu g) \cdot g^I$ and this makes $\mathcal{L}$ gauge covariant.

More mathematical way of looking at this is by seeing the fact that derivation $\partial_\mu$ is not adequate for derivation in charge space which is nontrivially glued together. This is because $\partial_\mu$ does not have any information about changing of the basis of charge space. In such case mathematics introduces covariant derivative $D_\mu$ that is a sum of space-time derivative $\partial_\mu$ and a term $A_\mu$ that tells how charge spaces in different regions are glued together. So adding $A_\mu$ term is just a proper way to derive in gauge covariant charge space and gauge covariance is just a result of using covariant derivative $D_\mu$ instead of only space-time derivative $\partial_\mu$ [3].

$$D_\mu = \partial_\mu + A_\mu$$

$$\psi^i i\sigma^\mu D_\mu \psi = \psi^i i\sigma^\mu \partial_\mu \psi + \psi^i i\sigma^\mu A_\mu \psi$$

Of course the $A_\mu$ in the covariant derivative has the same gauge transformation rule that we previously calculated since both presented ways are equivalent. Incidentally that transformation rule looks similar to $A'_\mu = A_\mu + \partial_\mu g$ from electromagnetism which identifies $A_\mu$ as an interaction potential. That is also evident from the $\psi^i i\sigma^\mu A_\mu \psi$ term which results to a coupling between $\psi$ and $A_\mu$ and is analogous to Lorentz force from electromagnetism.

So by $\psi^i i\sigma^\mu A_\mu \psi$ term inclusion, we introduced an interaction potential, and $\psi$ can feel a force as a result of this term. But this $A_\mu$ is an arbitrary function (external field), and $A_\mu$ cannot propagate so we still need to add a dynamical term for $A_\mu$ that will tell us how $A_\mu$ behaves on its own. This term should also be gauge covariant.

The most widely excepted dynamical term for $A_\mu$ is Yang-Mills term $\text{tr}(F_{\mu\nu} F^{\mu\nu})$ that uses tensor $F_{\mu\nu}$ called curvature of covariant derivative $D_\mu$ and is a priori gauge covariant [3], and also Lorentz covariant.

$$F_{\mu\nu} = D_\mu D_\nu - D_\nu D_\mu = (\partial_\mu A_\nu - \partial_\nu A_\mu) + (A_\mu A_\nu - A_\nu A_\mu)$$

So the new $\mathcal{L}$ with addition of interaction term and Yang-Mills terms looks like

$$\mathcal{L} = \psi_L^i i\sigma^\mu \partial_\mu \psi_L + \psi_R^i i\sigma^\mu \partial_\mu \psi_R$$

spin 1/2 dynamical term

$$- m \psi_L^i \psi_R - m \psi_R^i \psi_L$$

spin 1/2 mass term

$$+ \psi_L^i i\sigma^\mu A_\mu \psi_L + \psi_R^i i\sigma^\mu A_\mu \psi_R$$

interaction term

$$+ (h.c.) - \frac{1}{4 g^2} \text{tr}(F_{\mu\nu} F^{\mu\nu})$$

spin 1 dynamical term

where $g_s$ is a coupling constant. The trace $\text{tr}$ is acting on matrices $A_\mu$. This $\mathcal{L}$ is Lorentz and gauge covariant, and so it transforms well under rotations, boosts and smooth changes of the basis of charge space.

In this chapter we had three dimensional charge space, and we allowed orientation preserving unitary transformations of the basis of the charge space. So the theory we got is $SU(3)$ gauge covariant. That means that interaction fields $A_\mu$ are matrices from the group $SU(3)$ i.e. $A_\mu (x^\mu) \in SU(3)$. The group $SU(3)$ is eight dimensional and so the $A_\mu$ can be decomposed into eight fields which we call gluons. Gluons are boson particles that mediate strong interaction. Because of the self interaction part of Yang-Mills term the gluons are coupled to each other and can feel the strong force [4].
3.2 Weak interaction

As in previous chapter, we can now introduce new charge spaces (one, two dimensional) and consequential gauge covariances that result to new types of interactions. If we take two dimensional weak charge space and so SU(2) gauge covariance we introduce the weak interaction [2]. Let us present weak interaction in more detail and forget the strong interaction for now.

As said before, the gauge covariance of two dimensional weak charge space results to the existence of a new component. The group SU(2), that can act on the weak charge space, is three dimensional and so the corresponding interaction potential $A_\mu$ can be decomposed as

$$A_\mu = W_\mu^1 \tau_1 + W_\mu^2 \tau_2 + W_\mu^3 \tau_3$$

where $\tau_1 = [0 1 \ 1 0] / 2$, $\tau_2 = [0 -i \ -i 0] / 2$, $\tau_3 = [i 0 \ 0 -i] / 2$.

$\tau_a$ are matrices associated with the group SU(2) and are the generators of matrices allowed to change the basis of particles.

We can write the first four lines more compactly using covariant derivative $D_\mu$:

$$\psi^\dagger A_\mu \psi = \psi^\dagger \left( \begin{array}{c} c_1 \\ c_2 \end{array} \right) \left( \begin{array}{c} \varphi^1 \\\ \varphi^2 \end{array} \right) \left( \begin{array}{c} c_2 \\ c_1 \end{array} \right)$$

and we can understand what interaction term like $\psi^\dagger A_\mu \psi$ is.

$$W_\mu = \left( \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right)$$

We can see that $\varphi_{c1}$ and $\varphi_{c2}$ can be interpreted as separate wave functions, so each component of the charge space $[c_1 \ c_2]^T$ represents a particle. From the last equation we deduce that $W_3$ field couples the same type of particles $c_1 \leftrightarrow c_1$, $c_2 \leftrightarrow c_2$ and the $W_1$ couples different types of components of weak charge space $c_1 \leftrightarrow c_2$.

So we can only interact with another $c_1$ with $W_3$ boson and $c_1$ with $c_2$ only with $W_1$ as the mediator of the weak interaction.

But there is something very peculiar about the weak interaction. Experiments show that only left-handed particles (1/2-spinors) can feel the weak force, and right-handed cannot [1]. That means that we need to separate each spinor to its left and right component so that we can write interaction terms in a proper way.

Let us construct a minimal model to account for all properties that were mentioned so far. In the minimal model we only postulate one massive particle with spin $s = 1/2$, Lorentz covariance, SU(2) covariance of the weak charge space belonging to the one particle and the experimental fact that right-handed spinors do not interact. By these assumptions, our $s = 1/2$ particle has two charge components $[c_1 \ c_2]^T$ due to SU(2) covariance and each of those charge components $c_1$ and $c_2$ have two right-left components:

$$c_{\pm} = c_{\pm L} + c_{\pm R}$$ (handedness $H = L, R$) due to Lorentz covariance and each of those has another two components $c_{\pm H} = s_{\pm H1} \uparrow + s_{\pm H2} \downarrow$ because of spin $s = 1/2$.

We have three layers of indices (weak charge, handedness and spin) and, for readability, we can omit the spin layer but we bare in mind that $\sigma$ and $\bar{\sigma}$ act on the omitted spin indices. This results to the following Lagrange density

$$\mathcal{L} = c_{\pm}^L \bar{\psi}_L \gamma^\mu \partial_\mu c_{\pm} + c_{\pm}^R \bar{\psi}_R \gamma^\mu \partial_\mu c_{\pm} + m_{c_{\pm}} c_{\pm}^L \bar{\psi}_L + m_{c_{\bar{\pm}}} \bar{c}_{\pm} \gamma^\mu \partial_\mu c_{\bar{\pm}}$$

$c_{\pm}$ dynamical term

$c_{\pm}$ dynamical term

$c_{\pm L}$ coupling term

$c_{\pm R}$ coupling term

$c_{\bar{\pm}}$ mass term

$c_{\bar{\pm}}$ mass term

$(h.c.) - \frac{1}{4g_\sigma^2} \text{tr}(F_\mu F^\mu)$

$W_\pm$ and $W_3$ dynamical term

We can write the first four lines more compactly using covariant derivative $D_\mu = \partial_\mu + A_\mu$:

$$\mathcal{L} = \left[ c_{\pm}^L \ c_{\pm}^R \right]_{L,R} \bar{\psi}_L \gamma^\mu D_\mu \left[ \begin{array}{c} c_{\pm}^L \\ c_{\pm}^R \end{array} \right] + c_{\pm}^L \bar{\psi}_L \gamma^\mu \partial_\mu c_{\pm} + c_{\pm}^R \bar{\psi}_R \gamma^\mu \partial_\mu c_{\bar{\pm}}$$

Here we let matrix valued $A_\mu$ to act on a column $[c_{\pm}^L \ c_{\pm}^R]^T$. We can clearly see that only left-handed components can be mixed by SU(2) matrices of the weak gauge group because the derivative $D_\mu$ includes $A_\mu$ that allows that term to be unchanged by the action of SU(2). Right-handed components cannot mix because the terms concerning them do not include $A_\mu$ that would compensate for such mixing. So right-handed are not
written as vectors and do not carry weak charge. The left-handed appear as doublets since they have two components and the right-handed as singlets since they have only one component.

Now consider the mass terms of the form \(-m_i \epsilon_{L}^\alpha \epsilon_{R}^\alpha\). These terms include left- and right-handed components. But according to the weak theory we can mix left-handed \(c_i\) with \(g(x^\mu) \cdot [c_1 \ c_2]_L^T\) but not the right-handed because they are not vectors in charge space. This clearly means that the mass terms are not SU(2) gauge covariant. The mass terms of Dirac type are not consistent with the weak interaction (they are forbidden). The experimental fact that weak interaction discriminates between left- and right-handed particles results in the prohibition of mass as it is defined in Dirac equation. Of course mass in some particles is also an experimental fact so we need to correct our model to include both now mentioned experimental facts in a consistent manner.

3.3 Electromagnetic interaction

We conclude the gauge theory chapter with electromagnetism, the theory that inspired gauge theory. Let us include this well known interaction into our model. Electromagnetism has only one type of charge, and so we can understand how the idea of local gauge covariance arose.

The change of basis in this case is a bit inconceivable since the basis is only one vector. We know the results in the prohibition of mass as it is defined in Dirac equation. Of course mass in some particles is also forbidden). The experimental fact that weak interaction discriminates between left- and right-handed particles results in the prohibition of mass as it is defined in Dirac equation. Of course mass in some particles is also an experimental fact so we need to correct our model to include both now mentioned experimental facts in a consistent manner.

Let us only show how \(U(1)\) gauge covariance implies gauge freedom of classical electromagnetic potential so we can understand the idea of local gauge covariance arose.

\[
\begin{align*}
\psi^\dagger(\partial_\mu + i A'_\mu)\psi &= \psi^\dagger (e^{i\alpha})^\dagger (\partial_\mu + i A'_\mu) e^{i\alpha} \psi = \psi^\dagger (e^{-i\alpha} \partial_\mu e^{i\alpha} + e^{-i\alpha} e^{i\alpha} i A'_\mu) \psi \\
&= \psi^\dagger (\partial_\mu + i A'_\mu + i \partial_\mu \alpha) \psi = \psi^\dagger (\partial_\mu + i A_\mu) \psi
\end{align*}
\]

So by using covariant derivative \(\partial_\mu + i A_\mu\), as a result of \(U(1)\) covariance, we get classical equation for gauging the electromagnetic potential \(A_\mu\). But if we use gauge transformation on the potential \(A_\mu\), we have to transform the wave function as well \(\psi' = e^{i\alpha} \psi\). So the idea of gauge theory was to force some other gauge covariances to construct other interaction potentials.

Electromagnetic interaction is a bit different from other two forces included in the Standard model in a way that it does not have self interaction terms. This is because \(U(1)\) group is commutative (unlike matrices that were used in previous sections) and so the curvature tensor \(F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu) + (A_\mu A_\nu - A_\nu A_\mu) = \partial_\mu A_\nu - \partial_\nu A_\mu\) does not have terms that would cause \(A_\mu\) to interact with itself\(^5\) like in strong and weak interaction. This interaction is also different in the fact that charges of particles are not so evident as in previous sections where the components of multidimensional charge space implied the charge. To find out \(U(1)\) charge of a particle we use the operator of the charge \(Y\). Charge is an eigenvalue of \(Y\) so that \(Y\psi = e\psi\).

The use of charge operator is of course standard in all gauge models. The same way we defined weak \(A_\mu = W^\mu_\nu \tau_\nu\) we now define electromagnetic \(A_\mu = B_\mu Y\) where \(\tau_\nu\) and \(Y\) are charge operators, and \(W^\mu_\nu\) and \(B_\mu\) are the wave functions of bosons that mediate these interactions. From this form it is evident that eigenvalue of \(Y\) measures how strongly the particle is coupled to the interaction potential which is the definition of electrical charge.

\[
\psi^\dagger A_\mu \psi = \psi^\dagger B_\mu Y \psi = e \psi^\dagger B_\mu \psi
\]

4 \(U(1)\times SU(2)\times SU(3)\) covariant model

We introduced the three interactions that Standard model includes and can now compile this together and construct \(U(1)\times SU(2)\times SU(3)\) covariant model that is, a sort of, a prototype of the Standard model. This model cannot have mass terms because they are incompatible with left-right asymmetry of weak interaction.

A particle with spin \(s = 1/2\) can have a weak charge (is a weak doublet) and have two weak charge components or it can be uncharged (weak singlet) and only has one component. It can have a strong charge (is a strong triplet) and has three strong charge components or it can be uncharged (strong singlet) with only one component.

In this model we assume two spin \(s = 1/2\) particles, the lepton and the quark. They are both weak doublets and so have two weak charge components. The lepton is a strong triplet and so carries a strong charge. For readability we do not write strong charge components,\(^vi\) and we focus on weak charge components. We will name the weak charge components of lepton \(L\) the neutrino \(\nu\) and the electron \(e\), and we will name weak charge components of quark \(Q\) the up quark \(u\) and the down quark \(d\).

\(^{v}\text{group SU(1) has only one element and so SU(1) covariant theory is trivial and there is no resulting interaction}\\ ^{vi}\text{photon does not carry an electrical charge}\\ ^{vi}\text{components of the strong triplet (red, green, blue) are not observed in nature due to confinement [3] so omission of these indices for readability may be reasonable}
The next thing to assume is Lorentz covariance. This causes each particle to have left- and right-handed component. As we explained before the right-handed particles do not carry weak charge and so they are weak singlets. This is backed by experiments [1].

For propagation of interaction bosons whose existence is a consequence of gauge covariance we assume Yang-Mills terms. The Lagrange density of this model can be compactly written using covariant derivative $D_{\mu} = \partial_{\mu} + g_{\mu} A_{\mu}^{weak} + g_{\nu} A_{\mu}^{strong} - g_{\nu} A_{\mu}^{electromagnetic}$.

$$L = [\bar{\psi}_L (i\gamma^\mu D_\mu - m) \psi_L] + e [\bar{e}_L i\gamma^\mu D_\mu e_R + \nu_R i\gamma^\mu D_\mu \nu_R]$$

lepton dynamical and interaction terms

$$+ [\bar{u}_L i\gamma^\mu D_\mu u_L] + \bar{d}_R i\sigma^\mu \nu R d_R$$

quark dynamical and interaction terms

$$+ (h.c.) - \frac{1}{4} \text{tr}((D_\mu D_\nu - D_\nu D_\mu)(D^\mu D^\nu - D^\nu D^\mu))$$

Yang-Mills terms for gauge bosons

Here action of $A_\mu$ on uncharged particles is zero e.g. $A_\mu^{weak} \bar{u}_R = 0$ or $A_\mu^{strong} \nu_L = 0$ so that this compact expression is in accordance with previously presented ones. The reason we write this so compactly is to illustrate the logic of this type of models and because the stated $L$ has literally hundreds of terms if written in expanded form [4], and we do not wish to overwhelm the reader.

There is only one more step to the Standard model, the mass terms. In the stated $L$ all particles are massless. We need to introduce mass terms in a way that does not break SU(2) gauge covariance or Lorentz covariance.

5 Scalar particle

The purpose of this chapter is to find a theoretical justification for experimental observations of mass terms even though those terms are forbidden. One basic thing we can try is to identify mass terms as a byproduct of some interaction. This is sound because of the form that mass terms and interaction coupling terms take.

$$m \bar{\psi}_L \psi_L = \bar{\psi}_L^{1/2}(m) \psi_L = \bar{\psi}_L \psi_R \quad \text{analogously to} \quad \bar{\psi}_L i\sigma^\mu A_\mu \psi$$

For this to work the interaction potential must be $\phi = m$ on the energy scale we conduct experiments. Let us now present a new object $\phi$ to our model and we will determine its properties so that interaction terms between $\phi$ and 1/2-spinors will result to mass terms in low energy regime.

Let us find out the spin of this new object $\phi$. If the value of the $\phi$ at low energies should be equal to the mass $m$ then $\phi$ should transform the same way mass $m$ does under Lorentz transformations. Mass $m$ is a scalar so our $\phi$ has spin $s = 0$. $\phi$ is, what we call, a scalar particle.

In order for $\phi$ to respect Lorentz covariance there must exist a relativistic equation to determine its propagation. In the same way spin $s = 1/2$ particles obey Dirac equation, spin $s = 0$ particles obey Klein-Gordon equation\(^{vii}\) with the corresponding $L$.

$$\partial^\nu \partial_\nu \phi + m_\phi^2 \phi = 0 \quad \text{or equivalently} \quad L = (\partial^\nu \phi)^2 \partial_\nu \phi - m_\phi^2 \phi = 0$$

The $m_\phi$ is the mass of the particle $\phi$. But low energy solution of this equation is $\phi(x^\nu) = 0$, and we added $\phi$ with the intent that $\phi(x^\nu) = m$ at low energies. So we wish that $\phi$ would be constantly nonzero on all space-time at low energies. Because the $\phi$ will give masses to a lot of different particles let us scale our units of measurement so that $\phi = 1$ at low energies.\(^{vi}\) The simplest way we can force $\phi(x^\nu) = 1$ to be a low energy solution is by introducing another term to $L$, the self interaction term $(\phi^2 \phi)^2$.

$$L = \partial^\nu \phi^2 \partial_\nu \phi + m_\phi^2 \phi / 2 - m_\phi^2 (\phi \phi^2) / 4 = \partial^\nu \phi \partial_\nu \phi - m_\phi^2 \phi (\phi - 1)^2 / 4 + m_\phi^2 / 2$$

This $L$ is still Lorentz covariant and is the minimal modification of the original $L$ in order to get $|\phi| = 1$ at low energies, where $|\phi^2| = \phi^2$. You might have noticed that we flipped the sign of the mass term. This was done for convenience, coupling constants are written as prefactors of gauge fields so that we can write Yang-Mills terms as a single term and not as individual terms with each of its own prefactor connected with the coupling constant of that particular interaction, this form is equivalent to previous versions of $L$ and is connected by transformation $A_{\mu}^{weak} \rightarrow g_{out} A_{\mu}^{int}$.

\(^{vi}\)we have experimental evidence for Lorentz covariance, weak gauge covariance [2], weak left-right asymmetry [1] and clear evidence of the existence of mass and in this theoretical model these properties are logically contradictory

\(^{vii}\)relativistic equation for a scalar particle is easily constructed knowing the fact that $\partial^\nu \partial_\nu \phi$ transforms like a scalar under Lorentz transformations so that we can compose scalar objects on each side and get Klein-Gordon equation

\(^{viii}\)we use three units of measurement (kg, m, s) and two of them were already scaled by setting $\hbar = c = 1$ but we still have one unit unscaled and we are free to set $\phi = 1$ at zero energy i.e. vacuum expectation value of $\phi$ is 1
necessary to achieve a minimum of the potential \( V(|\phi|) = \partial^\alpha \phi \partial_{\alpha} \phi - \mathcal{L} \) at nonzero \( |\phi| \). We chose the prefactors so that the minimum of \( V(|\phi|) \) is at \( |\phi| = 1 \) and so that \( \phi \) at low energies, when it is restricted to \( |\phi| \approx 1 \), acts as a particle with mass \( m_{\phi} \) as is indicated in the figure. So although the mass term of \( \phi \) is not signed correctly we still get a particle with some positive mass \( m_{\phi} \) if the energy scale is low enough.

In this way we have a vacuum solution with \( |\phi(x')| = 1 \). Despite \( V(|\phi|) \) reflectional symmetry we have a vacuum solution that is not symmetric under reflection \( \phi \rightarrow -\phi \). This property is called spontaneous symmetry breaking. The vacuum that respects reflectional symmetry is \( \phi(x') = 0 \) but it is unstable because even the smallest perturbation shifts the value to a minimum \( \phi = \pm 1 \). Of course this symmetry breaks only at low energies when \( \phi \) uniformly takes some nonzero value and this low energy break of symmetry allows us to have a field with nonzero vacuum expectation value that we need to disguise mass terms as interaction coupling terms [2]. Let us emphasize that the sign of the minimum \( \phi = \pm 1 \) does not really matter.

Because of reflectional symmetry \( \phi \rightarrow -\phi \) of \( \mathcal{L} \) the dynamic of \( \phi \) is identical in \( \phi = 1 \) or \( \phi = -1 \) option.

In hope that introduction of \( \phi \) to our model will bring mass terms in low energy regime we have determined the spin and Lagrange density of \( \phi \). Other properties of \( \phi \) can be determined when we look at the coupling between \( \phi \) and fermions. All we are doing here is fitting the properties of \( \phi \) so that it satisfies our wishes that interaction with \( \phi \) results to acquisition of mass.

### 5.1 Fermion mass terms

Let us find out how the coupling terms between fermions and \( \phi \) should look like if we wish they resemble mass terms. If we want to couple differently handed particles, as it is the case with mass terms \( \psi_{\lambda}^L \bar{\psi}_{\lambda}^R \), we hit an obstacle since all left-handed particles in our model carry a weak charge and all right-handed particles are chargeless. This is a problem because we can use gauge SU(2) covariance to mix left-handed components but we cannot mix right-handed ones. This problem, that we encountered before, can now be resolved by turning \( \phi \) into a weak doublet \( [\phi_1, \phi_2]^T \). This means that we gave \( \phi \) a weak charge. In this way when we mix left-handed components we simultaneously mix \( \phi_1 \) and \( \phi_2 \) and these two actions eliminate each other and SU(2) gauge covariance remains unbroken. Let us look at the d quark mass term to illustrate this.

\[
Q^L_1 \phi d_R = \begin{bmatrix} u_L^1 & d_L^1 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} d_R \xrightarrow{SU(2)} \begin{bmatrix} u_L^1 & d_L^1 \end{bmatrix} \cdot g \cdot \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} d_R = u_L^1 \phi_1 d_R + d_L^1 \phi_2 d_R
\]

Using the fact that \( \phi \) has a weak charge we can use SU(2) gauge transformation on \( \phi \) and mix \( \phi_1 \) and \( \phi_2 \) in a way that \( \phi_1 = 0 \) and \( \phi_2 = H \in \mathbb{R} \). With this we have chosen SU(2) gauge. We said that \( \phi \) has a vacuum solution where \( \phi = 1 \) and with the chosen gauge this means that \( \phi = \begin{bmatrix} 0 & 1 \end{bmatrix}^T \) at zero energy. That means that at low energies \( m_{\phi_1} Q^L_1 \phi d_R = m_{\phi_2} d_L^1 d_R \) and this is the term that makes d quark massive with mass \( m_d \).

So we have molded our \( \phi \) so that the coupling between \( Q_L \) and \( d_R \) with \( \phi \) as the mediator results to mass term for d in SU(2) covariant way. Analogously we can give mass to electron in such a way with \( m_e Q^L_1 \phi L_R \) term.

What about U(1) covariance? The term \( Q^L_1 \phi L_R \) will be U(1) covariant if the total charge of \( \mathcal{L} \) of all factors of the term is zero. With this we have chosen SU(2) gauge. We said that \( \phi \) has a vacuum solution where \( \phi = 1 \) and with the chosen gauge this means that \( \phi = \begin{bmatrix} 0 & 1 \end{bmatrix}^T \) at zero energy. That means that at low energies \( m_{\phi_1} Q^L_1 \phi d_R = m_{\phi_2} d_L^1 d_R \) and this is the term that makes d quark massive with mass \( m_d \).

So if we want our mass terms to arise this way, we set some constraints about the charge of the particles. The \( Y \) charge of \( d_R \) and \( e_R \) is determined by the \( Y \) charge of \( Q_L \) and \( L_R \) and \( \phi \).

Now we can add mass terms for \( d \) and \( e \) in U(1)×SU(2) covariant way because \( \phi = \begin{bmatrix} 0 & H \end{bmatrix}^T \) and \( d \) and \( e \) are second components of their doublets. The same does not apply to \( u \) and \( \nu \) that are first components of their doublets. To give them mass terms in covariant way they have to be coupled to \( \bar{\phi} = \begin{bmatrix} H & 0 \end{bmatrix}^T \). If we do not want to introduce a different scalar field to our model, than we need to transform the existing one to meet these demands. As before we can calculate \( Y(\bar{\phi}) = Y(Q_L) - Y(u_R) \) which means that \( \phi \) has different \( Y \) charge than \( \phi \). The only way we can transform \( \phi \rightarrow \phi \) so that \( Y(\bar{\phi}) \neq Y(\bar{\phi}) \) is to flip the sign of the charge by \( Y \) charge conjugation \( \psi \rightarrow \psi^* \). The \( \bar{\phi} \) should also transform as SU(2) doublet to maintain SU(2) covariance. The \( \bar{\phi} \) that meets all of these demands is \( \bar{\phi} = i r_2 \phi^* \). It is SU(2) doublet so that \( Q^L_1 i r_2 \phi u_R \) and \( L^L_1 i r_2 \phi e_R \) are U(1)×SU(2) covariant [4].
We call the terms on the left-hand side the Yukawa terms and, as we see, they reduce to mass terms at low energies. Now all this looks a bit awkward, but it is a convenient way to introduce mass terms to fermions in a covariant manner. We achieved that some particular coupling terms, between and fermions, reduce to fermion mass terms at low energies. By low energies we mean energies of the order of $m_e$, the mass of the scalar particle $\phi$. We must also stress the fact that each term $m_e \psi_{\mu}^\dagger \phi \psi_R$ has its own coupling constant $m_\phi$ that corresponds to particle $\psi$ mass at low energies. This is highly unusual because we are used to interactions having the same coupling constant for all particles. $\phi$ should not be coupled to some particles more strongly than to the others. But on the other hand, we can look at masses as some internal values of particles that are the charges for interaction with $W$. The same way we use $Y$ for interaction with electromagnetic field.

As said, these terms also cause some restrictions on the values of $Y$ charge. If we know the $Y$ charge of the left-handed doublets $Q_L$ and $L_L$ and of $\phi$, then the $Y$ charges of the right-handed particles are completely determined. This is a side effect of introducing these terms in U(1) covariant way.

$$Y(u_R) = Y(L_L) + Y(\phi)$$

$$Y(e_R) = Y(L_L) - Y(\phi)$$

5.2 Boson mass terms

The existence of this kind of scalar particle $\phi$ has interesting consequences for the bosons as well. As we have seen, the $\phi$ must carry a weak and electric charge. That means that it is coupled to $W_\mu^+ \tau_a$ and $B_\mu$ fields through the covariant derivative in its dynamical term. Let us look at dynamical term of $\phi$ in low energy regime when $\phi = [0 \ 0]^T$. At this stage we must choose the electric charge $Y$ of $\phi$ and we choose$^{xx}$ $Y \phi = +\phi/2$.

$$D_\mu \phi = (\partial_\mu + A_\mu^{\text{weak}} + A_\mu^{\text{electromagnetic}}) \phi = (\partial_\mu + W_\mu^a \tau_a + B_\mu Y) \phi$$

$$= W_\mu^a \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + W_\mu^3 \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + W_\mu^3 \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + B_\mu \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} W_\mu^1 - iW_\mu^2 \\ B_\mu - W_\mu^3 \end{bmatrix}$$

and with $W_\mu^\pm = W_\mu^1 \pm iW_\mu^2$ and $Z_\mu = W_\mu^3 - B_\mu$, since $(W^-)^\dagger = W^+$ and $Z^\dagger = Z$.

The terms $W^+ W^- + Z^a Z_\mu$ are the low energy interaction terms between $W_\mu^\pm$ and $Z_\mu$, and $\phi$. Terms like $(A_1^a)^3 A_\mu$ are spin $s = 1$ mass terms [4], and so we see that in this way the $\phi$ gives mass not only to fermions but to some bosons as well. A consequence of $\phi$ being charged is massiveness of $W_\mu^\pm$ and $Z_\mu$. We have introduced $W_\mu^\pm$ before but we have not mentioned $Z_\mu$ so far. It is a mixture of $W_\mu^3$ and $B_\mu$ generator. The other possible mixture of $W_\mu^3$ and $B_\mu$ is

$$\gamma_\mu = \frac{W_\mu^3 + B_\mu}{2}$$

and this one stays massless. So $\phi$ dynamical term implies a new basis of spin $s = 1$ bosons $(W_\mu^1, W_\mu^2, W_\mu^3, B_\mu) \rightarrow (W_\mu^\pm, W_\mu^3, B_\mu, \gamma_\mu)$, and the new $W_\mu^\pm$ and $Z_\mu$ acquire mass while $\gamma_\mu$ remains massless. We call this the electroweak break.

It turns out that interactions mediated by massive bosons (like $W_\mu^\pm$ and $Z_\mu$) have a short range [4]. So electroweak break causes three of the four electroweak interaction fields to become short-ranged and only one remains long-ranged, the $\gamma_\mu$. This interaction field $\gamma_\mu$ is of course a photon, the only experimentally known massless boson in the electroweak sector [4]. This means that $B_\mu$ is not really low energy photon and $Y$ is not low energy electric charge. The $\gamma_\mu$ is the real wave function of the photon. To find out the real electric charge operator $q$ let us look at electroweak interactions of the electric charge. A particle $\psi$ is the coupling between the particle $\psi$ and the photon $\gamma_\mu$.

$$W_\mu^3 \tau_3 + B_\mu Y = (\gamma_\mu + Z_\mu) \tau_3 + (\gamma_\mu - Z_\mu) Y = Z_\mu (\tau_3 - Y) + \gamma_\mu (\tau_3 + Y) = \gamma_\mu u + \gamma_\mu q$$

$$\psi^\dagger A_\mu^a \psi = [c_1^\dagger \ c_2^\dagger] [W_\mu^a \tau_a + B_\mu Y] [c_1 \ c_2]$$

and using the calculation (A) on page 5

$$= \left( c_1^\dagger W_\mu^\pm c_2 + c_1^\dagger W_\mu^3 c_2 \right) + \left( u(c_1^\dagger c_2^\dagger Z_\mu c_1 + u(c_2^\dagger c_2^\dagger Z_\mu c_2) \right) + \left( q(c_1^\dagger c_2^\dagger c_1 \gamma_\mu c_1 + q(c_2^\dagger c_2^\dagger c_2 \gamma_\mu c_2) \right)$$

So electric charge operator $q$ is a sum of $Y$ operator and $\tau_3$ operator. Now when we know $Y$ is not electric charge, we will call $Y$ hypercharge, and we call $\tau_3$ weak isospin. The eigenvalues of $\tau_3$ are $\pm 1/2$ for particles

$^{xx}$we can choose $Y(\phi)$ freely because of U(1) covariance, with this we have chosen a gauge where $a$ in the gauge transformation $exp(i\alpha Y)$ is chosen in a way that $Y \phi = +\phi/2$ on all space-time

$^{xvi}$because of this mixing of generators of two different groups we must take different coupling constants ($g_\mu$ and $g_Y$) into account, and that difference results to $Z_\mu$ having slightly different mass than $W_\mu^3$. 

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in weak doublet\textsuperscript{xviii} and 0 for weak singlets. So value of isospin $\tau_3$ specifies the weak charge of the particle and together with hypercharge $Y$ they determine the electric charge $q$. Now we can set the hypercharge of our particles so that their electric charges $q = Y + \tau_3$ will match experimental values.

6 Standard model

Now we have all the ingredients needed to construct the Standard model of particle physics. Standard model is essentially the model presented so far but with three generations of fermions [4]. This means that we do not only postulate one lepton $L$ and one quark $Q$ as we stated before, but three leptons $L^{i=1,2,3}$ and three quarks $Q^{i=1,2,3}$. This only increases the number of fermions. Each member of a higher generation has greater mass (coupling to $\phi$) than the corresponding particle of the previous generation. This mass hierarchy causes particles of higher generation to decay to lower ones which explains why everyday matter is composed of particles of first generation. We also postulate a scalar particle $\phi$ with all the properties stated in previous chapters. We call such type of scalar object a Higgs boson [2].

If we want left- and right-handed components of a fermion to have the same electric charge, than the hypercharge $Y$ of the scalar particle $\phi$ should really be $Y(\phi) = +1/2$, as we have chosen before. This becomes obvious when we compare the relations (\textsuperscript{1}) on page 9 with the $q = Y + \tau_3$ and the fact that $\tau_3 = \pm 1/2$ for elements of a weak doublet. If $Y(\phi) \neq +1/2$, than the left- and right-handed components of the same particle would carry a different electric charge, and this is not the case in nature. Using $Y(\phi) = +1/2$ with (\textsuperscript{1}) and setting $Y(Q_L) = 1/6$ and $Y(L_{R}) = -1/2$ gives us the table to the right. This way we fitted our model parameters so that the electric charges of our particles correspond to the ones observed in nature.

Let us review our steps and list all the assumptions we made so that we get a concrete feeling what the Standard model is. We postulate:

- **Gauge covariance**: assumption that three charge spaces exist, one, two and three dimensional, and that bases of these charge spaces are arbitrary. Associated groups, that are allowed to transform the charge spaces, are U(1), SU(2) and SU(3) respectively. To properly derive in gauge covariant charge space we need to use covariant derivative $D_{\mu} = \partial_{\mu} + g_{\mu} B_{\mu} Y + g_{\mu} W^{\mu}_{\alpha} \tau_{\alpha} + g_{\mu} C^{\mu}_{\nu} T_{\nu}$, where $a = 1, 2, 3$ and $r = 1, 2, \ldots, 7, 8$. The $Y$, $\tau_{\alpha}$ and $T_{\nu}$ are generators of groups U(1), SU(2) and SU(3) respectively, and they transform charged particles and annihilate chargeless particles. This postulate introduces $1 + 3 + 8 = 12$ spin $s = 1$ interaction fields, the gauge bosons $B_{\mu}$, $W_{\mu}$ and $C^{\mu}_{\nu}$.

- **Lorentz covariance**: assumption that all terms of $\mathcal{L}$ are Lorentz covariant which in turn allows only some specific types of dynamical and mass terms for fermions and bosons. We have chosen terms that are as close as possible and of the lowest possible order.

\begin{align*}
\text{spin} &= 0 \quad L_{\text{scalar}} = (D^\mu \phi)(D_\mu \phi) - m^2_{\phi} \phi^2 (\pm \alpha \phi^2)^2 \quad \text{(Klein-Gordon)} \\
\text{spin} &= 1/2 \quad L_{\text{fermion}} = i \bar{\psi} \gamma^\mu \sigma^\nu D_\mu \psi + i \bar{\psi} \gamma^\mu \sigma^\nu D_\mu \psi_R - m_{\psi} \psi^\dagger \psi_R - m_{\psi} \psi_R \psi_L + \text{(h.c.)} \quad \text{(Dirac)} \\
\text{spin} &= 1 \quad L_{\text{boson}} = -\text{tr}(D_\mu D^\mu - D_\mu D_R)(D^{\mu R} D^{\mu R} - D^{\mu R} D^\mu R)/4 \quad \text{(Yang-Mills)}
\end{align*}

Very important consequence of Lorentz covariance is the distinction between left- and right-handed 1/2-spinors.

- **Weak interaction is a chiral theory** which means that weak interaction does not respect parity symmetry i.e. only left-handed fermions are coupled to weak field.

- **Quarks**: we assume the existence of three quarks $Q$ with a strong charge, a weak charge and with hypercharge $Y(Q_L) = 1/6$.

- **Leptons**: we assume the existence of three leptons $L$ without strong charge, with a weak charge and with hypercharge $Y(L_{R}) = -1/2$.

- **Higgs boson**: this is a scalar particle $\phi$ with properties that precisely match those wanted to explain mass terms at low energies on gauge covariant way: has nonzero vacuum expectation value, carries a weak charge, $Y(\phi) = 1/2$, is coupled to fermions, $d^c, e^c$ are coupled to $\phi$ and $u^c, \nu^c$ to a transformed version of it $\tilde{\phi} = i\tau_2 \phi$, and has different coupling constants for different fermions that are experimentally measured as masses (proportionally).

The model presented in this text is not completely in accordance with experiments. If we look at the table of electroweak charges we see that $\nu^c_R$ does not carry weak charge or hypercharge. With the fact that leptons

\textsuperscript{xviii}for example $\tau_5 \nu_L = \frac{1}{2} \begin{bmatrix} 1 & \frac{+1}{0} & \frac{0}{0} & \frac{-1}{0} \end{bmatrix}$, $[\tau_3^c] = \frac{1}{2} \begin{bmatrix} 1 & \frac{0}{0} & \frac{-1}{0} \end{bmatrix}$, $\nu_L = \frac{1}{2} \begin{bmatrix} \frac{+1}{0} & \frac{0}{0} \end{bmatrix}$ and $\tau_3 \epsilon_L = \frac{1}{2} \begin{bmatrix} \frac{+1}{0} & \frac{0}{0} \end{bmatrix}$.
do not carry a strong charge, this means that $\nu_R$ does not interact with any particle except $\phi$. We call it the sterile neutrino. This particle is experimentally unproven [6].

On the other hand we have evidence that mass terms for neutrinos exist [7], and neutrino mass terms need left- and right-handed neutrino ($-m_\alpha \nu_L^\dagger \nu_R^\dagger$). The evidence that suggests the existence of neutrino mass terms are neutrino oscillations. This is an effect when propagating neutrino $\nu_i$ in oscillatory fashion. This can be explained in our model using Yukawa terms $L_L^i \phi \nu_R$. To understand this let us look at Yukawa terms for three generations of $Q, L$ particles in more detail. All possible Yukawa terms are

$$L_{\text{Yukawa}} = -M^{\nu}_{ij} (Q_L^i)^\dagger \tau_2 \phi \nu_R^j - M^{\nu}_{ij} (Q_L^j)^\dagger \phi \nu_L^i - M^{\nu}_{ij} (L_L^i)^\dagger \tau_2 \phi \nu_R - M^{\nu}_{ij} (L_L^j)^\dagger \phi \nu_L^i + (h.c.)$$

where $M^{\nu}$ are some matrices whose components $M^{\nu}_{ij}$ determine the couplings between $\nu_L^j$ and $\nu_R^k$ with $\phi$ as the mediator. We call $M^{\nu}$ a mass matrix. These mass matrices are not necessarily diagonal, which we would expect so that each particle would have a definite mass. Consequently the “true fermions” with definite masses are actual linear combinations of those in $L$, or conversely the fermions in $L$ are linear combinations of the true fermions. So when a particle propagates, its three components with different masses propagate at different speeds, which causes interference. This in turn causes oscillations in flavor of neutrinos when they propagate freely [7].

For the proper completion of the chapter let us state the Lagrange density $L$ that follows from the postulates of the Standard model.

$$L = \sum_{i=1}^{3} \left[ (L_L^i)^\dagger i\sigma^\mu D_\mu L_L^i + (\nu_R^i)^\dagger i\sigma^\mu D_\mu \nu_R^i + (\nu_L^i)^\dagger i\sigma^\mu D_\mu \nu_L^i + (d_R^i)^\dagger i\sigma^\mu D_\mu d_R^i + (u_R^i)^\dagger i\sigma^\mu D_\mu u_R^i + (e_R^i)^\dagger i\sigma^\mu D_\mu e_R^i \right]_{\text{lepton dynamical and interaction terms}}$$

$$+ \left[ (Q_L^i)^\dagger i\sigma^\mu D_\mu Q_L^i + (u_L^i)^\dagger i\sigma^\mu D_\mu u_L^i + (d_L^i)^\dagger i\sigma^\mu D_\mu d_L^i + (L_L^i)^\dagger i\sigma^\mu D_\mu L_L^i + (e_L^i)^\dagger i\sigma^\mu D_\mu e_L^i + (R_L^i)^\dagger i\sigma^\mu D_\mu R_L^i \right]_{\text{quark dynamical and interaction terms}}$$

$$- M^{\nu}_{ij} (L_L^i)^\dagger \tau_2 \phi \nu_R^j - M^{\nu}_{ij} (Q_L^j)^\dagger \phi \nu_L^i - M^{\nu}_{ij} (L_L^i)^\dagger \tau_2 \phi \nu_R^j - M^{\nu}_{ij} (Q_L^j)^\dagger \phi \nu_L^i + (h.c.)_{\text{lepton Yukawa terms}}$$

$$- \frac{1}{4} \text{tr}((D_\mu D_\nu - D_\nu D_\mu)(D^\mu D^\nu - D^\nu D^\mu))_{\text{gauge bosons terms}}$$

$$+ (D^\mu \phi) \partial_\mu \phi - m_\phi^2 (\phi \phi - 1)^2/4_{\text{Higgs bosons terms}}$$

7 Experimental evidence for Higgs boson

We have experimental evidence for mass terms of fermions ($m_\psi\psi_L\psi_R$) and the weak bosons ($m_{W^ \pm}W^{+\mu}W^{\mu} + m_{Z^0}Z^{\mu\nu}Z_{\mu\nu}$). Our model postulates a scalar field $\phi$ whose interaction terms reduce to these terms at low energies. We need a proof that mass terms really emerge in this way. The easiest way is to find the excitation of the field $\phi = [0 \ H]^T$, the particle corresponding to the $H$. This particle should have some very specific properties. Finding this particle with all its properties proves that mass terms emerge with $\phi$ as the mediator.

The mass of the Higgs boson $H$ is a free parameter in our model. Our theory tells us nothing about $m_\phi$. The early limits of the mass of Higgs boson in the 1970s came from the absence of observation of Higgs related effect in nuclear physics and neutron scattering experiments. This sets limit $m_\phi > 18\text{MeV}$. This limit was pushed upward by the years and when in 1983 the $Z$ boson was produced it became obvious that Higgs boson should be heavier than $Z$ otherwise $Z$ would be able to decay to Higgs boson and that was not observed, so $m_\phi > 91\text{GeV}$. The LEP (Large Electron-Positron collider) pushed this limit to $m_\phi > 114\text{GeV}$ until its shutdown in 2000. The search continued in Tevatron until 2011 when they excluded the possibility of Higgs boson to have mass between $147\text{GeV}$ and $180\text{GeV}$. In 2010 the LHC (Large Hadron Collider) started its operation, the machine specifically build to prove or disprove Higgs boson. LHC was operational until 2013 until its shutdown, to prepare the collider for higher energy and luminosity. During that time the limits were narrowed until the new particle was confirmed in 2013 with mass $125\text{GeV}$ [5].

Is the particle found in LHC a Higgs boson? To ask differently, does this particle has the properties of the Higgs boson and is so responsible for mass terms of fermions and weak bosons? The production and decay of the Higgs boson is highly dependent of its mass. Now when we know the mass of the candidate for Higgs boson, we can look at how it is most likely to arise and most likely to decay. The most common production processes, if produced in LHC, is gluon fusion (figures below) and vector boson fusion, where two fermions emit two weak bosons that produce Higgs boson [5]. The strength of the coupling between Higgs boson and a particle is the particle mass. This is why the main mediators in its production and decay are the heavy top and bottom quark, and the weak bosons. The most probable decays is to fermion and antifermion (most notably bottom quark), or to two massive gauge bosons that subsequently decay to leptons (lower figure) or quarks [5]. The two most probable processes involving Higgs boson, if it were produced in LHC, are shown on the
figure. So now we can probe if the found particle is produced and decays in this way. Let us look if the found particle has the properties of the Higgs boson, one property at a time.

The particle should have zero spin, so that its vacuum expectation value is scalar, like mass. It was soon realized that the particle found has either $s = 0$ or $s = 2$, because there were events when it decayed to two photons. The $s = 2$ option was later excluded when analyzing decays to $Z$ and $W$ bosons [8]. It was also confirmed that that the found particle has even parity, as it is expected for Higgs boson [9].

The particle should couple to mass i.e. the strength of interaction with Standard model particles is proportional to their masses. The couplings can be measured by measuring cross sections and decay rates, and they are in good accordance to the Standard model [10].

The particles decay channels should be as predicted for Higgs boson. The $\gamma\gamma$, $\tau\tau$, $WW$ and $ZZ$ decay channels were observed. Some decay rates are a little higher than expected, in particular $H \rightarrow \gamma\gamma$. The $bb$ channel is not yet confirmed [10].

Uniqueness of the Higgs boson. Is there only one Higgs-like particle or are there many of them. This is not yet answered. After the 2015 LHC restart at the full planned energies 13 − 14GeV, searches for multiple Higgs particles (as predicted in some theories) will take place.

So we can say with some certainty that Higgs boson exists and that it is responsible for masses of particles. Masses are a consequence of the $\phi$ coupling to fermions and weak bosons and the fact that $\phi$ has nonzero vacuum expectation value which is possible due to spontaneous symmetry breaking.

8 Conclusion

We presented a model of particle physics in a way that is theoretically consistent and that its predictions match the experimental observations. The problem of mass terms highlighted some theoretical inconsistencies that were mended with introduction of some new object, the Higgs boson. This made the model awkward in some aspects (Yukawa terms) and it still has some glitches (neutrino mass terms). It is a fact that Standard model leaves a lot of questions unanswered. Specifically concerning mass terms, we may ask ourselves what specifies the couplings between fermions and Higgs boson i.e. why do particles have such masses as they do. We might even ask ourselves why are they coupled in the first place. In this aspect we might regard Standard model as semi-empirical. It is not its intent to answer these questions. Its intent is to show a simplest way how these things that we observe in nature are possible. The minimal model that matches the observations, but with a lot of assumptions. Further experiments will show how close Standard model is to reality, and they might even shed some light to what the answers to our questions in this chapter are. We need to wait and see.

References


