Cooling by Heating

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Abstract

The concepts of energy sufficiency and efficiency are of great importance nowadays. Therefore the importance of sunlight as an alternative energy is in increase. A surprising new device, which uses sunlight for refrigeration has been presented recently. Although the idea is not a total novelty, as through presentation of laser-cooling is showed in the seminar, it raises huge hopes for technological application. The main part consists of quantum dots, which have been in the center of studying since their invention in 1990s and have been applicable in many devices already. Through the presentation of laser-cooling, quantum dots and especially the refrigeration device, one can see how knowledge from different parts of physics can be used to create an impressive device with potential to be applicable in common life.
1 Introduction

As through the whole history, the mankind has been facing problems of energy sufficiency and efficiency. Nowadays, when conservative sources of energy are slowly but systematically losing its monopoly in global market the use of alternative energy sources is becoming more and more important. One of the main contenders of clean energy is solar power. The energy of solar photons has already been implemented in solar cells generating electrical energy. But solar energy can be used directly to produce heat and drive a generator as well, such as in solar furnaces. However, a very new and surprising approach for application of solar power has been proposed recently. A simple solid state device which uses photons to perform the task of refrigeration directly, bypassing the need to first generate another form of energy. Although the idea of cooling directly by heating is not totally revolutionary, as it has already been integrated in relatively old evaporative cooling mechanism and laser cooling devices, fresh approaches are emerging. As these applications avoid the transformations between different forms of energy, which are normally accompanied by losses, it gives us hope that such cooling mechanisms would be more efficient. [1]

The main part of this seminar is dedicated to aforementioned new method of cooling by photons. However, first few lines present the "mother" of ideas for cooling by using photons - laser cooling. Furthermore as a background for the main subject in this seminar we discuss another important technological achievement of our time - quantum dots. [2]

2 Laser cooling

The basic ideas for slowing down the atoms by irradiation reach back in the early years of the past century. The theoretical basics of reducing atom's velocity by light irradiation were presented already in 1917 by Albert Einstein and proven by Ottos Frisch experiments in 1933. However the idea of such cooling was reborn in late 1980’s due to the technological development. Before trying to explain the nature of process, we need to understand, what is the origin of temperature, or how we distinguish cooler from hotter systems. The easiest way to explain this is to imagine free atoms in a gas. They move around the space with different velocities in different directions, and they are changing these quantities all the time. The kinetic energy of these atoms is the origin of system’s temperature. The higher the average kinetic energy of atoms, the higher the temperature. Therefore, if we on average somehow manage to slow down these atoms, the temperature would decrease as well. This effect is achieved by photon irradiation of an atom moving antiparallel to the light beam. It follows that by understanding the scattering force of a photon acting on an atom during the process, we would practically
understand the idea of such cooling. Scattering force is a momentum transferred to the atom in a period of time during scattering of a photon. [2]

The average scattering force is in the direction of propagation of the light and equals to the product of photon momentum $\hbar k$ and the photon scattering rate. The photon wavevector $k$ has magnitude $2\pi/\lambda$, where $\lambda$ is the wavelength of light. The average force reaches maximum when the light is resonant with atomic transition to excited state. The scattering force fluctuates because the photons scatter at random times and because the direction of the re-emitted photon, and hence the direction of the recoil momentum due to this re-emission, is random as well. However, if we irradiate an atom from a certain direction all the time, the net momentum added to the atom would be parallel to the light beam. This follows from the fact, that absorbed photon adds a momentum in particular direction, but on the other hand, emitted photon adds on average momentum equally in all directions. [2]

These conclusions sum up in the real process as follows. A laser beam which frequency is close to but lower than the atomic resonance frequency irradiates the atom. If the atom is moving against the laser beam, then the frequency of light "sensed" by the atom is Doppler shifted toward resonance. Hence, the scattering force is higher for an atom moving against the laser beam, and the atom’s velocity is damped. In this way the velocity of an atomic beam can be reduced substantially. Without another beam with the same intensity and frequency in the opposite direction, the atom would eventually turn around and start moving parallel to the beam. Therefore one can see that in three dimensions at least three pairs of (equal intensity and frequency) laser beams are needed. During the process the atoms are substantially trapped in smaller and smaller space as we can see in Figures 2. As the average atom speed decreases, the Doppler effect becomes smaller, and laser beam frequency slowly limits to the frequency of transition rate. By slowly shifting the frequency of laser beam, one can almost totally calm down the atoms. However, due to the inherent fluctuations of scattering force and thermic energy, the velocity is never dumped to zero. [2]

The theoretical minimum temperature that can be obtained is given by a balance between the dissipation and fluctuations. Assume the laser intensity is below saturation, even when tuned to resonance. We also assume that the recoil energy, given by $(\hbar k)^2/2m$ is less than $\hbar \gamma$. The recoil energy is the kinetic energy that an initially motionless atom of mass $m$ would have due to its recoil after emitting a photon of wavevector $k$. The minimum temperature $T_{\text{min}}$ is achieved when the laser frequency is tuned below the atomic resonance frequency by an amount.
equal to $\gamma/2$, in which case:

$$kT_{\text{min}} = \frac{1}{2} \hbar \gamma$$

For a decay rate $\gamma$ of $10^8$/s, the minimum temperature is about 0.38 mK.[2]

Laser cooling has been having an important role in many experiments. It opens ways for studying collisions between very cold atoms or ions and atom-surface collisions at low temperature. Furthermore it can help us obtain unique states of condensed matter. One possibility, for example, is observing liquid and solid plasmas.

3 Quantum dots

Although quantum dot emerged and breakthrough almost 25 years ago, it still plays an important role in modern technology. Furthermore it is the main part of the refrigeration device presented in following section as well. For better understanding of refrigeration process in that device the knowledge of quantum dots is essential.

Quantum dots are one of the technical structures with a common name artificial atoms. The artificial atoms are particles of metal or pools of electrons in a semiconductor that are only a few hundred angstroms in size, coupled to electrical leads through tunnel junctions. They have shown some unique and spectacular properties, for instance that the current through it can vary by many orders of magnitude when its charge is changed by a single electron.

The basic idea is to confine the electrons into the movement in only two or even one dimension. One way to confine electrons in a small region is by employing material boundaries by surrounding a metal particle with insulator, for example. Alternatively, one can use electric fields to confine electrons to a small region within a semiconductor. Figure 3 shows one type of quantum dots, called the controlled-barrier atom. Although we know three basic types of quantum dots, it is the only relevant for further discussion.[3]

The confinement of space is accomplished with electric fields in semiconducting GaAs (labelled with red). It has a metal ”gate” on the bottom (blue) with an insulator (white) above
Figure 3: The form of artificial atoms - the controlled-barrier atom. Areas shown in blue are metallic, white areas are insulating, and red areas are semiconducting. The inset shows a potential similar to the one in the controlled-barrier atom, plotted as a function of position at the semiconductor-insulator interface. The electrons must tunnel through potential barriers caused by the two to constrictions.

it. With combination with GaAs as semiconductor, for the insulator is mainly used AlGaAs. Because of the strong electric field at the AlGaAs-GaAs interface, the electrons’ energy for motion perpendicular to the interface is quantized, and at low temperatures the electrons move only in the two dimensions parallel to the interface. The special feature is the pair of electrodes (blue) on the top surface of the GaAs (red). When a negative voltage is applied between these and the source or drain, the electrons are repelled and cannot accumulate underneath them. Therefore the electrons are confined in a narrow channel between the two electrodes. The shape of electrodes implies the space where electrons are confined by creating potential barriers similar to the one shown in Figure 3. For an electron to travel from the source to the drain, it must tunnel through the barriers. That is why this type of artificial atom is called the controlled-barrier atom, as one can control the height of these barriers by varying the voltage on electrodes.

Being familiar with the structure of artificial atoms, it is now appropriate to point out some of its important characteristics. Looking for parallels with natural atoms, we would like to find out what is the energy needed to remove or add an electron into the quantum dot. In nature these processes are accompanied with photon absorption or emission. As we will see in the next section, the transition of electrons due to the absorption and emission of a photon appears in this new refrigeration device as well. However in the artificial atom this is stimulated by the changes of gate voltage. Therefore we can obtain these energies by measuring the current through the artificial atom as a function of the voltage between the gate and the atom. We obtain the results, that the conductance displays sharp resonances that are almost periodic in gate voltage. Furthermore, if we calculate the capacitance between the artificial atom and the gate we can show that the period is the voltage necessary to add one electron to the confined pool of electrons. This phenomena is very well theoretically explained by the classical Coulomb blockade model. The highest conductance appears when the charge in electron pool equals to \( Q = -(N + 1/2) e_0 \), where \( N \) is a prime number. Through quantum mechanical aspect, this state suffice to fifty per cent probability to find an electron in or outside the potential well. The most favourable classical perception would be, that the electron jumps in and out of the pool all the time, because it is totally indecisive considering where it would prefer to stay. On the other hand, the conductance practically drops to zero in case of \( Q = -Ne_0 \). Therefore it is reasonably to conclude that artificial atoms share with natural atoms the quantization of charge as a common characteristic.
Because of the small size of the artificial atom, beside the charge quantization, the quantization of energy appears as well. This is a totally quantum mechanical effect, which appears at confinement of particle in sufficiently small space. If there is a lot of electrons in artificial atoms, about $10^7$, than the level spectrum is effectively continuous. However in the controlled-barrier atom, there are only about 30 to 60 electrons, similar to the natural atoms. Therefore we face a discrete energy spectrum to add an extra electron in the latter object. One can measure the spectrum of energy levels directly by observing the tunnelling current at fixed gate voltage $V_g$ as a function of the voltage between drain and source $V_{ds}$. The change of flow appears when this energy rises above the first quantized energy level of an atom. During rising of $V_{ds}$, more and more energy levels are overcome, and measured current is higher. Therefore if we measure $dI/dV_{ds}$ we can obtain energy level spacings by looking for maximums. Another important characteristic is, that by increasing the gate voltage one can decrease all energy levels in the atom by $eV_g$.\[3\]

At this point we can obtain a very important conclusion, which would be relevant for our discussion in the next section. Considering aforementioned qualities of artificial atoms, we can by manipulating with shape of electrodes, gate voltage and voltage between source and the drain create a quantum dot with random relevant energy levels and random number of electrons. In the following section we would face a structure of two quantum dots, with only one electron in each of two relevant energy levels. Later on, the reader should already know the answer to the question “Is it possible to create such a structure?”, as we have already explained it here.

Quantum dot has already find its place in many technical applications. In electronic devices it has been proven to operate like a single electron transistor. It has also been suggested for implementations of qubits for quantum information processing. However, we can find quantum dots in photovoltaic, light emitting and photo detector devices as well. Although it has been a long time, since their invention, quantum dots still posses the potential for further applications.\[4\]

4 Refrigeration by replacing hot and cold electrons

In this model we consider a refrigeration on nano-sized solid state device, imposed by flow of photons. The device consists of two leads or so called electron reservoirs, connected by two neighbouring quantum dots. Each quantum dot has a lower and upper energy level while electrons in the leads obey Fermi-Dirac distribution $f(\epsilon) = 1/(\exp((\epsilon - \mu)/k_B T) + 1)$, where $\epsilon$ and $\mu$ stand for energy and chemical potential over Boltzmann constant $k_B$. The basic idea is, as shown in Figure 4, that under influence of the high temperature photons, cold electrons(those under Fermi level) are pumped from the left to the right lead transversing through the junction of the lower energy levels of the quantum dots, while an equal amount of hot electrons are pumped the other way at the junction of the higher energy levels. The result is a net replacement of hot electrons by cold electrons in the right lead, implying its refrigeration.\[1\]

An important role in this system is played by quantum dots. The proper circulation of electrons from one lead to another (marked with dashed grey arrow in Figure 4) depends on choice of levels in particular quantum dot. At equilibrium there will be no flux between them so that occupation probability in each level of each quantum dot equals to the probability to find an electron with such energy in the nearby lead. As shown in the Figure 4 the device is structured so that the upper energy level of the left quantum dot $\epsilon_g + \epsilon_2$ is higher than corresponding $\epsilon_2$ in the right quantum dot. The opposite occurs with the lower levels. This means that for the equilibrium state, where both leads have the same temperature $T_l = T_r$, ...
the occupation probability for electron in the upper level of the right quantum dot is higher than in the left quantum dot. This directly follows from the Fermi-Dirac distribution, as a higher $\epsilon - \mu$ implies a lower occupation probability of level $\epsilon$. The opposite is the case with lower levels of quantum dots. The exchange of electrons between quantum dots is modulated by photons from the Sun. As mentioned before, by this construction we obtain the movement of hot ($\epsilon > \mu$) electrons from the right lead to the left one, and the opposite for cold ($\epsilon < \mu$) electrons. The result is obviously the refrigeration of the right lead and heating of the left lead. All aforementioned movements of electrons are presented with grey arrows in Figure 4.

![Figure 4: The theoretical scheme of device - solar refrigerator, consisting of two electron reservoirs (leads), connected by two quantum dots. The leads contain electrons at different temperatures, which obey Fermi-Dirac distribution. Each quantum dot has only two relevant energy levels, which can be occupied by electrons. The gray arrows present the possible electron movement, which is induced by solar photons (curly red arrows) at the junction between quantum dots. The overall electron current through the device is shown by the dashed gray arrow.](image)

The final goal of further analysis is to obtain the coefficient of refrigeration efficiency. Therefore we would like to quantify the total electrical current and therefore the overall heat current between the leads. The first step is to quantify the transition rules of an electron to transit between lead and a quantum dot or between two quantum dots. Neglecting the effect of quantum mechanics and coupling between dots and lead, the transition probabilities can be described in a classical way. According to, we assume the transition probability $k_{\epsilon \rightarrow d}$ for electron to move from the lead into empty (unoccupied by another electron) energy level in quantum dot, is proportional to the probability $f(\epsilon)$ (Fermi-Dirac distribution) to find an electron in the lead with energy $\epsilon$. Considering this formulation, the probability of the inverse process $k_{d \rightarrow \epsilon}$ (electron jumping from the dot to lead) is proportional to the probability that energy level $\epsilon$ in the lead, is free. Therefore the corresponding rate is proportional to $1 - f(\epsilon)$. The time scale of aforementioned jumps is implied by proportional constant $\Gamma$ of the processes. We assume that the movement from the lead to quantum dot has the same time scale as the inverse process. However this depends on technical performance of device and is not part of this discussion. Extracting the aforementioned findings writing down these equations follows:

\[ k_{\epsilon \rightarrow d} = \Gamma f(\epsilon) \]  \( (2) \)
\[ k_{d \rightarrow \epsilon} = \Gamma (1 - f(\epsilon)) \]  \( (3) \)
\[ f(\epsilon) = \left[ \exp\left(\frac{\epsilon - \mu}{T}\right) + 1 \right]^{-1} \]  \( (4) \)

The temperature $T$ in above $f(\epsilon)$ transforms into $T_r$ or $T_l$ whether we consider the transition rate from the right or left lead.

The next step is to consider the electron transitions between quantum dots. As mentioned before, each quantum dot has a higher and lower energy level, but the gap between those levels
is not the same in both dots. On the other hand the energy difference between higher and lower energy levels is the same and equal to $\epsilon_g$. In this analysis we neglect both: the possibility for electron to jump between levels within a quantum dot, and to move e.g. from right high level to left low level and other possibilities. The explanation could be, that we consider so high energy gap $\epsilon_1 + \epsilon_2$, so that solar photons are unable to modulate such transitions. Therefore the transitions of electrons can occur only between two lower or upper energy levels. These transitions are mediated by photons. An electron can jump to a higher energy level if it absorbs a photon which energy equals to the energy difference of the two levels involved. The corresponding transition rate $k_\uparrow$ is then proportional to the number of photons with such energy, so called the Bose-Einstein distribution $n(\epsilon_g)$. The proportional constant $\Gamma_s$ sets the overall time scale of the process, similar as $\Gamma$ in previous transition between the lead and quantum dot. It represents the device material characteristics. On the other hand the transition rate for the opposite process of electron jumping from higher to lower level emitting a photon is $k_\downarrow = \Gamma_s(1 + n(\epsilon_g))$. This expression takes into account probabilities for absorption and induced photon emission. It follows from quantum mechanical derivation for discussed process \[5\]. Writing down these conclusions in coherent way gives us \[1\]

\[ k_\uparrow = \Gamma_s n(\epsilon_g) \quad \quad \quad (5) \]
\[ k_\downarrow = \Gamma_s (1 + n(\epsilon_g)) \quad \quad \quad (6) \]
\[ n(\epsilon_g) = \left[ \exp(\epsilon_g/T_s) - 1 \right]^{-1} \quad \quad \quad (7) \]

In further analysis we make two additional assumptions. First, transitions between lower and higher energy levels within the same quantum dot are neglected. As a result the flows of electrons between the leads through lower or upper levels are physically decoupled from each other, and described by an independent equation. Second, due to Coulomb repulsion, there can be only one electron in the quantum dot at the time. By taking these assumptions into account, we realize it is reasonable to consider probabilities $p_0$, $p_l$ and $p_r$ to find none in both levels, or a single electron in the left or right energy level and their time dependences. Before writing the master equation $\hat{p}(t) = M \cdot p(t)$ with $p(t) = (p_0, p_l, p_r)^T$ we are going to illustrate how to obtain a time dependence of probability on example of $p_0$ for transitions through lower levels $\epsilon_1$ and $\epsilon_1 - \epsilon_g$. Trying to write an expression for $\dot{p}_0$ we have to take into account all possible electron movements affecting its change. The probability for left and right level being empty, would decrease with time (\`\`\` before probabilities for these transitions) if any electron jumps from the left lead to left quantum dot, or from the right lead to the right energy level. The possibility for those transitions equals to $k_{\ell\rightarrow d}^{\epsilon_1-\epsilon_g}$ or $k_{\ell\rightarrow d}^{\epsilon_1}$. Multiplying these transition rates with probability for both energy levels being empty $p_0$, we obtain the first factor: $\dot{p}_0 = (-k_{\ell\rightarrow d}^{\epsilon_1-\epsilon_g} - k_{\ell\rightarrow d}^{\epsilon_1})p_0 + \ldots$. Furthermore if there is an electron in left level with probability $p_l$, the probability $p_0$ would increase with time if electron jumps from left level $\epsilon_1 - \epsilon_g$ to left lead. Therefore follows the second factor $\cdots + k_{d\rightarrow l}^{\epsilon_1-\epsilon_g} + \ldots$ And at last lets consider the case of electron being in the right energy level with probability $p_r$ and than leaving it according to the transition rate $k_{d\rightarrow l}^{\epsilon_1}$. Obviously the third factor follows as $\dot{p}_0 = \cdots + k_{d\rightarrow l}^{\epsilon_1}$. Summarizing all three factors we obtain the full expression for $\dot{p}_0$. Following the same principle the expressions for $\dot{p}_l$ and $\dot{p}_r$ as a function of $p_0, p_l, p_r$ can be obtained easily as well. After that the master equation follows:

\[
M = \begin{pmatrix}
-k_{\ell\rightarrow d}^{\epsilon_1-\epsilon_g} - k_{\ell\rightarrow d}^{\epsilon_1} & k_{d\rightarrow l}^{\epsilon_1-\epsilon_g} & k_{d\rightarrow l}^{\epsilon_1} \\
k_{\ell\rightarrow d}^{\epsilon_1} & -k_{d\rightarrow l}^{\epsilon_1-\epsilon_g} - k_\uparrow & k_\downarrow \\
k_{\ell\rightarrow d}^{\epsilon_1} & k_\uparrow & -k_{d\rightarrow l}^{\epsilon_1-\epsilon_g} - k_\downarrow 
\end{pmatrix}.
\]
In the next step the resulting average electron (particle) current through the dots is of our main interest. Namely we are not interested in unstable movements which occur for a short period of time and can be easily neglected, but focused on main contribution to refrigeration. Average current can be, respectively, obtained through the condition of stable state. In stable state \( p_0, p_l, p_r \) probabilities are time independent and therefore \( \dot{p} = 0 \). A focused reader can notice that matrix \( M \) simplifies with one simple trick. We can see that the third row is the sum of the first and second. Therefore we can modify matrix with linear transformations so, that all places in the last row are equal to zero. We get the system of two equations and three variables. The fact that the sum of all probabilities has to equal to one, provides us with third equation. Therefore we obtain the system of three independent equations and three variables.

\[
X = \begin{pmatrix}
-k_{l\rightarrow d}^{\epsilon_1} - k_{l\rightarrow d}^{\epsilon_2} & k_{d\rightarrow l}^{\epsilon_1} & k_{d\rightarrow l}^{\epsilon_2} \\
0 & -k_{d\rightarrow l}^{\epsilon_1} - k_{d\rightarrow l}^{\epsilon_2} & k_{d\rightarrow l}^{\epsilon_2} \\
1 & 1 & 1
\end{pmatrix} \quad \mathbf{X} \cdot \mathbf{p} = (0, 0, 1) \quad (9)
\]

Solving this system provides us with results for stable state \( p_0, p_l, p_r \) probabilities.[1]

Furthermore we define \( J_1(J_2) \) as the particle current between the energy level \( \epsilon_1(\epsilon_2) \) and the right lead. As mentioned before, the current of electrons moves from level \( \epsilon_1 \) to the right lead \( (J_1) \), and then from the right lead to level \( \epsilon_2 \) \( (J_2) \)(look Figure 4). Using the same logic while obtaining the matrix \( M \), we can write dependence of particle currents on transition rates \( k_{\epsilon_1} \) and probabilities \( p_{0,l,r} \) as:

\[
J_1 = p_r k_{d\rightarrow l}^{\epsilon_2} - p_l k_{d\rightarrow l}^{\epsilon_1} \quad (10)
\]

\[
J_2 = p_l k_{d\rightarrow l}^{\epsilon_2} - p_r k_{d\rightarrow l}^{\epsilon_1} \quad (11)
\]

Because of the condition of stable electron movement \( (\dot{p} = 0) \), we know that particle current between levels \( \epsilon_2, \epsilon_2 + \epsilon_g \) and left lead equals to \( J_2 \), and similarly between left lead, \( \epsilon_1 - \epsilon_g \) and \( \epsilon_2 + \epsilon_g \) to \( J_1 \). So frankly we have already described all currents flowing through device. Replacing \( p_l \) and \( p_r \) with its equalities obtained in the linear system \( (9) \) we get the expression for current dependent only on the energy levels in quantum dots \( (\epsilon_1, \epsilon_2, \epsilon_g) \), temperatures of the left (right) lead and the sun \( (T_l, T_r, T_s) \), and last but not the least the material constants \( (\Gamma, \Gamma_s) \). The final expression is very long and complicated and furthermore does not allow any obvious conclusions so it has not found place in this seminar.[1]

The motion of electrons between the leads gives rise to an associated heat exchange. For example, an electron leaving the right lead by moving to the upper energy level \( \epsilon_2 \) withdraws an amount of heat equal to \( dQ = \epsilon_2 - \mu > 0 \) and effectively cools down the lead. The amount of heat taken by electron follows from the more general expression \( T dS = dQ = dU - \mu dN \) describing the change of entropy of a reservoir when amount of energy \( dU \) and particles \( dN \) is added[6]. Similarly, an electron moving towards the right lead from the lower energy level \( \epsilon_1 \) adds an amount of heat \( \epsilon_1 - \mu < 0 \) and therefore cools down the lead as well. With aforementioned expressions for steady state particle currents \( J_1 \) and \( J_2 \), we thus obtain the following explicit results for stationary heat fluxes \( \dot{Q}_r \) (from the right lead to device), \( \dot{Q}_l \) (from the left lead to the device), and the heat flux from the photon reservoir (e.g. the Sun) \( \dot{Q}_s \) [1]:

\[
\dot{Q}_r = (\epsilon_1 - \mu)(-J_1) + (\epsilon_2 - \mu)(-J_2) \quad (12)
\]

\[
\dot{Q}_l = (\epsilon_1 - \epsilon_g - \mu)(J_1) + (\epsilon_2 + \epsilon_g - \mu)(-J_2) \quad (13)
\]

\[
\dot{Q}_s = \epsilon_g J_1 + \epsilon_g (-J_2) \quad (14)
\]
The derivation of above expressions is very simple. One simply takes a look at the particle currents going in or out of the observed lead (‘+’ stands for intake and ‘-’ for out-take) and multiplying it with corresponding energy of electron in transition. The operation principle of the refrigerator can be formulated as follows: the photon source (Sun) is the power source which provides the energy ($\dot{Q}_s$) to draw the heat from the right lead ($\dot{Q}_r$) which is then dumped in the left lead ($\dot{Q}_l$). The performance of such refrigerator is characterized by the coefficient of refrigeration performance (COP), or in common words efficiency [6]:

$$\eta_{COP} = \frac{\dot{Q}_r}{\dot{Q}_s}. \quad (15)$$

Before further analysis we are going to rewrite this expression using information that gives us the first and the second law of thermodynamics. From the equations (16), (17) and (18) we can see that the conservation of energy $\dot{Q}_l + \dot{Q}_r + \dot{Q}_s = 0$ appears. Writing down the second law of thermodynamics, saying that the entropy of an isolated system never decreases gives us:

$$\dot{S}_i = -\frac{\dot{Q}_s}{T_s} - \frac{\dot{Q}_r}{T_r} - \frac{\dot{Q}_l}{T_l} \geq 0.$$ 

By combining these expression and implementing it into (15) we obtain:

$$\eta_{COP} = (1 - \frac{T_l}{T_s} - \frac{T_l\dot{S}_i}{\dot{Q}_s}) \frac{\dot{Q}_r}{T_r - T_l}. \quad (16)$$

The obtained expression can be simply explained as follows. If one takes a look at the first factor, one can notice that without $T_l\dot{S}_i/\dot{Q}_s$ it would present a Carnot efficiency engine working between two reservoirs with temperatures $T_s$ and $T_l$. In the same way, looking at the second factor, one can see it presents maximal COP for a refrigerator driven by some reversible work source.\[7\] Therefore we can easily conclude that overall maximal COP would be obtained when $\dot{S}_i = 0$, as Carnot efficiency is the maximal possible and the second factor can not be changed. But this condition implies the total equilibrium, where temperatures in the right and left lead are the same all the time. This is definitely not something we would like to happen. By appropriate choice of parameters e.g. $\mu, \Gamma, T_{l,r,s}$ one can achieve high efficiency as well. It turns out, that this appears if heat currents are proportional to each other. Furthermore, with appropriate choice of $\mu = (\epsilon_1 + \epsilon_2)/2$ the net electrical current between leads $J_1 + J_2$ drops to zero. This is very favourable, as it means the same amount of electrons comes in and goes out of the leads and therefore no electric charging appears. By these conditions we obtain [8]:

$$\dot{Q}_r = \frac{\epsilon_2 - \epsilon_1}{2} (J_1 - J_2) \quad \dot{Q}_s = \epsilon_g (J_1 - J_2), \quad (17)$$

The relations between the power output ($\dot{Q}_r$) and the input energy ($\dot{Q}_r$) of an heat engine are known as tight coupling. It implies that $\eta_{COP}$ equals to $(\epsilon_2 - \epsilon_1)/2\epsilon_g$. The tight coupling principle has already been proven as a very good option for restriction of parameters in [8].

Furthermore, it is certainly possible to generally investigate the efficiency dependence on device parameters. This can be simply done with numerical analysis. However, while doing it, one needs to pay attention to make analysis within the region of parameters which imply cooling. Therefore restrictions $\dot{Q}_r \geq 0, \eta_{COP} \geq 0$ and $T_s \geq T_l \geq T_r$ have to be taken into account. As already mentioned in previous pages and shown in Figure [4] we require $\epsilon_1 \leq \mu \leq \epsilon_2$ as well. Considering these conditions it is possible to follow with numerical investigation of refrigeration efficiency. In the left graph in Figure [4] we plot the maximum temperature difference $T_l - T_r$ for which cooling takes place, as a function of $\epsilon_2 - \epsilon_1$. The dashed lines present the best dependence, obtained by general numerical choice of parameters. We can see the maximal value appears very close to the point under which the cooling is not possible. On the other
hand we can see that with the choice of \( \epsilon_1 = -\epsilon_2 \) (full lines), maximal temperature difference is a monotonically decreasing function of \( \epsilon_2 - \epsilon_1 = 2\epsilon_2 \). This restriction stands for so called tight coupling, mentioned before. In the inset there is a graph of a temperature \( T_s \) that is required for cooling. However in the right plot in Figure 5 we plot the COP as function of the cooling rate \( Q_r \). Again, we did it for general numerical choice of parameters (dashed lines) and for parameters obeying tight coupling condition (full lines). During analysis other parameters were being put on some random value. We can see that dashed lines are closed loops, starting and ending in the origin. This means that \( Q_s \) does not vanish when the system stops refrigerating: the device continues to consume energy without any cooling. In contrast, for a tightly coupled device, both \( Q_r \) and \( Q_s \) vanish at that point. In this case COP does not drop to zero, as in the first case. In the same graph we can see, that for the same current \( Q_r \), strong coupling provides higher COP. To sum up, from these findings we can conclude, that condition (tight coupling) in which heat currents are proportional to each other and even implies zero net electrical current, provides us with better results, for our refrigeration device.

5 Conclusion

In conclusion, a novel electronic photon driven nanorefrigerator, which cools an electron reservoir (lead) by replacing its hot electrons with cold ones has been presented. The process in this device is driven by absorption of photons and a microscopic analysis shows that maximal efficiency for cooling can be reached by the careful fine-tuning of the systems parameters. Under the so called tight coupling condition which implies proportionality of the heat fluxes, maximal efficiency can be reached. Surprisingly this condition implies zero net electric current in device, which means no electric charging can occur and that is another advantage for technological application. However the presented device has a great potential for application in information technology, for example for cooling down hardware components. In comparison
with photovoltaic systems it probably provides higher efficiency rates, as in the latter, the solar power has to be at first transformed into electricity and in the next step used to propel heat engines. Furthermore, this seminar additionally supported the statement that light beams can be used for refrigeration, through presentation of laser cooling. As the sunlight warms the whole planet, the intuitive perception would be, there is no way, it can be used for cooling as well. This seminar definitely refutes such perception. The laser cooling and aforementioned refrigeration device are good examples of how should one think out of the box in order to create something new.

References


