Abstract
Gamma ray bursts (GRBs) are short flashes of gamma rays, that can release up to $10^{51}$ ergs of energy and are observable up to redshifts of $z = 10$. Their energy spectra cover a wide range of wavelengths, from gamma rays to radio. GRBs emit two jets in the opposite directions, which create shockwaves while interacting with circumburst medium. From this shocks light is emitted in the form of afterglows. Depending on the observer’s position, one can see both the gamma ray burst and the afterglow, or only the so called orphan afterglow.
1 Introduction

Gamma ray bursts are short and very bright flashes of gamma rays that last from less than a second up to several hundreds or sometimes even thousands of seconds. They are completely unpredictable, the next GRB can happen at any time. We can detect them with gamma ray detectors placed outside Earth’s atmosphere.

To date GRBs are the most energetic explosions observed in the entire Universe. One burst can release up to $10^{51}$ erg of energy\(^1\), which corresponds to approximately $5.5 \times 10^{-4}$ rest masses of the Sun\(^2\). Not two GRBs are the same, however they share similar characteristics. Distribution of GRBs in the sky is isotropic, their origin is cosmological, which means GRBs typically occur at distances, larger than 100 Mpc [1].

Most of the GRBs last a few seconds, while GRBs shorter than a second have also been observed. For this reason we divide GRBs into two classes: long GRBs - they occur after the death of a young, massive and rapidly rotating star; and short GRBs - they occur when two very massive objects (two neutron stars or a neutron star and a black hole) merge [2]. After the burst of gamma rays we can in most cases also observe an afterglow in a wide range of frequencies (from X-rays to radio frequencies). Afterglows usually last from days, weeks, months or even years [4]. Depending on the position of an observer, this afterglow can be seen on-axis or off-axis. Afterglows seen off-axis, in which initial gamma ray burst is not detected, are called orphan afterglows.

In this seminar I will first present the standard GRB fireball model with jetted emission, then I will explain the origin of the GRB afterglow, some basic properties of orphan afterglows and their rate of detectability.

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\(^1\)1 erg equals $10^{-7}$ J.

\(^2\)The rest mass of the Sun is $M_\odot c^2 = 1.8 \times 10^{-47}$ J.
2 Standard GRB model

GRBs are different from most of the observable objects in the Universe, due to their emitted energy spectra. Unlike other sources of radiation, which mostly radiate as black bodies, GRBs follow a power-law distribution of energies

\[ dN \sim E^{-\xi}dE, \]

where \( \xi \) is typically 1.3 and 2.5. The most general property of GRBs is non-thermal emission with maximal radiated energy in the energy range between 100 keV and 1 MeV, which corresponds to hard X-rays and soft gamma-rays, respectively [1]. Energy spectrum of GRB 990123 is shown in figure 1.

![Energy spectrum of GRB 990123](image)

Figure 1: Energy spectrum of GRB 990123 [5].

GRB explosion creates two relativistic narrow ejecta (jets), which point in the opposite directions. The Lorentz factor \( \Gamma \) of these jets is typically between 100 and 1000. Jets that propagate with velocities near the speed of light, have an important effect on the intrinsic spectra: they cause a large Doppler shift. For example, a photon, observed at a few MeV, may have been radiated at a few keV. High redshifts, measured from GRBs spectra, lead to the conclusion, that GRBs are of cosmological origin [2].

2.1 Jetted emission

If GRBs were spherical explosions, their emission would be isotropic and their emitted energy would be far greater than observed. The evidence for jetted outflows follow from long-term observation in radio frequencies. A few weeks after the initial burst, when the outflow slows done and is no longer relativistic, its emission becomes isotropic in radio frequencies. With this we can measure the total kinetic energy of the ejecta, which does not exceed \( 10^{51} \) erg.

Ejecta of the burst slow down through interaction with surrounding medium, which also makes it expand sideways. Initial jet is emitted into a cone of half opening angle \( \theta_j \) (figure 2). Opening angles vary between 1° and 25°, with a clear concentration around 3°[1]. Initial energy per solid angle \( \varepsilon \) and Lorentz factor \( \Gamma \) are uniform inside half opening angle \( \theta_j \), but drop quickly outside \( \theta_j \). Due to relativistic effects, light from a blob of matter
with a bulk Lorentz factor $\Gamma$ is beamed into a beaming angle $1/\Gamma$. Special relativity shows that particles, moving with the speed near the speed of light, radiate most of their energy in the same direction, in which they move. Therefore most of the energy is Doppler beamed into an angle of angular size $\theta \sim 1/\Gamma$. Due to a large Lorentz factor $\Gamma$ this angle is initially relatively small. At that time observer sees only a small fraction of the jet and cannot determine, if the ejecta is beamed or isotropic. When jet propagates into surrounding medium and it slows down, the beaming angle $1/\Gamma$ increases and the emitting surface of the afterglow, that is observed by an outside observer, increases with time. At later times, when $\Gamma$ decreases enough, we can see that there is no matter at larger off-axis angles, which means that emission is not spherical.[1].

Figure 2: Relativistic outflow is beamed into a half opening angle $\theta_j$. When ejecta slows down, a break in the light curve occurs[3].

The slowing of the ejecta causes a change in hydrodynamical behaviour and an achromatic break in the brightness of the afterglow, which is shown in figure 2. When $\Gamma \sim 1/\theta_j$, relativistic beaming effect becomes less effective. Jet then expands into an opening angle $1/\Gamma[4]$. The break in the light-curve indicates a transit between ultra-relativistic and relativistic or none-relativistic phase of the jet. After that $\Gamma$ decreases exponentially with radius.

Due to the jetted emission we can detect only events, which are directed towards the Earth. If we assume that GRB luminosity is independent of its redshift $z^3$ and consider a lognormal probability density for the luminosity $L$ with mean $\mu$ and width $\sigma$ given by

$$p(L) = \frac{1}{\sqrt{2\pi}\sigma L} \exp\left(-\frac{(\log L - \mu)^2}{2\sigma^2}\right),$$

we can estimate the fraction of detectable GRBs as a function of redshift $z$:

$$F(z) = \frac{dN_{\text{detectable}}}{dN_{\text{GRB}}(z)} = \int_{L_{\text{lim}}(z)} p(L) dL,$$

$^{3}$Redshift $z$ is used for measuring of cosmological distances in the Universe. Due to expansion of the Universe the light, emitted by objects that are far away, shifts towards longer wavelengths. If $\lambda_o$ is the observed wavelength and $\lambda_e$ is the emitted wavelength, then $1 + z = \lambda_o/\lambda_e$. 


where \( dN_{\text{detectable}} \) is the number of detectable GRBs, \( dN_{\text{GRB}(z)} \) is the number of all GRBs at given \( z \) and \( L_{\text{lim}(z)} \) is the limiting luminosity of our detector. \( F(z) \) shows a steep decrease when \( L_{\text{lim}(z)} \) increases, which reduces the probability of detecting a GRB at larger redshifts[6].

### 2.2 Fireball model and shocks

When a gamma ray burst occurs, all of the energy observed in gamma rays is released into a small space with a size around a few thousand kilometres in just a few seconds. Huge amount of energy inside a small volume implies the formation of a fireball, which consist of positrons, electrons and photons. This ”soup” of particles and light is opaque, meaning only a few photons can escape from it. In order for gamma rays to escape from the central object, its material must be accelerated to relativistic speeds. The compact source has such a high energy density, that gamma rays collide and make electron-positron pairs. The pairs then annihilate and form high-energy gamma rays, which can escape from the central object.

When the fireball begins to expand the energy associated with the internal random motions of moving particles and photons is transferred into the bulk outward flow. We assume that GRB jets behave like they are a part of the spherical fireball. Outflowing material then crashes into surrounding medium. Due to low densities of both relativistic outflow and surrounding medium, direct interactions are highly unlikely. For the conservation of both the energy and the angular momentum, particles must be connected via long-range forces, such as those created by magnetic fields at the edge of the fireball.

![Fireball model with production sites of internal shocks (prompt emission) and external shocks (afterglow)](image)

Interaction with circumburst medium produces a shockwave. Shocks are places where kinetic energy of the outflow is transferred into radiated energy. Shocks are also the reason for none-thermal spectrum of GRBs. There are two interactions that cause shockwaves, both shown in figure 3.
Internal shocks happen when a shell in the outflow with a higher Lorentz factor $\Gamma$ catches up with another shell, whose $\Gamma$ is smaller, and the shells merge. Internal shocks take place inside of the moving ejecta and are highly relativistic. They explain rapid time variability of the GRB prompt emission light-curves.

External shocks occur when outflowing mass runs into circumburst medium near the explosion site, which causes the outflowing material to slow down. In the process of deceleration some of the outflow’s kinetic energy is transferred into motion of particles. This process can radiate light. With external shocks we can explain conversion of the bulk outflow into radiated energy and the origin of the afterglows. Typically external shocks take place after internal shocks and are mildly relativistic [2].

2.3 Acceleration of particles

Particles can radiate light, if a given particle is accelerated or decelerated. Hot fireballs of energy and plasma eventually start to expand and their internal thermal energy is transferred into bulk kinetic outflow with relativistic speeds. If a relativistic shell in the ejecta, which is moving faster, catches up with a shell, that is moving slower, the faster shell starts to slow down, which creates an internal shock. From the inner (faster) shell’s point of view it looks like particles from outer (slower) shell ”fall” toward the inner shell. When particles enter the shocked region, their trajectories change in comparison with their initial radial trajectories. This process releases internal thermal energy, which can be radiated.

With internal shocks electrons are accelerated to relativistic speeds. Charged electron is accelerated by magnetic field, that changes its trajectory and increases its kinetic energy. After multiple changes of trajectory electron gains a lot of energy and can travel with speed, which is much larger than the speed of the shock [2]. Electrons, accelerated by shocks, emit synchrotron radiation. If an electron interacts with nearby photons, it decelerates. In that case it emits light, which is called inverse Compton scattering.

All of the electrons that are accelerated within shocks lead to a power-law distribution of photon energies (equation (1))[4].

3 Afterglows

Afterglow is a delayed emission that covers a wide range of frequencies, from X-rays to radio. Afterglows are described by synchrotron emission from accelerated electrons when a relativistic shell collides with circumburst medium. In principle afterglow is all of the radiated light from GRB, which is not connected to the same emission processes as the prompt emission. The properties of each afterglow depend on the surrounding medium. Afterglow spectra and light-curves are consistent with synchrotron radiation from power-law distribution of electrons [8]. Considering this distribution we can determine light-curve seen by the observer [4]:

$$F(t, \nu) \propto t^{-\alpha} \nu^{-\beta}. \quad (4)$$

3.1 Origin of the emission

When a relativistic shell of the jetted outflow slows down in circumburst medium, its kinetic energy converts into random motion of electrons and neutrons. This creates an external shock, which moves forward. External shocks radiate light at longer wavelengths...
contrary to internal shocks and prompt emission.

Afterglow begins at a distance from the central object, where most of the outflow’s energy is transferred into surrounding medium. Its luminosity increases and reaches its maximum when the distance from the center $R$ reaches a typical deceleration radius

$$R_{\text{dec}} \propto \left( \frac{3E_0}{4\pi nm_pc^2\Gamma^2} \right)^{1/3}.$$  

$(5)$

$E_0$ is initial energy of the burst, $n$ number density of surrounding medium, $m_p$ mass of a proton and $c$ the speed of light. From here on luminosity decays quickly with time[1].

### 3.2 Spectra of afterglows

The peak of afterglow’s light occurs at typical synchrotron frequency $\nu_m$. At frequencies, larger than $\nu_m$, spectrum decays as a power-law $F(\nu) \propto \nu^{-\beta}$, where $\beta = (p - 1)/2$. $p$ describes the energy distribution of electrons and has a typical value between 2 and 2.5. At higher frequencies the slope of the spectrum becomes steeper, due to electrons radiating a lot of their kinetic energy. The joint of both curves is called cooling break and it happens at cooling frequency $\nu_c$ (figure 4).

At frequencies that are lower than $\nu_m$, the dominant part of the spectrum is low-energy synchrotron radiation of electrons, where $\beta = -1.3$.

![Figure 4: X-ray to radio frequency spectrum of a GRB 970508 afterglow 12.1 day after the initial burst[1].](image)

$\Gamma$ decreases with time, therefore spectra also evolve with time. The peak moves from X-ray frequencies into visible and radio light in time scales from minutes to weeks. Flux before the maximal brightness increases with power-law, after the maximum it decreases with a different power-law. Evolution of spectra with time is given by $F(t) \propto t^{-\alpha}$, where $\alpha = 3(p - 1)/4$ after the maximum.

Spectra of afterglows also show absorption and emission properties. Absorption usually takes place at lower energies, in visible, IR and UV part of the spectrum and is a consequence of absorption in medium between the Earth and the GRB. From absorption lines
we can infer some properties of gas and dust surrounding the GRB and of its host galaxy. In the first few seconds of the GRB more than $10^{55}$ high-energy photons escape from it. These photons cause a full ionization of lighter elements (H, He) in circumburst medium, while heavier elements (Fe, Ni) retain a few of their electrons. High-energy photons move some of the electrons to excited states, but they quickly move back to lower-energy state, and by doing so they emit low-energy photons at given wavelengths[2]. In spectra we can therefore detect lines from fine-structure levels of the ground state Fe and Ni, that vary significantly with time. This process, called UV-pumping, was observed in GRB 100901A and is particularly helpful when determining distance of the absorber from the location of the burst[9].

When observing afterglows we first detect an afterglow in the X-ray part of the spectra. This helps us with precise determination of the location of the GRB on the sky. X-ray afterglow is followed by UVOIR (UV, optical, IR) afterglow. Its light-curve is usually not correlated with GRB itself, which leads to a conclusion, that UVOIR afterglows are consequences of different emission mechanisms and (or) emission sites than high-energy prompt emission. At radio wavelengths afterglows peaks only a few days or even weeks after the prompt emission, which helps us with determining the total energy of the GRB[2].

3.3 Host galaxies

Light from GRB afterglow travels through its host galaxy. On its way it can be absorbed in gas or dust. From absorption lines of the afterglow we can estimate metallicity$^4$ of the host galaxy. Typical metallicities are lower than the Sun’s metallicity.

Long GRBs host galaxies are typically blue, faint and irregular, with sizes comparable to those of Small and Large Magellanic Clouds. Their metallicities are very low and they are found at redshifts $z > 1$. Host galaxies of short GRBs are massive elliptical galaxies that are found at lower redshifts $z = 0.1$ to $z = 0.2$. After the afterglow we can often see the faint galaxy. From this observations we can determine the precise location of the galaxy[2].

4 Orphan afterglows

Jetted emission of GRBs has an important consequence: the existence of orphan afterglows. Orphan afterglows are physically exactly the same as normal afterglows, but it depends on the position of the observer, if they are detectable or not. An observer, who is off-axis of the GRB jet, cannot see the initial jet, but can observe the orphan afterglow, an afterglow without prompt gamma-ray emission. Observer at $\theta_{\text{obs}}$ outside the opening angle $\theta_j$ can only see the afterglow after $1/\Gamma$ becomes larger than the angle between jet’s axis and observer’s line of sight[1]. The geometry of orphan afterglows is shown in figure 5.

Most of the GRB jets are not pointed towards the Earth, therefore we can expect a large number of orphan afterglows. However, no orphan afterglows have been discovered jet. With detection of orphan afterglows we would be able to estimate the opening angles of GRB jets, the true rate of GRB and confirm the standard model of GRBs[11].

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$^4$Metallicity measures amount of metals - elements, heavier than hydrogen and helium.
4.1 Basic properties

Orphan afterglows are slow transients seen off-axis that become visible on time scales from days to years in optical, near IR and radio frequencies, when the prompt emission in gamma and X-rays has already ceased. Considering a typical opening angle of 3°, there should be approximately 730 GRBs, pointing in other directions, for every GRB pointing towards the Earth.

A GRB observed off-axis at observable angle $\theta_{\text{obs}} > \theta_j$ will be undetected, because an observer cannot see the prompt emission. Afterglow emission can be observed only after $1/\Gamma \sim \theta_{\text{obs}}$. At that time the brightness of the orphan afterglow reaches its maximum. Flux already rises before the maximum, when the edge of the jet is getting closer to $\theta_{\text{obs}}$. After the maximum afterglow for an off-axis observer is the same as it would be, if the observer was on-axis (if $\theta_{\text{obs}}$ was smaller than $\theta_j$). Figure 6 represents on-axis and off-axis afterglow light-curves.

Figure 6: Light-curves of on-axis ($\theta_{\text{obs}} = 0°$ and 5°) and off-axis GRB afterglows($\theta_{\text{obs}} = 10°$, 15°, 20° and 25°)[10].
Orphan afterglows are detectable at any frequency, but do not contain the prompt emission of gamma rays. We have to study them differently than normal afterglows, where we first detect the prompt emission of gamma rays and then do follow-up observations at other wavelengths. As explained in section 3.3, high-energy photons from the burst ionize most of the elements in the surrounding gas. When observing orphan afterglows, we could observe the medium, that has not been affected by the prompt emission of photons. Therefore, we can say that we could study the ”pristine” gas of the host galaxy. There is a chance we could detect orphan afterglows with large synoptic sky surveys as transients. Up to date they were not observed due to low sensitivity of detectors, however, we do expect to observe them in the future surveys. One of the greatest challenges is how to differ orphan afterglows from other transient sources. There is a clear advantage of observation in radio frequencies. Maximum of brightness only happens after a few days. That enables detection and follow-up observations of GRB radio emission, when the flux at higher frequencies is already under the detector’s sensitivity[12].

4.2 Detectability of orphan afterglows

If we assume that all of the GRBs release the same amount of energy, post jet-break evolution of the afterglow is more or less universal. After the jet-break the afterglow can be observed at larger $\theta_{\text{obs}}$ over time. Post jet-break light-curve decays very quickly, therefore most of the orphan afterglows will be dim and undetectable. From the light-curve it is possible to determine maximal flux at an observing angle $\theta_{\text{obs}}$, from which we can estimate the total number of orphan afterglows detectable with a given limiting magnitude of instrument and as a function of $\theta_{\text{obs}}$ and $z$.

Jet-break happens at

$$t_j = 0.7(1 + z)\left(\frac{E_{51}}{n_0}\right)^{1/3} \left(\frac{\theta_j}{0.1}\right)^2 \text{days},$$

where $Q_x$ denotes value of quantity $Q$ in units $10^x$ its value in cgs units.

Due to relativistic beaming effect an observer at $\theta_{\text{obs}}$ outside of initial opening angle of the jet ($\theta_{\text{obs}} > \theta_j$) detects the afterglow emission over the time $t_\theta$, when $\Gamma = 1/\theta_{\text{obs}}$:

$$t_\theta = A \left(\frac{\theta_{\text{obs}}}{\theta_j}\right)^2 t_j.$$

A is of order unity and its value changes from one model to another. Here we take $A = 1$. Approximately at $t_\theta$ afterglow emission is also maximal. From here on the light-curve of the orphan afterglow is the same as in case of a normal, on-axis afterglow.

Maximal flux is strongly dependant of $\theta_{\text{obs}}$ and independent of $\theta_j$. It decays quickly when the observer moves away from jet axis. If we know the maximal flux $F_{\nu}^{\text{max}}(\theta_{\text{obs}})$, we can estimate the maximal observing angle $\theta_{\text{max}}(z, m)$, at which the afterglow is brighter than the limiting magnitude $m$.

An observer at $\theta_{\text{obs}}$ will detect the afterglow if $\theta_{\text{obs}} < \theta_{\text{max}}(z, m)$ and will be able to observe it over $t_{\text{obs}}(z, \theta, m)$ until the signal becomes lower than the limiting magnitude[11]:

$$t_{\text{obs}}(z, \theta, m) \sim \frac{A t_j}{\theta_j^2}(\theta_{\text{max}}^2 - \theta_{\text{obs}}^2).$$
The estimated rate of observed orphan afterglows over the sky from \( z = 0 \) to 10 is:

\[
R_{OA} = \int_0^{10} \frac{n(z)}{1+z} \frac{dV(z)}{dz} dz \int_{\theta_j}^{\theta_{max}(z,m)} d\theta.
\]

(9)

\( n(z) \) is rate of GRBs per unit volume and unit proper time and \( dV(z) \) is differential volume element at redshift \( z \). We assume that GRB rate is proportional to star formation rate (SFR) and is given by \( n(z) \propto 10^{0.75z} \) for \( z \leq z_{peak} \) and \( n(z) \propto 10^{0.75z_{peak}} \) for \( z_{peak} < z < 10 \). At \( z_{peak} \) the star formation rate peaks (SFR peaks at \( z = 1 \) and \( z = 2 \)).

Usually the detector’s exposure time is shorter than \( t_{obs}(z, \theta, m) \) and we can estimate the number of detectable orphan afterglows in a single snapshot of the sky:

\[
N_{OA} = \int_0^{10} \frac{n(z)}{1+z} \frac{dV(z)}{dz} dz \int_{\theta_j}^{\theta_{max}(z,m)} t_{obs}(z, \theta, m) \theta d\theta.
\]

(10)

At limiting magnitude \( m = 25 \) in visible wavelengths the median observing angle is between 5° and 7°, which is comparable to the opening angle. Therefore most of the orphan afterglows will be "near misses" of an on-axis event. \( \theta_{max}(z_{peak}) \) becomes larger \( \sim 10^\circ \) for \( z_{peak} = 1 \) and \( \sim 14^\circ \) for \( z_{peak} = 2 \). At limiting magnitude \( m = 27 \) the median of the angular distribution is 12°. This is larger than most GRB beaming angles, but still narrow, corresponding only to 2% of the sky. Estimated rate of detectable orphan afterglows is around 35 per year.

The number of detectable orphan afterglows decreases with various factors. We must take into account extinction between GRB and the Earth and the ability for identification of a weak transient on top of a background host galaxy. Other obstacles include weather conditions and the ability to distinguish orphan afterglows from other transients[11].

5 Conclusion

Gamma ray bursts are the most energetic events ever recorded in the Universe. They are connected to deaths of a young and massive stars or to mergers of two compact objects. GRBs show power-law distribution of energies, connected to the movement of relativistic particles. Their emission is jetted, typically the ejecta are beamed into a half opening angle of 3°. With internal shocks in the ejecta itself and external shocks of the ejecta with surrounding material we can explain non-thermal spectra and the origin of the afterglow. Afterglows occur when the ejecta interacts with cirbumburst medium. They are an important evidence for jetted emission and they also show power-law spectra. After the afterglow fades away, we can often observe the host galaxy. Depending on the observer’s position, one can see both the prompt emission of gamma rays and the afterglow, or one can only sees the afterglow. In that case the observer is off-axis of the initial jet and sees an afterglow, called the orphan afterglow. None have been detected yet. With future synoptic sky surveys we expect to detect at least 35 of orphan afterglows per year.

GRBs and their afterglows are conveniently useful for exploration of the early, young Universe, first populations of stars and metallicities of the first galaxies. If we use GRBs as standard candles, we can observe the Universe much further than \( z = 2 \), which is especially helpful with determination of cosmological parameters. GRBs themselves are not standard candles, but we can standardize them with proper correlations (for example Amati’s correlation between peak and isotropic energy) and use them as they were
standard candles to determine cosmological parameters.

There are a few quite important factors, that affect the true rate of GRB. All of the telescopes are blind for some parts of the sky at any time. Sensitivity of the detectors is often too low, because we can detect GRB only to a limited magnitude, meaning we can observe them to a limited distance. For determining the redshift z, we must observe the afterglow, which is many times too faint to make a high quality spectrum. Often we cannot separate absorption lines from continuum of the spectra. Due to the jetted emission, we miss most of the GRBs, which are not pointed towards the Earth.

Literature


[10] LSST Science Collaborations and LSST Project 2009, LSST Science Book
