Control of quantum systems by dissipation

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Abstract
Quantum computer could contribute a great deal in many areas. An important use of quantum computer for physicists is quantum simulation, which is presented in the first part of the seminar. The greatest obstacle in quantum computation is decoherence, which is induced by environment. Therefore, it is important to be able to control the coupling of system with environment. The idea of dissipative quantum state preparation and dissipative quantum computing is to use such coupling for obtaining pure states of quantum systems or even result of quantum computation.
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1 Introduction

This seminar deals with the dissipative control of quantum systems. The topic consists of two parts. The first part is about dissipative quantum computation and the second one is about dissipative preparation of pure quantum states. The field is an intersection of quantum information theory [1] and the theory of open quantum systems [2]. The dissipative quantum computation employs dissipative processes for performing quantum computational algorithms. In the first part of seminar we will briefly review the idea of quantum computation, its algorithms and how they can be employed for simulating quantum processes. For a physicist this is probably the most interesting use of a quantum computer as it could be used for the verifications of physical models and could help solve some yet unsolved problems that can not be efficiently simulated on a classical computer. We will also describe Feynman’s scheme for simulating dynamics of quantum system, which will then be used in dissipative approach to quantum computing. For the reader who would want to obtain general knowledge of quantum computation and its use in other fields, we would recommend reading [1].

The second section of the seminar will be dedicated to the discussion of the evolution of open quantum systems. In this section we will present dynamical quantum maps and the Lindblad equation, which are the tools used in the derivation of the dissipative quantum computation and in analysis of dissipative quantum state preparation. This section is followed by the description of a proposed scheme for dissipative quantum computation. Last section is dedicated to the review of the dissipative preparation of the entangled states of two qubit systems.

2 Quantum simulations

This section is divided in two parts. The first part of this section is based on [1]. Here we will present some of the basics of the quantum computation to get acquainted with quantum circuits and to show that it is possible to simulate arbitrarily complicated interactions with a set of few universal quantum gates. This fact makes simulation of systems easier as we have to be able to perform just a few basic manipulations of quantum system in order to simulate any interaction.

The second part is dedicated to quantum simulations and the presentation of Feynman’s clock idea, a method for simulating time evolution of quantum system.

2.1 Quantum computation

Quantum computation deals with unitary manipulation of two-level quantum systems called qubits. We would like to simulate an arbitrary unitary evolution of a system. There are two technically important features of quantum computation that make this problem easier. The first one is that any unitary transformation can be decomposed into operations which act non-trivially only on two components of a state vector; the second important feature of quantum computing is that any unitary evolution can be represented with only few quantum gates. Such a set of quantum gates is said to be universal [1].

\[ \text{If we act with two level matrix on two nonzero components of a state vector, we can choose the matrix so that one component of the state vector is set to zero, after the multiplication. Therefore we can change the state-vector so that all components are zero except for one. But because the matrices are unitary, we can also get an arbitrary state from the state where only one component of the state vector is 1, using inverse matrices. Thus we have shown that it is possible to change a general state of a state vector into arbitrary state, using only two-level matrices, which implies that arbitrary unitary transformation can be decomposed into set of two-level matrices.} \]
2.1 Quantum computation

The first statement is that arbitrary matrix can be decomposed in product of matrices which act non-trivially only on two components of vector (two-level matrices) [1]

\[ U = V_1 \ldots V_n. \] (1)

Here \( V_i \) are two-level matrices and \( U \) is arbitrary unitary matrix. One can show that the decomposition can be achieved with at most \( d(d-1)/2 \) two-level matrices, where \( d \) is dimension of Hilbert space [1]. Another simplification can be made by reducing general two level matrix operation to a controlled-NOT gate (introduced in the example) and single qubit manipulation [1]. We shall demonstrate the above general idea on an example of two qubits. The goal is to perform transformation of the state vector described by two level matrix

\[ U = \begin{bmatrix}
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}, \] (2)

using controlled-NOT operation and single qubit operations.

Initially we have general two qubit state \( |\psi\rangle \)

\[ |\psi\rangle = \psi_{00}|00\rangle + \psi_{01}|01\rangle + \psi_{10}|10\rangle + \psi_{11}|11\rangle. \] (3)

The operation (2) exchanges the states of the second qubit if the first one is in a state \( |0\rangle \). The operation is schematically represented by a quantum circuit

\[ \begin{array}{c}
1 \\
2
\end{array} \xrightarrow{\text{controlled-NOT}} \begin{array}{c}
1 \\
2
\end{array} \] (4)

When this transformation is applied, the state vector changes to

\[ |\psi\rangle' = \psi_{01}|00\rangle + \psi_{00}|01\rangle + \psi_{10}|10\rangle + \psi_{11}|11\rangle. \] (5)

A controlled-NOT operation is defined so that it flips the second qubit, if the first one is in a state \( |1\rangle \) and leaves the state of the second qubit unchanged, if the first qubit is in a state \( |0\rangle \). This can be presented with a quantum circuit as

\[ \begin{array}{c}
1 \\
2
\end{array} \xrightarrow{\text{controlled-NOT}} \begin{array}{c}
1 \\
2
\end{array} \] (6)

The corresponding matrix is

\[ C_{\text{not}} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{bmatrix}. \] (7)
The goal is to implement operation (4) using a controlled-NOT operation and the single qubit operation. A single qubit operation that is needed is $X$, which flips the state of the qubit. The operation (2) can be represented with quantum circuit using only the two previously mentioned operations as (see [1])

$$1 \xrightarrow{X} 1 \xrightarrow{X} 2$$

$X$ flips the state of the first qubit. Therefore, controlled-NOT operation exchanges the amplitudes of a states initially corresponding to a $|0\rangle$ state of the first qubit. The state of the first qubit is flipped once again and the desired state (5) is obtained.

With this example we illustrated that general unitary matrix, can be decomposed in controled-NOT operation and single qubit operations. Further simplification is achieved by noting that general unitary operation can be approximated with few operations [1]. For instance controlled-NOT, Hadamard $\frac{\pi}{3}$ and $\frac{\pi}{2}$ gates [1]. Detailed discussion on this can be found in [1].

The point of this discussion was to show the fact that we can simulate general time evolution with few simple qubit manipulations, and to get the taste of how quantum algorithms are produced.

### 2.2 Feynman’s clock

The goal of a quantum simulator as envisioned by Feynman is to find easily controlled quantum system with which it would be possible to simulate any other quantum system [3]. Such quantum device would be very important for development of physics and would probably help solve some yet unanswered questions, such as high-temperature superconductivity [3].

The problem of simulating many-body quantum processes with classical computer is that the number of the degrees of freedom of quantum system grows exponentially with the number of subsystems. This is due to the mathematical structure of quantum mechanics. For $N$ two level systems we have to store and manipulate $2^N$ complex numbers [3].

In this part we will introduce the idea behind Feynman’s clock as it will be used in the discussion of dissipative quantum computing. We will base our discussion upon [4] and deal with Schrödinger equation, as the idea is easier to understand through unitary evolution and can be immediately applied to dissipative processes. The basic idea is to add an auxiliary quantum system, the states of which serve for imprinting the time evolution of another system. This is done by finding the appropriate Hamiltonian of composite system, so that the time evolution of the system is incorporated in the ground state of new Hamiltonian. The direct simulation of time evolution of initial system is replaced by searching the ground state of new Hamiltonian.

Our goal is to simulate time evolution of system governed by arbitrary Hamiltonian

$$i\frac{\partial}{\partial t} |\psi(t)\rangle = H(t)|\psi(t)\rangle.$$  \hspace{1cm} (9)

Throughout the seminar $\hbar$ is set to 1. We can write the solution of equation (9) as

$^2$The first component of the state vector is replaced by sum of both components divided by $\sqrt{2}$, and the second component is obtained by subtracting the second component from the first one and dividing the result by $\sqrt{2}$ [1].

$^3$Leaves first component of single qubit state vector unperturbed and multiplies the second component with $\exp(i\pi/4)$ [1].
\[ |\psi(t)\rangle = U(t, t_0) |\psi(t_0)\rangle, \]  
(10)

where \( |\psi(t_0)\rangle \) is the initial state of a system. Time evolution operator can be decomposed into

\[ U(t_k, t_0) = U(t_k, t_{k-1}) \ldots U(t_1, t_0). \]  
(11)

The time is divided into \( T+1 \) time slices and an auxiliary quantum system, with \( T+1 \) basis states \( |n\rangle \), is introduced. We define \( |\psi_n\rangle = U(t_n, t_0) |\psi(t_0)\rangle \) and \( U_n = U(t_{n+1}, t_n) \). The state of the auxiliary system denoted by \( |n\rangle \), corresponds to time \( t_n \). The goal is to find composite Hamiltonian, the ground state of which is the superposition of states of our system at time \( t_n \) - \( |\psi_n\rangle \) entangled with the state of an auxiliary system \( |n\rangle \) (see [4]).

\[ H_{tot} = \left(I - |\psi_0\rangle\langle \psi_0|\right) \otimes |0\rangle\langle 0| + \frac{1}{2} \sum_{n=0}^{T} \left(I \otimes |n\rangle\langle n| - U_n \otimes |n+1\rangle\langle n+1| - U_n^\dagger \otimes |n\rangle\langle n+1| + I \otimes |n+1\rangle\langle n+1| \right). \]  
(12)

The first part of \( H_{tot} \) is used to enforce the desired initial state \( |\psi_0\rangle \) of the system [4]. If we have some other initial state, this term increases the energy. The eigenstate of Hamiltonian (12) with eigenvalue 0 is (see [4])

\[ |\phi\rangle = \frac{1}{\sqrt{T+1}} \sum_n |\psi_n\rangle \otimes |n\rangle. \]  
(13)

If we measure that auxiliary system is in a state \( |n\rangle \), we know that the system is in a state \( |\psi_n\rangle \). This state corresponds to the state of the wave function, evolving according to equation (9), at time \( t_n \). Schematically the state (13) is represented in figure (1). With this approach the time evolution of a system (9) is replaced by seeking the ground state of a system (12).

Figure 1: Schematic representation of the state (13) for four time slices. The state of the auxiliary system corresponding to certain time slice is denoted by the green dot and the number of the time slice. The state of the system corresponding to the same time slice is denoted by series of arrows.

The largest difficulty of quantum computation or simulation is decoherence. Decoherence is induced by the coupling of the system with environment. It causes non-unitary evolution of quantum system. Therefore, 

\footnote{We will take that we have basis states of auxiliary system from \( |0\rangle \) to \( |T\rangle \) and not to \( |T-1\rangle \), as in [4] was done, to obtain complete analogy with dissipative case.}
the state of the system is disturbed during computation and evolution ceases to be unitary. The problem of decoherence increases as we try to use larger quantum systems since decoherence times become shorter due to the increasing mass of the system.

3 Dynamics of open quantum system

This section is based on [2]. Dissipative quantum computation is achieved by the controlled coupling of quantum system to environment. Therefore, understanding the dynamics of open quantum system is important. If quantum system is isolated from environment the evolution of a systems state is described by von-Neuman equation

\[ \dot{\rho} = -i[H, \rho], \]  

(14)

which can be derived directly from Schrödinger equation (see for instance [2]). Here \( \rho \) is the density matrix of the system, \([..., ...]\) denotes a commutator and \( H \) system Hamiltonian. The state of system coupled with environment is described by reduced density matrix (all observables acting non-trivially only on system can be computed from the reduced density matrix) [2]. Reduced density matrix is obtained by tracing out environment degrees of freedom. We would like to obtain an equation that describes the evolution of a system coupled with environment. If we make some approximations, the evolution can be described by appropriate master equation. Detailed derivation of quantum Markovian master equation can be found in [2]. We will briefly review the derivation and illustrate the physical meaning of quantities on an example.

We assume, that the evolution of the composite system is described by von-Neuman equation.

\[ \dot{\rho}_S = -i tr_E[H, \rho], \]  

(15)

where \( \rho \) is density matrix describing the composite state of the system S and the environment, and \( \rho_S \) is the reduced density matrix of the system. The first assumption is that the composite system can initially be written as a direct product of initial density matrices of the system S and the environment [2]. This means that initially two systems are not entangled. We can characterize the evolution of composite system with the evolution operator \( U(t, 0) \). By tracing out the environment degrees of freedom we obtain equation for the reduced density matrix of the system (see [2]).

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5The environment and the system coupled with environment, which will be denoted by S. See the figure 2.
where $\rho_E$ is the density matrix of the environment. By writing environment density matrix as $\rho_E = \sum_\alpha \lambda_\alpha |\psi_\alpha\rangle$, where $|\psi_\alpha\rangle$ form basis we obtain the representation of time evolution (see [2])

$$\rho_S(t) = \sum_{\alpha,\beta} W_{\alpha,\beta}(t) \rho_S W_{\alpha,\beta}^\dagger(t); \quad W_{\alpha,\beta} = \sqrt{\lambda_\beta} |\psi_\beta\rangle \langle U(t, 0)|\psi_\alpha\rangle.$$  \hspace{1cm} (17)

We get the sum over $\alpha$ due to the trace and the sum over $\beta$ due to the decomposition of density matrix. Two additional assumptions are made. The first one is that the decay time of environment correlations is much shorter than the time scale on which the state of the system $S$ changes considerably [2]. The second assumption is that the state of the system $S$ does not affect the state of the environment [5]. $V(t)$ is dynamic map and is defined by upper relation [2]. The dynamic map is connected to Liouville operator by $V(t) = \exp(\mathcal{L}t)$ [2]. Further reshaping of equation yields the next result (see [2])

$$\dot{\rho}_S(t) = \mathcal{L}\rho_S = -i[H', \rho_S] + \sum_{k=1}^{N^2-1} \gamma_k \left( A_k \rho_S A_k^\dagger - \frac{1}{2} A_k^\dagger A_k \rho_S - \frac{1}{2} \rho_S A_k^\dagger A_k \right).$$  \hspace{1cm} (18)

This is Lindblad equation. $H'$ consists of the system $S$ Hamiltonian, but it can also contain some corrections due to the interaction [2]. The operators $A_k$ are Lindblad operators which are to be determined in each specific case from the interaction Hamiltonian. The first part of equation (18) is a standard von-Neuman equation. $N$ is the dimension of Hilbert space of the system $S$, $\gamma_k \geq 0$ are constants connected with dissipation relaxation times, as we shall see in the example [2]. The crucial difference between von-Neuman equation and Lindblad equation is irreversibility due to non-unitary evolution of reduced density matrix. Due to the irreversibility $\rho(t)$ as $t \rightarrow \infty$ approaches time-independent state called a steady state. Let us consider the decay of a two level system. Lindblad equation is derived considering the dipole interaction of the two level system with environment in thermal equilibrium [2]. In this case Lindblad operators are $\sigma^+$ and $\sigma^-$ [2]. If we disregard the corrections to unitary part of Lindblad equation due to the interaction, the unitary part does not contribute to the evolution of reduced density matrix [2]. The Lindblad equation governing the evolution is (see [2])

$$\dot{\rho}_S(t) = (\alpha + \gamma_0)(\sigma^- \rho(t) \sigma^+ - \frac{1}{2} \{\sigma^+ \sigma^-, \rho(t)\}) + \alpha (\sigma^+ \rho(t) \sigma^- - \frac{1}{2} \{\sigma^- \sigma^+, \rho(t)\}),$$  \hspace{1cm} (19)

where $\gamma_0$ is spontaneous emission rate and $\alpha$ is connected with emission and absorption due to the finite temperature [2]. The equation (19) holds only in the weak-coupling limit [2]. The steady state is (see [2])

$$\rho_0 = \begin{pmatrix} \frac{1}{2}(1 - \frac{1}{2^{2\gamma_0+1}}) & 0 \\ 0 & \frac{1}{2}(1 + \frac{1}{2^{2\gamma_0+1}}) \end{pmatrix}.$$  \hspace{1cm} (20)

The top diagonal element is the probability of the system being in the excited state, and the bottom element is the probability of the system occupying the ground state. The off-diagonal elements die off exponentially with the rate $(2\alpha + 2\gamma_0)/2$ and the diagonal elements approach steady value twice as fast [2]. If the temperature is 0 so is the constant $\alpha$ and from equation (20) follows, that the excited state is not occupied. If on the other hand the temperature is infinite, one can see that both states are occupied with probability 0.5.
4 Dissipative quantum computation

In this part we focus on our primary topic, which is dissipative quantum computation. The idea of dissipative quantum computation is to find appropriate system environmental coupling, so that the result of some unitary quantum evolution of the system (represented by a quantum circuit) can be obtained by evolution under the influence of the environment instead [6]. The key property is that this can be achieved in a polynomial time with respect to the time needed for operations in quantum circuit [6]. The unitary part of the evolution of reduced density matrix in (18) is not needed. The goal is to obtain Lindblad equation that has similar properties as (12). Similarly as in the second part of the second section we want to simulate the time evolution of a quantum state, at a given time \( t \) defined by \( |\psi_t\rangle = U_t U_{t-1}...U_1 |000...000\rangle \) [6]. We are particularly interested in the final state \( \psi_T \) reached at the end of computation. We choose two sets of Lindblad operators (see [6])

\[
\mathcal{L}_i = |0\rangle_i \langle 1| \otimes |0\rangle_i \langle 0|; \quad \mathcal{L}_t = U_t \otimes |t+1\rangle \langle t| + U_t^\dagger \otimes |t\rangle \langle t+1|,
\]

where \( i \) denotes the site of a qubit and \( |t\rangle \) denotes a state of an auxiliary system, which serves as a time register. It can be shown that the steady state of Lindblad equation with such Lindblad operators is (see [6])

\[
\rho_0 = \frac{1}{T+1} \sum_t |\psi_t\rangle \langle \psi_t| \otimes |t\rangle \langle t|.
\] (22)

This means that \( \mathcal{L}(\rho_0) = 0 \) and this is the only state with such property [6]. On the average, the computation has to be carried out \( T+1 \) times, to obtain the wanted result \( |\psi_T\rangle \). If we carry out the measurement of the auxiliary system we get exactly the same probabilities for corresponding states as for wave function (13). The density matrix (22) in this case is analogous to the state (13).

We are also interested in how fast the state \( \rho_0 \) will be reached. Time at which we reach steady state (22) with accuracy \( \epsilon \) is given by

\[
\|\rho(t) - \rho_0\| < \epsilon. \tag{23}
\]

The time \( t \) at which the inequality (23) is met, depends on the number of time slices \( T \). If we have some initial state \( \rho(0) \), the time evolution can be described as \( \rho(t) = \exp(\mathcal{L}t)\rho(0) \), where \( \exp(\mathcal{L}t) \) is a superoperator, which acts on a matrix. Provided we can diagonalize operator \( \mathcal{L} \), the time evolution of density matrix can be written as

\[
\rho(t) = \sum_j c_j \exp(-\lambda_j t) \rho_j, \tag{24}
\]

where \( \rho_j \) are matrices and they are eigenvectors with eigenvalues \( \lambda_j \). We arrange \( \lambda_j \), so that \( \lambda_0 < re(\lambda_1) \leq re(\lambda_2) \leq re(\lambda_3) \ldots \) and \( \lambda_0 = 0 \). Constants \( c_j \) are obtained by writing initial density matrix as linear combination of \( \rho_j \) matrices. The state \( \rho_0 \) is an eigenvector with eigenvalue 0 and because of that this state will be preserved if it is present in the initial state.

The information for (23) is determined by the spectral gap \( \Delta = re(\lambda_1) \), which tells us how fast the state which dies off slowest, dies off. In due time we have two dominant contributions to the reduced density matrix \( \rho(t) \approx \frac{1}{2}(\rho_0 + c_1 \exp(-\lambda_1 t)\rho_1) \). This implies that \( \epsilon \approx \|c_1 (\rho_1 - \rho_0) \exp(-\Delta t)\| \) and the time that we

\( \footnote{U_i \text{ are unitary transformations which act non-trivially only on nearest neighbors of the qubit chain [6].}} \)
have to wait to achieve the desired accuracy is $t \sim -\frac{1}{\Delta} \ln(\epsilon)$. In our case, the spectral gap is $\Delta = \frac{\pi^2}{(2T+3)^2}$, so the time needed for obtaining the result of quantum computation scales polynomially with respect to the number of time slices $t \sim T^3$, where $T$ is the number of steps needed for carrying out quantum computation by quantum circuit. We get $T^2$ due to the spectral gap and $T$ for the average number of times we have to carry out the experiment in order to successfully obtain the final state $|\psi_T\rangle$.

An important advantage of dissipative quantum computing and state preparation is that the desired state is obtained regardless of the initial state and that with controlled coupling we remove the problem of quantum decoherence.

5 Dissipative state preparation

In this section we present an experiment in which a desired entangled state was obtained with the help of dissipative process [7]. The reduced density matrix of a system in contact with environment tends to become separable, and diagonal in preferred basis due to the decoherence. Here we are interested in the preparation of entangled states with the help of dissipative processes. The controlled dissipation is used only in two steps of each cycle of preparation of entangled states.

![Figure 3: Schematic presentation of experiment setup. The ions are trapped and unitary transformation of states is obtained by application of laser pulses. The laser pulses are applied to the single ions as well as to the sequence of the ions, for instance when applying Mølmer-Sørensen transformation. The controlled dissipative manipulation is performed only on environment qubit in the last part of each dissipative map. The picture adapted from [7].](image)

We will center our discussion on entangling two qubits, as described in [7]. In this experiment $Ca^+$ ions were used. By $|0\rangle$ and $|1\rangle$ we denote two electronic Zeeman levels [7]. The transition between states is achieved by coupling ions to a laser with a wavelength 729 nm [2].

The system is composed of 3 qubits of which one acts as an environment qubit and serves in preparation of the entangled state of 2 remaining qubits. We wish to prepare one of the four two-qubit Bell-states which can be characterized by the eigenvalues of two operators $X_1X_2$ and $Z_1Z_2$ [7]. $X_i$ and $Z_i$ are Pauli matrices acting on the $i$-th qubit [7]. Bell-states form orthonormal basis and are also the most entangled states of two qubits [7]. We can present the evolution of reduced density matrix with dissipative maps. The goal is to find appropriate dissipative maps which will bring us to the desired final state, and then to find the practical implementation of the evolution described by these maps. We choose the desired final state to be the Bell-state.
\[ \rho_S = |\psi^\rangle \langle \psi^-|; \quad |\psi^-\rangle = \frac{1}{\sqrt{2}} (|01\rangle - |10\rangle). \]  
(25)

As mentioned before, Bell-states form an orthonormal basis of two-qubit states. So it is enough to ensure that application of the dissipative maps to any Bell-state will result in the \(|\psi^-\rangle \langle \psi^-|\) Bell-state. This can be achieved by the application of two dissipative maps. The first map pumps to -1 eigenspace of \(Z_1Z_2\) and the second can be used for pumping to -1 eigenspace of \(X_1X_2\) (see figure (4))(from here on \(\pm 1\) eigenspace will denote state with respect to operators \(Z_1Z_2\)). The map for pumping into -1 eigenspace of \(Z_1Z_2\) is (see [7])

\[
E(\rho_S) = E_1 \rho_S E_1^\dagger + E_2 \rho_S E_2^\dagger; \quad E_1 = \sqrt{p} X_2 \frac{1}{2} (1 + Z_1Z_2), \quad E_2 = \frac{1}{2} (1 - Z_1Z_2) + \sqrt{1 - p} \frac{1}{2} (1 + Z_1Z_2). \]  
(26)

We will examine the action of this map with \(p = 1\) on pure Bell-state. We set the initial state of the system to

\[ |\phi^-\rangle = \frac{1}{\sqrt{2}} (|00\rangle - |11\rangle). \]  
(27)

If we apply \(E_2\) to this state we get 0 and therefore the second part of the map (26) does not contribute. \(|\phi^+\rangle\) is an eigenstate of \(Z_1Z_2\) with eigenvalue 1. The operator \(X_2\) changes the state of the second qubit from 0 to 1 and vice versa. The application of \(E_1\) therefore yields the desired state (25). To obtain the state (25) from other Bell-states we have to apply additional dissipative map which pumps into -1 eigenspace of operator \(X_1X_2\) [7].

![Figure 4: Schematic representation of actions of two dissipative maps. The first map pumps to -1 eigenspace of operator \(Z_1Z_2\) and the second to -1 eigenspace of \(X_1X_2\). Application of both maps yields a state \(|\psi^-\rangle\). Picture taken from [7].](image)

Now we can move to the implementation of these operations. Exact implementation is more complicated and can be found in supplementary information of [7]. Quantum circuit can be seen in figure (5). The first part of the quantum circuit is the implementation of previously theoretically described mapping to -1 eigenspace of operator \(Z_1Z_2\). In figure (5) this part is denoted by \(Z_1Z_2(p)\) and is composed of four steps, which will be presented as follows. The idea is to entangle the states of two qubits corresponding to
different Bell-states with environment qubit, so that when the state of environment qubit is changed by dissipation, the probability of the two-qubit system being in desired state increases. The first step is the entangling of the system and the environment qubit. Initially the state of the environment qubit is set to $|1\rangle$ [7]. If the state of the system lies in +1 eigenspace of $Z_1 Z_2$ operator, the state of the environment qubit is set to $0\rangle$, otherwise it is left unperturbed [7]. This can be achieved by applying the Mølmer-Sørensen operation which is done by application of bi-chromatic light field [4]. The second step is the rotation of state corresponding to +1 eigenspace into state in -1 eigenspace. The operator can be written as $C(p) = |0\rangle(0\rangle \otimes U_{X_2}(p) + |1\rangle(1\rangle \otimes I_2$ [7]. First term implies that if the environment qubit is in state $|0\rangle$, the transformation of the state of second qubit is made, which causes the rotation into -1 eigenspace and is connected with transition probability $p$ [7]. The second term implies that if environment qubit is in state $|1\rangle$, the state of the system remains unchanged.

This is followed by applying inverse map $M^{-1}$, which is once again achieved by application of the same Mølmer-Sørensen operation. If there was no rotation (second step) we would obtain the initial state after application of the second Mølmer-Sørensen operation. Due to the rotation the part of two-qubit state, corresponding to the state of the environment qubit $|0\rangle$, remains present. In general, the state of two qubit system corresponding to the state of the environment qubit $|1\rangle$ is composed of a part corresponding to +1 and -1 eigenspace, but the probability of it being in +1 eigenspace decreases compared to the initial state.

![Quantum circuit diagram](image)

**Figure 5:** Quantum circuits representing implementation of the dissipative maps producing state $|\psi^\perp\rangle$. Picture taken from [7].

The last part of the process is dissipative. The environment qubit is dissipativelly set to $|1\rangle$ [7]. Then we can get dissipative map for pumping from +1 to -1 eigenspace of $X_1 X_2$ by exchanging operators $X$ and $Z$ [7]. The results of experiment can be seen in figure [6][9].

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7 For mapping the states to environment qubit with respect to the $\pm1$ eigenspace of $X_1 X_2$ operators, the Mølmer-Sørensen operation is presented by unitary transformation $U_{X_2}(\theta) = \exp(-i\frac{\theta}{2}(\sum_i X_i)^2)$ [7]. Mølmer-Sørensen operator also changes the state of the two qubit system, but the key feature is the entangling process.

8 This is achieved by coupling of the environment qubit with artificial environment [7].

9 The interpretation of measurement corresponds to the former discussion, although the actual pumping in experiment was done with respect to the $X_1 X_2$ and $Y_1 Y_2$ operators. (Supplementary information of [7])
6 Conclusion

We have introduced some aspects of the dissipative control of quantum systems. This new approaches could bring some technical advantage over standard quantum computation schemes, and could prove even more fruitful in the preparation of entangled states. The dissipative quantum computation could be used for simulating open quantum systems. The idea of dissipative quantum computation is interesting as it uses the openness of quantum systems, which in unitary quantum computation causes errors, for obtaining the result of unitary evolution.

Figure 6: Results of previously described experiment. On the left is the result of the experiment when pumping probability is set to 1 and on the right the result when pumping probability is set to 0.5. Red line denotes theoretical expectation for population of $|\psi^-\rangle$ and red triangles denote measured values. Other lines and dots denote theoretical predictions and measured data for other Bell-states. Starting point is the equal mixture of four Bell-states. In first graph it can be seen that after the application of first dissipative map populations for two states in +1 eigenspace of $Z_1Z_2$ operator drop off completely. After the application of the second map the state with +1 eigenvalue of $X_1X_2$ operator also drops off, and we are left with state $|\psi^-\rangle$ as desired. Picture taken from [7]
References


