Summary

Seminar talks about neutrinos or more exactly about the difference between Dirac and Majorana neutrinos. It starts with definition of neutrinos and how do they appear at weak decays and then continue with laboratory experiments which try to detect neutrinos. There is also an explanation what neutrino Majorana is. Majorana neutrino is a particle and an antiparticle at the same time like a photon or Z boson. In the end we go through well-known see-saw relation which explains why neutrinos are light.
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Introduction

The origin of neutrino mass is a very old problem. Pauli predicted the existence of the neutrino for the first time in 1930, he already suspected its near-zero mass. Even today, direct measurements always end up with upper limits of the mass value. This fact and the discovery of the parity violation had led people at one time to suspect whether the neutrino is a two-component Weyl spinor. Experimentally, only the left-handed neutrino $\nu_L$ and its charge conjugate $\nu_L^c$ were observed in conventional thinking, it should be the right-handed anti-neutrino. Experimentally, there are no evidences that the right-handed neutrino exists, but if it does, the proof of its existence is hard to come by. In the Standard Model, it is electrically neutral and constitutes a weak isospin singlet. If the neutrino is massive, as was verified by the discovery of the neutrino oscillation, $\nu_R$ has to exist. Because it can only fly a subluminal speed and hence there always exists a Lorentz frame moving faster than particle. In that frame, the neutrino is flying backward but the orientation of rotation, that is, its spin direction, does not change, and hence $\nu_L$ becomes $\nu_R$ in this frame. In the Standard Model, the neutrino is assumed to have zero mass. $\nu_R$ is sterile because it does not feel the electroweak force; hence there is no way to prove its existence.
Experiments

For neutrino’s masses existing upper limits are:

\[
\begin{align*}
 m(\nu_e) &< 2 \text{ eV} \\
 m(\nu_\mu) &< 190 \text{ keV} \\
 m(\nu_\tau) &< 18.2 \text{ MeV}
\end{align*}
\] (1)

We have three major sites of the world’s long-baseline oscillation experiments. Two detectors are placed at a distance of over 100 km apart. GPS is used to find the accurate position of the far detector and direct the neutrino beam to a pinpoint target to synchronize timing with the near detector. K2K is the first successful accelerator-based experiment. The group extracted the 12-GeV proton beam from KEK/PS (proton synchrotron) and generated a low-energy $\nu_\mu$ beam at the right value of $\frac{L}{E_\nu} \sim 520$ ($L = 250\text{ km}$, $\langle E_\nu \rangle = 1.3\text{ GeV}$).

Fig 1. World neutrino oscillation experiment sites. From top clockwise: K2K (KEK to Kamioka, 250 km). T2K (Tokai to Kamioka, 295 km.) in Japan, MINOS (Main Injector Neutrino Oscillation Search Fermilab to Soudan, 735 km) in the United States and CNGS (CERN to Gran Sasso in Italy, 730 km) in western Europe.[1]
Simple theoretical framework

Assume that we have three charged leptons, $e, \mu$ and $\tau$ [4]. Each of three leptons $\ell$ is coupled to a neutral state called $\nu_\ell$ (neutrino associated with the lepton $\ell$) via the intermediate vector boson discovered at CERN with a mass around 83 GeV. The Hamiltonian density for charged current weak interaction is then ($G_f$ is a Fermi constant):

$$H_{cc} = \frac{G_f}{\sqrt{2}} W^\mu (83 \text{ GeV}) \sum_{\ell=e, \mu, \tau} \bar{\nu}_\ell \gamma_\mu (1 - \gamma_5) \nu_\ell + \text{h.c.} \quad (2)$$

The set \{\nu_\ell\} is said to be the set of flavor eigenstates or weak interaction eigenstates.

Now assume that the freely propagating neutrinos of definite mass are not the $\nu_\ell$ but a set of three neutral states $\nu_m$ with masses $M_m$, and that the flavor eigenstates $\nu_\ell$ are linear combinations of these eigenstates $\nu_m$.

$$\nu_\ell = \sum_m U_{\ell m} \nu_m \quad (3)$$

Here the $U$ is the neutrino mixing matrix and is analog of the Kobayashi-Maskawa matrix in the charged current Hamiltonian between quarks. We have

$$H_{cc} = \frac{G_f}{\sqrt{2}} W^\mu (83 \text{ GeV}) \sum_{\ell=e, \mu, \tau} \bar{\nu}_\ell \gamma_\mu (1 - \gamma_5) \sum_m U_{\ell m} \nu_m + \text{h.c.} \quad (4)$$

CPT symmetry

Here we shall make use only of basic quantum mechanics and discrete symmetries. We shall take our time to review C - charge conjugation, P - parity and T – time reversal operators and then define precisely what we call a Majorana neutrino.

Suppose there exists a massive neutrino with negative helicity $\nu_-$. However our theory is CPT invariant: this means that there exists also the CPT mirror image of $\bar{\nu}_+$. 

\[
\begin{pmatrix}
\nu_- \\
\bar{\nu}_+ \\
\end{pmatrix}
\]
Fig 1. Mirror image of $\nu_-$ and antineutrino with positive helicity $\bar{\nu}_+$.  

But our neutrino $\nu_-$ has a mass: its velocity is smaller than the speed of light so an observer can move faster. Then, in the frame of this observer, the neutrino is running the other way around but it is still spinning the same way. In other words, we have converted with a Lorentz boost our neutrino with negative helicity into a neutrino with positive helicity $\nu_+$.  

\[
\begin{array}{c}
\text{Lorentz boost} \\
(\nu_-, \bar{\nu}_+) \quad (\quad, \nu_+) \\
\text{CPT}
\end{array}
\]

Fig 2. Adding Lorentz boost.  

Now the question is: are these two states with positive helicity $\nu_+$ and $\bar{\nu}_+$ the same? If we make the assumption that $\nu_+$ is not the same particle as $\bar{\nu}_+$ then $\nu_+$ has its own CPT mirror image $\bar{\nu}_-$. This new state can be connected to $\bar{\nu}_+$, by a Lorentz transformation.  

\[
\begin{array}{c}
\text{Lorentz boost} \\
\text{or} \\
(\nu^D, \bar{\nu}^D) \\
\text{CPT}
\end{array}
\quad
given
\begin{array}{c}
(\nu^D, \bar{\nu}^D) \\
\text{CPT}
\end{array}
\]  

Fig 3. Dirac neutrino $\nu^D$ with external electromagnetic field $B, E$.  

We have got four states with the same mass. This set of states is called a Dirac neutrino $\nu^D$.  

In general, a Dirac neutrino will have a magnetic dipole moment and an electric dipole moment. The two states ($\nu^D, \bar{\nu}^D$) either by a Lorentz transformation or by the action of an external electromagnetic field $B, E$.  

If $\nu_+$ is identical to $\bar{\nu}_+$, we are left with only two states with the same mass.
The set of two states is called a Majorana neutrino $\nu^M$. In the rest frame, a CPT transformation applied to either of the two spin states simply reverses its spin. Then a $\pi$ rotation brings the neutrino back to the original states.

That is what we mean by saying that a Majorana neutrino is its own antiparticle: it goes back to itself under the action of CPT transformation followed by a $\pi$ rotation. [2]

**The see saw mechanism.**

Lagrangian of a gauge theory contains Yukawa couplings between a fermion field $\psi$ and a scalar Higgs fields $\Phi$ of the form:

$$\bar{\psi} \Phi \psi$$

Such couplings are allowed by gauge invariance. During the spontaneous symmetry breaking, the field $\Phi$ acquires some vacuum expectation value $\langle \Phi \rangle_0$. The Yukawa coupling then gives:

$$\langle \Phi \rangle_0 \bar{\psi} \psi$$

which is a mass term, the vacuum expectation $\langle \Phi \rangle_0$ being proportional to the mass.
Consider here our left-right symmetric model [3], the spontaneous symmetry breaking is a two step process. First, the gauge group $SU(2)_L \times SU(2)_R \times U(1)$ is broken down to $SU(2)_L \times U(1)$ and the field $W_R$ acquires a mass $M(W_R)$ which is of the order of the symmetry breaking scale $\langle \Phi \rangle_R$,

yielding:

$$M(W_R) \approx \langle \Phi \rangle_R.$$ (7)

Then the group $SU(2)_L \times U(1)$ is broken down to the $U(1)$ factor of electromagnetism and the usual $w_L$ field acquires its mass $M(W_L)$. Measured 83 GeV at the Cern collider,

yielding:

$$M_{W_L} \approx 83\text{GeV} \approx \langle \Phi \rangle_L.$$ (8)

Fig 6. Contribution of the $W_R$ to the amplitude $\langle K^0|H|\bar{K}^0 \rangle$.

No effects of right- handed currents have been observed experimentally, so we expect the mass of the $W_R$ to be very large. For example, the $W_R$ would contribute to the amplitude $\langle K^0|H|\bar{K}^0 \rangle$ through the graph of Figure (6.) and from the experimental value of the mass difference between the $k^0_L$ and the $K^0_S$ mesons, one can set limit:

$$M_{W_R} > 1.6\text{TeV}$$ (10)

The breaking scale of $SU(2)_R$ is thus expected to be much bigger than the one of $SU(2)_L$:

$$\langle \Phi \rangle_R \gg \langle \Phi \rangle_L$$ (9)

Let us now construct Yukawa couplings. The neutrino fields $\psi_L$ and $\psi_R$ in the left-right symmetric model have $SU(2)_L \times SU(2)_R \times U(1)$ quantum numbers. As we have said earlier:

the field $\psi_L$ is in an $SU(2)_L$ doublet $[\psi_L]$ with $I_L = \frac{1}{2}$, $I_R = 0$,

the field $\psi_R$ is in an $SU(2)_R$ doublet $[\psi_R]$ with $I_L = 0$, $I_R = \frac{1}{2}$.
Therefore, the bilinear products involving $\psi_L$ and $\psi_R$ have the following quantum numbers:

$$\bar{\psi}_R \psi_L \text{ has } I_L = \frac{1}{2} \text{ and } I_R = \frac{1}{2}$$ (11)

$$\overline{(\psi_L)^C} \psi_L = \psi_L \Omega \psi_L \text{ has } I_L = 1 \text{ and } I_R = 0$$ (12)

$$\overline{(\psi_R)^C} \psi_R = \psi_R \Omega \psi_R \text{ has } I_L = 0 \text{ and } I_R = 1$$ (13)

These terms with constant coefficients are forbidden by the $\text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)$ invariance of the Lagrangian. However, allowed terms (with total $I_L = I_R = 0$) may be constructed by coupling these bilinears to Higgs fields $\Phi$, $\Delta_L$, and $\Delta_R$ with properly chosen quantum numbers. These Yukawa couplings are:

$$\bar{\psi}_R \Phi \psi_L \text{ where } \Phi \text{ has } I_L = \frac{1}{2} \text{ and } I_R = \frac{1}{2}$$ (14)

$$\overline{(\psi_L)^C} \Delta_L \psi_L \text{ where } \Delta_L \text{ has } I_L = 1 \text{ and } I_R = 0$$ (15)

$$\overline{(\psi_R)^C} \Delta_R \psi_R \text{ where } \Delta_R \text{ has } I_L = 0 \text{ and } I_R = 1$$ (16)

If we now take the fields in the general free Lagrangian

$$-L = \bar{\psi} i \gamma_\mu \partial^\mu \psi + M_D [\bar{\psi}_R \psi_L + \text{ h.c.}] + \frac{M_L}{2} [(\bar{\psi}_L)^C \psi_L + \text{ h.c.}] + \frac{M_R}{2} [(\bar{\psi}_R)^C \psi_R + \text{ h.c.}]$$ (17)

the first term represents the kinetic term, and the rest are mass terms. The third and fourth terms are referred to as the Majorana mass terms. To be the neutrino fields in the Yukawa couplings, we see that once the Higgs fields acquire vacuum expectation values, there will be Dirac and Majorana neutrino mass terms with:

$$M_D \sim \langle \Phi \rangle \hspace{1cm} (18)$$

$$M_L \sim \langle \Delta_L \rangle \hspace{1cm} (19)$$

$$M_R \sim \langle \Delta_R \rangle \hspace{1cm} (20)$$

Let us now compare the three parameters $M_L$, $M_D$, and $M_R$:

The expectation value $\langle \Delta_L \rangle$ of the $I_L = 1$ Higgs field affects the $\rho$ parameter of the neutral current neutrino scattering, which is measured to be very close to 1. If we take as in SM $\rho = 1 = \frac{M_\nu}{M_2 \cos \theta_W}$ as a phenomenological constraint, it means that $\langle \Delta_L \rangle$, hence $M_L$, vanishes. It vanishes because left handed neutrino is related to term that changes weak hypercharge by 2 units, which is not possible with the standard Higgs interaction. Turning to $M_D$, we note that in the left-right symmetric model the same
spontaneous symmetry breaking which leads to the neutrino Dirac mass also leads to the Dirac masses of the charged fermions. Now, charged particles cannot have Majorana masses, since, for example, a mass term such as:

$$\overline{(e_L)^C} e_L = e_L^C \Omega e_L$$

(21)

would violate electric charge conservation. Therefore, for the charged fermions, the Dirac masses are the physical masses. Thus, we expect the neutrino Dirac mass $M_D$ to be of the order of the “observed” masses of the related charged lepton and quarks. Finally, since $\langle \Delta_R \rangle$ gives $W_R$ its very large mass, we expect that $M_R \sim \langle \Delta_R \rangle$ is also very large, and, in particular, that

$$M_R \gg M_D$$

(22)

Hence, the neutrino mass matrix from the Lagrangian

$$-L = \overline{V} y^\mu \partial_\mu V + \overline{V}[M]V$$

(23)

Where $V$ is the column vector

$$V = [f \bar{f}] , \quad f = \frac{\psi_L + (\psi_L)^C}{\sqrt{2}}, \quad F = \frac{\psi_R + (\psi_R)^C}{\sqrt{2}}$$

(24)

And $[M]$ from Eq. (17) is the real symmetric matrix

$$[M] = \begin{bmatrix} M_L & M_D \\ M_D & M_R \end{bmatrix}$$

(25)

Which is called the neutrino mass matrix and can be diagonalized by a rotation in the two-dimensional space of the vector $V$. Let $\nu'$ and $N$ be the eigenvector fields, with eigenvalues $M'_\nu$ and $M_N$ so our Lagrangian eq. (18) looks like

$$-L = \overline{\nu'} y^\mu \partial_\mu \nu' + \overline{N} i y^\mu \partial_\mu N + M_{\nu'} \nu' \nu' + M_N N N$$

(26)

Physics content of this Lagrangian is now clear, it is free Lagrangian for two particles $\nu'$ and $N$, with masses $M_{\nu'}$ and $M_N$.

We put results Eq. (23) in neutrino mass matrix

$$M = \begin{bmatrix} 0 & M_D \\ M_D & M_R \end{bmatrix}$$

(27)
with $M_R$ domination $M_D$.

The eigenvalues are then

$$M_N \equiv M_R \quad (28)$$

and

$$M_{\nu'} \equiv -\frac{M_D^2}{M_R} \quad (29)$$

In term of the fields $f$ and $F$ of Eqs. (25) the corresponding eigenvector fields are:

$$N \equiv F + \frac{M_D}{M_R} f \quad (30)$$

And

$$\nu' \equiv f - \frac{M_D}{M_R} F \quad (31)$$

In order to get positive mass for the light neutrino, we take the physical neutrino field to be the field $\nu$ related to $\nu'$ by:

$$\nu = \gamma_5 \nu' \quad (32)$$

Or, in other words, we had negative eigenvalue of mass and by redefining $\nu = \gamma_5 \nu$ we make it positive. Note that $\nu' \nu' = -\bar{\nu} \nu$, but $\bar{\nu}' \nu^\mu \partial_\mu \nu' = +\bar{\nu} \nu^\mu \partial_\mu \nu$. Thus, in terms of $\nu$ and $N$, the Lagrangian of Eq. (27) becomes:

$$-L = \left[ \bar{\nu} i \gamma^\mu \partial_\mu \nu + M_\nu \bar{\nu} \nu \right] + \left[ \bar{N} i \gamma^\mu \partial_\mu N + M_N \bar{N} N \right] \quad (33)$$

With

$$M_\nu \equiv \frac{M_D^2}{M_R} \quad \text{and} \quad M_N \equiv M_R \quad (34)$$

If $M_D \sim M_{q \text{ or } e}$, a typical quark or charged lepton mass, as we expect, then from Eq. (34) we see that

$$M_\nu M_N = M_{q \text{ or } e}^2 \quad (35)$$
This is the famous see-saw relation! If there is indeed a heavy neutral lepton $N$, as Eqs. (34) and (35) suggest, then one can understand from the see-saw relation why the ordinary neutrino is light. Conversely, if one believes in the mechanism that we have described, the fact that the neutrino is light suggests that we should find sooner or later a heavy neutral lepton $N$. [4]

**Conclusion**

In the seminar we discussed about Dirac and Majorana neutrinos. We started with standard Lagragian and added to Lagragian the Majorana part Eq. (17), which bring us to neutrino mass matrix. To get the meaning of mass matrix we had to calculate vacuum expectation value for left handed and right handed fields. In order to simplify our result we took $\rho = 1$ as in Stardad Model and from that we obtained our see-saw relation.

For theory it would be more appropriate that Majorana neutrino exist. But for now we still don’t know what kinds of neutrinos exist in nature. It can happen that both Majorana and Dirac neutrinos exist (but only one of them interact weakly) or only one of them exist. More time and experiments are required to confirm or reject our theory.

**Dirac equation and notations**

The time component of the four vectors is $x_4 = ct$. The Dirac equation of a free fermion is:

$$\left[i\gamma_\mu \partial^\mu - M\right]\psi(x) = 0$$

(37)

For given momentum $p$, this equation has four linearly independent solutions:

$$u(p, s) = \sqrt{\frac{E + M}{2M}} \left[\frac{1}{E + M}\right] \chi_s, \quad v(p, s) = \sqrt{\frac{E + M}{2M}} \left[\frac{\sigma p}{E + M}\right] \chi_s$$

(368)

where $s = \frac{1}{2}$
With:

\[
\begin{align*}
\chi_{s=\frac{1}{2}} &= \begin{bmatrix} 1 \\ 0 \end{bmatrix}, & \chi_{c,s=\frac{1}{2}} &= -\begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
\chi_{s=-\frac{1}{2}} &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, & \chi_{c,s=-\frac{1}{2}} &= \begin{bmatrix} 1 \\ 0 \end{bmatrix}
\end{align*}
\]  

(39)

Reference:


