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Seminar 4

KALUZA-KLEIN THEORY

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Abstract

Physicists have tried to find the theory which would unify all the known interactions under a fundamental law for a long time. One of the first attempts was the Kaluza-Klein theory which uses the postulation of extra space dimensions as the basic idea. In this seminar the 5-dimensional Kaluza-Klein theory is described thoroughly and at the end the extension to higher dimensions is mentioned. The description of possibility of Kaluza-Klein theory to be the right unifying theory is also included.

Contents

1 Introduction

The original Kaluza-Klein theory was one of the first attempts to create an unified field theory i.e. the theory, which would unify all the forces under one fundamental law. It was published in 1921 by German mathematician and physicist Theodor Kaluza and extended in 1926 by Oskar Klein. The basic idea of this theory was to postulate one extra compactified space dimension and introduce nothing but pure gravitiy in new $(1+4)$-dimensional space-time. It turns out that the 5-dimensional gravitiy manifests in our observable $(1+3)$-dimensional space-time as gravitational, electromagnetic and scalar filed. Kaluza and Klein obviously managed to unify gravity and electromagnetism but the theory had some major flaws. For example, the calculated mass and electric charge of electron did not correspond to experimental facts. Further more, the theory did not contain any of the nuclear forces simply because they were not known at the time of the development of the theory.

Despite the inconsistencies of the theory, it was never completely abandoned. Over many decades physicists were trying to improve the Kaluza's and Klein's concept, resulting in many new unified filed theories, for example well known string theory. They all share the same assumptions, that is the compactified extra space dimensions of space-time, and are commonly known as Kaluza-Klein(like) theories.

Figure 1: *The founders of original KK theory, Theodor Kaluza (left) and Oskar Klein (right)* [1]

1.1 General coordinate transformations

Before we can proceed with the theory, we need to get familiar with some concepts we will use. First is the general coordinate transformation, which tells us how to transform physical quantities between different frames of reference. It is useful to write the square distance between the two neighboring points $P(x)$ and $Q(x+dx)$, the line element, in reference frame $S(x)$,

$$ds^2 = dx^\mu g_{\mu\nu}(x) dx^\nu, \tag{1.1}$$

where $g_{\mu\nu}(x)$ is the metric tensor. Notice, that the Einstein summation convention is used. Since the interval between P and Q is independend of the choise of reference frame, the line element (1.1) is invariant under the transformation between reference frames so in $S'(x')$ one can write

$$ds^2 = dx'^\mu g'_{\mu\nu}(x') dx'^\nu. \tag{1.2}$$

The general coordinate transformation $dx^\mu = \frac{\partial x^\mu}{\partial x'^\nu} dx'^\nu$ and the equality of equations (1.1) and (1.2) yields

$$g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu} g_{\alpha\beta}(x) \frac{\partial x^\beta}{\partial x'^\nu} = e^\alpha_{\ \mu} g_{\alpha\beta}(x) e^\beta_{\ \nu}, \tag{1.3}$$

where

$$e^\mu_{\ \nu} = \frac{\partial x^\mu}{\partial x'^\nu} \tag{1.4}$$

is the definition of the *vielbein*, which will be needed in a great extent later in the theory for converting vectors and tensors from one frame into another. The inverse vielbein $f^\nu_{\ \mu}$ can also be defined, so that the equations

$$\begin{aligned} e^\alpha_{\ \nu} f^\mu_{\ \alpha} &= \delta^\mu_{\ \nu}, \\ e^\mu_{\ \alpha} f^\alpha_{\ \nu} &= \delta^\mu_{\ \nu} \end{aligned} \tag{1.5}$$

must be valid.

For better understanding I will present a simple example. Consider the relation between Cartesian and polar coordinates,

$$\begin{aligned} x^0 &= \rho cos\varphi, \\ x^1 &= \rho sin\varphi. \end{aligned} \tag{1.6}$$

The metric tensor in Cartesian coordinate system $S(x^0, x^1)$ is of the form $g_{\mu\nu} = diag(1,1)$. If we want to obtain the metric tensor in polar coordinate system $S'(\rho, \varphi)$, we must first calculate the vielbeins. This is easy, by differentiating equations (1.6) we obtain

$$e^0_{\ \rho} = cos\varphi \quad e^1_{\ \rho} = sin\varphi$$

$$e^0_{\ \varphi} = -\rho sin\varphi \quad e^1_{\ \varphi} = \rho cos\varphi$$

and using (1.3) we can calculate metric tensor

$$g'_{\mu\nu} = diag(1, \rho^2).$$

Using the identity

$$g_{\alpha\mu} g^{\mu\beta} = \delta^\beta_{\ \alpha}, \tag{1.7}$$

the inverse metric tensor $g^{\mu\nu}$ can be obtained,

$$g'^{\mu\nu} = diag(1, 1/\rho^2).$$

1.2 Principle of equivalence

Another very important concept is the principle of equivalence, stated by Albert Einstein in 1907. We will formulate the principle with the help of a thought experiment. Consider a closed laboratory witout any windows with a scientist inside. If we place the laboratory in outer space with no forces acting on it, the scientist will float (left side of Figure 2).

Figure 2: *Free falling downwards is equivalent as being in the absence of gravity and other forces* [2]

If now someone on the outside suddenly pulls the laboratory upward with a constant acceleration, the scientist will be pushed to the bottom of the laboratory with the constant force (right picture of Figure 3). On the other hand, we can place the laboratory in a uniform gravitational field and the effect on the scientist be completely the same (left picture of Figure 3). From this we can conclude that the effects of acceleration and gravity are indistinguishable [3].

Furthermore, if we drop the laboratory to fall freely in a uniform gravitational field, the scientist and the object inside will once again float, as in

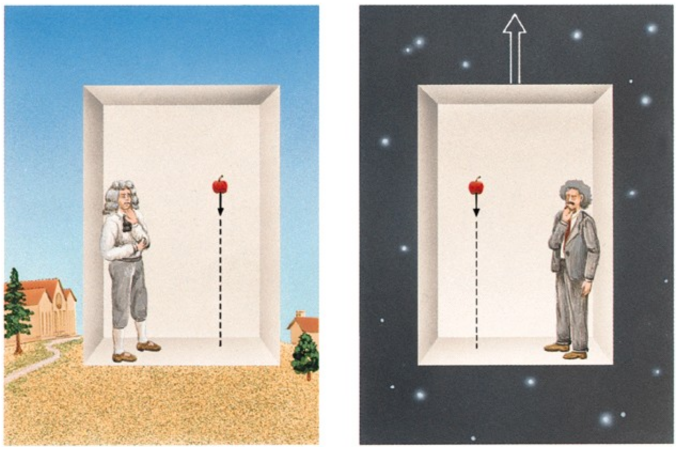

Figure 3: *The scientist inside the closed laboratory cannot tell whether the laboratory is accelerating or it rests in a uniforn gravitational field* [4]

the case of absence of all forces (right side of Figure 2). The effect of gravity could obviously be eliminated by going to freely falling reference frame. In such a frame our scientist will observe all the objects obeying the usual laws of motion in the absence of gravity and the frame is therefore inertial and a special theory of relativity can be used in it.

Throughout the whole section I was using only the uniform gravitational field. What happens if we would like to make a generalization to a completely arbitrary non-uniform gravitational field? All we have to do is to include the term *local* into our conclusions. For a non-uniform gravitational field we would state: *At every point in a reference frame with an arbitrary gravitational field it is possible to chose a locally inertial (freely falling) reference frame*. The term locally is implying that we are constricted to such small regions of space-time that gravitational field seems to be uniform. We can also say that the space-time is locally flat and is having the simple Minkowski metric,

$$\eta_{\mu\nu} = diag(1, -1, -1, -1). \tag{1.8}$$

1.3 Notations

In this section I will shortly discuss the notation, which will be used later on. We will be dealing with different reference frames, some having an arbitrary metric tensor $g_{\mu\nu}$ describing curved space-time, and others having Minkowski metric η_{ab} describing flat space-time. Greek indices $(\alpha, \beta, \gamma, ...)$ will denote curved index, while Latin indices $(a, b, c, ...)$ will denote flat index. Besides that, letters from the beginning of both the alphabets $(a, b, c, ... \; and \; \alpha, \beta, \gamma, ...)$

will indicate general index, from the middle of the alphabets the observed $(1+3)$ dimensions ($m, n, ...$ *and* $\mu, \nu, ... = 0, 1, 2, 3$) and letters from the bottom of the alphabets ($s, t, ...$ *and* $\sigma, \tau, ...$) will indicate compactified extra space dimensions.

1.4 Spin connection field

In the last section before the begining of the concrete Kaluza-Klein theory I will present the spin connection field, a very important concept in this theory. The equivalence principle tells us that the Einstein's general relativity, which we will be using, must be invariant under local Poincaré transformations. In other words, the fermionic field (spinor) Ψ must transform under frame rotations as an $SO(1,3)$ spinor,

$$\Psi(x) \to \Psi'(x') = M(x)\Psi(x),$$

where $M(x) = L(x) + S(x)$ is a group generator and is a function of position. This is called *local* or *gauge* transformation. The Dirac action for fermions

$$S_D = \int d^4x \mathscr{L}_D(\Psi, \partial_\mu \Psi)$$

must be invariant under these transformations, which means that its variance must vanish. Considering the position dependance of $M(x)$ this certainly does not hold. To provide the invariance of the action, the *covariant derivative* $\nabla_\mu \Psi$ must be introduced in such a manner that the equation

$$\delta S_D = \delta \int d^4x \mathscr{L}_D(\Psi, \nabla_\mu \Psi) = 0$$

is valid [5]. This yields

$$\nabla_\mu \Psi = (\partial_\mu - \frac{i}{2} S^{ab} \omega_{ab\mu})\Psi, \tag{1.9}$$

where $S^{ab} = \frac{i}{4}\{\gamma^a, \gamma^b\}_-$ with γ^a being Dirac gamma matrices and

$$\omega_{ab\mu} = f^{\alpha}{}_{b} e_{a\beta} \Gamma^{\beta}_{\mu\alpha} - f^{\alpha}{}_{b} \partial_\mu e_{a\alpha} \tag{1.10}$$

is so-called spin connection [6]. $\Gamma^{\beta}_{\mu\alpha}$ is the Christoffel symbol with definition

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2} g^{\alpha\gamma}(\partial_\mu g_{\nu\gamma} + \partial_\nu g_{\mu\gamma} - \partial_\gamma g_{\mu\nu}). \tag{1.11}$$

Because of the $M(x)$ being an $SO(1,3)$ group generator, the spin connection field can be interpreted simply as the angular momentum gauge field, but it can be shown that the orbital angular momuntum $L(x)$ is not needed and only spin $S(x)$ component is important. Spin connection filed is a compensating field which raises to cancel all the unwanted effects of local transformations enabling the existance of local symmetries [5].

2 Kaluza-Klein theory

We are now familiar with all necessary concepts to derive and understand the Kaluza-Klein theory. We will begin by constructing the Kaluza-Klein action in $d = (1 + (d-1))$ dimensions which will describe the massless fermionic field and only pure gravtity. Such an action is

$$S_{KK} = S_D + S_E = \int d^d x E(\frac{1}{2}\Psi^\dagger \gamma^0 \gamma^a p_{0a} \Psi + h.c.) - \alpha \int d^d x E R, \qquad (2.1)$$

where $h.c.$ means Hermitian conjugate, S_E is Einstein action describing gravity, $E = det(-g_{\alpha\beta})^{\frac{1}{2}} = det(e^a{}_\alpha)$, α is gravitational coupling constant and R is Ricci scalar, defined as

$$R = f^{\alpha[a} f^{\beta b]} (\partial_\beta \omega_{ab\alpha} - \omega_{ca\alpha} \omega^c{}_{b\beta}), \qquad (2.2)$$

with [...] meaning antisymmetrization, $f^{\alpha[a} f^{\beta b]} = f^{\alpha a} f^{\beta b} - f^{\alpha b} f^{\beta a}$. Using the definition for covariant derivative (1.9) in knowing the form of the momentum operator we can write

$$p_{0a} = f^\alpha_a (p_\alpha + \frac{1}{2} S^{cd} \omega_{cd\alpha}). \qquad (2.3)$$

From now on, our goal will be to derive, how the fields in higher dimensions manifest in our observed $(1+3)$-dimensional space-time. First, we will follow the original Kaluza and Klein idea in 5 dimensional space-time and later on we will extend the theory to higher dimensions.

2.1 Kaluza-Klein theory in $(1+4)$-dimensions

The original Kaluza-Klein theory was derived with one extra spacial dimension. We have to find the appropriate metric tensor for 5 dimensional space-time, which is of the form

$$\hat{g}_{\mu\nu} = \left(\begin{array}{c|c} g_{\mu\nu} & g_{\mu 5} \\ \hline g_{\nu 5} & g_{55} \end{array} \right), \qquad (2.4)$$

where hat $(\hat{})$ denotes the 5 dimensional quantity. The 5th dimension is postulated to be compactified, rolled-up in a small circle, which provides us the explanation for the unobservability of the extra dimension. Thus the 5-dimensional space-time has the topology $M^4 \times S^1$, where M^4 is 4-dimensional Minkowski space-time and S^1 is a circle. The simplest way to imagine space with one extra dimension is to imagine a small circle (extra dimension) in every point of 3-dimensional space as shown in Figure 4, where for simplicity instead of the 3-dimensional space the 2-dimensional plane is used.

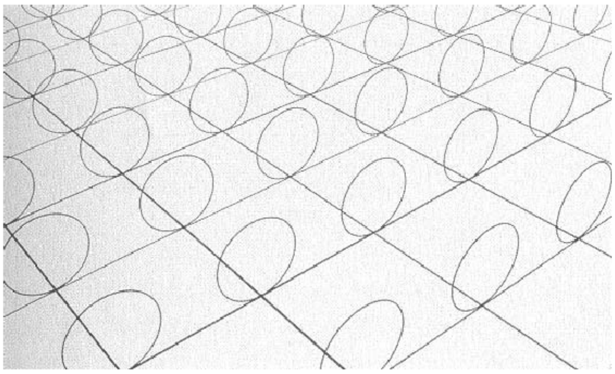

Figure 4: *In every point of space-time there is one extra dimension, roled-up into a circle* [7]

We can choose the basis vectors $\mathbf{e}_\mu$ and $\mathbf{e}_5$, where the latter is in direction of 5th dimension. Vector $\mathbf{e}_5$ will not in general be orthogonal to $\mathbf{e}_\mu$, thus

$$\mathbf{e}_\mu \cdot \mathbf{e}_5 = g_{\mu 5} \neq 0.$$

The 4-dimensional basis is decomposed into a parallel and orthogonal part

$$\mathbf{e}_\mu = \mathbf{e}_{\mu\perp} + \mathbf{e}_{\mu\|}, \quad \mathbf{e}_{\mu\perp} \cdot \mathbf{e}_5 = 0$$

and because $\mathbf{e}_5$ and $\mathbf{e}_{\mu\|}$ are parallel, one can write $\mathbf{e}_{\mu\|} = \frac{g_{\mu 5}}{g_{55}}\mathbf{e}_4$. The first component of metric tensor can now be obtained,

$$g_{\mu\nu} = \mathbf{e}_\mu \cdot \mathbf{e}_\nu = g^{(4)}_{\mu\nu} + \frac{g_{\mu 5} g_{\nu 5}}{g_{55}},$$

where $g^{(4)}_{\mu\nu}$ is metric of 4-dimensional Minkowski space-time. With definitions

$$B_\mu = \frac{g_{\mu 5}}{g_{55}},$$

$$\Phi = g_{55},$$

the final form of metric tensor (2.4) is obtained,

$$\hat{g}_{mn} = \left(\begin{array}{c|c} \eta_{mn} + \Phi B_m B_n & \Phi B_m \\ \hline \Phi B_n & \Phi \end{array} \right), \tag{2.5}$$

where we have considered $g^{(4)}_{\mu\nu} = \eta_{mn}$ for locally flat space-time [8]. The inverse metric tensor is calculated with the help of identity (1.7),

$$\hat{g}^{mn} = \left(\begin{array}{c|c} \eta^{mn} & -B^m \\ \hline -B^n & \frac{1}{\Phi} + B_{m'}B^{m'} \end{array}\right). \tag{2.6}$$

We will also need to calculate the vielbein and inverse vielbein. This will be done using the relations (1.3) for transformation from Minkowski metric to metric (2.5) and relation (1.5). For vielbein we obtain

$$\hat{e}^{a}_{\ \alpha} = \left(\begin{array}{c|c} \delta^m_\mu & 0 \\ \hline \sqrt{-\Phi}B_\mu & \sqrt{-\Phi} \end{array}\right) \tag{2.7}$$

and for inverse vielbein

$$\hat{f}^{\alpha}_{\ a} = \left(\begin{array}{c|c} \delta^\mu_m & -B_m \\ \hline 0 & \frac{1}{\sqrt{-\Phi}} \end{array}\right). \tag{2.8}$$

At this point I will present the so-called *cylinder condition*, which takes care of unobservability of the 5th dimension. It states that all metric components are independent of extra dimension,

$$\partial_5 \hat{g}_{\alpha\beta} = 0.$$

The consequence of this condition is that (1+3) dimensions are also independent of extra dimension. With the help of definition (1.4), taking cylinder condition into account, we can see the reason for $e^m_{\ 5}$ and $f^\mu_{\ 5}$ components of vielbein (2.7) and inverse vielbein (2.8).

If we consider the translation along extra dimension, $x'^\mu = x^\mu$, $x'^5 = x^5 + \epsilon(x)$, we can calculate the transformation of $g_{\mu5}$ component of metric tensor using equation (1.3). Considering cilinder condition, we obtain

$$g'_{\mu5} = \frac{\partial x^\nu}{\partial x'^\mu}\frac{\partial x^5}{\partial x'^5}g_{\nu5} + \frac{\partial x^5}{\partial x'^\mu}\frac{\partial x^5}{\partial x'^5}g_{55},$$

which yields

$$B'_\mu = B_\mu + \partial_\mu \epsilon. \tag{2.9}$$

B_μ obviously transforms like the electromagnetic potential. Because the gauge transformations in electrodynamics are local $U(1)$ transformations, we can identify B_μ with electromagnetic potential by postulating the extra dimension to be geometrically a circle. Therefore, any movement along x^5 can be interpreted as an Abelian gauge transformation of B_μ [5].

2.1.1 5-dimensional gravity

I will consider the terms of the action in equation (2.1) separately. In this section I focus on 5-dimensional gravity and how it manifests in observed (1+ 3)-dimensional Minkowski space-time. The Einstein acion in 5 dimensions reads

$$S_E = -\hat{\alpha} \int d^5 x E \hat{R}, \tag{2.10}$$

where hat again denotes 5-dimensional quantity and $E = \sqrt{-\Phi}$. Placing definition for spin connection (1.10) into (2.2), the equation for Ricci scalar can be rewritten in the form

$$R = g^{\alpha\beta}(\partial_\gamma \Gamma^\gamma_{\alpha\beta} - \partial_\beta \Gamma^\gamma_{\alpha\gamma} + \Gamma^\delta_{\alpha\beta}\Gamma^\gamma_{\delta\gamma} - \Gamma^\delta_{\alpha\gamma}\Gamma^\gamma_{\beta\delta}).$$

Considering definition (1.11), a straightforward calculation leads to the 5-dimensional Ricci scalar

$$\hat{R} = R^{(4)} - \frac{1}{4}\Phi F_{\mu\nu}F^{\mu\nu} - \frac{2}{\sqrt{-\Phi}}\partial_\mu \partial^\mu \sqrt{-\Phi} \tag{2.11}$$

where $R^{(4)}$ is 4-dimensional Ricci scalar and $F_{\mu\nu} = \partial_\nu B_\mu - \partial_\mu B_\nu$. The action (2.10) now looks very similar to action for gravitational field, electromagnetic field and one additional unrecognizable scalar field. Setting this scalar field to constant value, $\Phi = -1$, integrating over the compactified dimension x^5 and substituting $B_\mu = \frac{A_\mu}{\sqrt{\alpha}}$ provides us with effective 4-dimensional action

$$S_{E_{eff}} = -\alpha \int d^4 x R^{(4)} - \int d^4 x \frac{1}{4} F_{\mu\nu}F^{\mu\nu} = S_E + S_{EM}, \tag{2.12}$$

where $\alpha = L\hat{\alpha}$ is 4-dimensional gravitational coupling constant and $L = 2\pi R$ is the length of the compactified dimension [5]. The equation (2.12) clearly shows us that we have managed to describe 4-dimensional gravity and electromagnetic field beginning only with 5-dimensional gravity (2.10).

2.1.2 5-dimensional fermionic field

Similarly as in previous section we will now determine the effective 4-dimensional Dirac action. We begin with massless fermionic field from action (2.1), which in 5 dimensions have the form

$$S_D = \int d^5 x \sqrt{-\Phi}(\frac{1}{2}\Psi^\dagger \gamma^0 \gamma^a f^\alpha_{\ a}(p_\alpha + \frac{1}{2}S^{cd}\omega_{cd\alpha})\Psi + h.c.). \tag{2.13}$$

Again, we set $\Phi = -1$ and insert the inverse vielbein (2.8) and we obtain the action

$$S_D = \int d^5x(\frac{1}{2}\Psi^\dagger\gamma^0(\gamma^m(p_m - \frac{p_5}{\sqrt{\alpha}}A_m) + \gamma^5 p_5 + \gamma^a f^\alpha_a \frac{1}{2} S^{cd}\omega_{cd\alpha})\Psi + h.c.). \quad (2.14)$$

The last term of (2.14) can be expanded as $\gamma^a f^\alpha_a \frac{1}{2} S^{mn}\omega_{mn\alpha} + \gamma^a f^\alpha_a S^{m5}\omega_{m5\alpha}$. The first term is describing coupling with gravity, which is extremely small and has not been measured yet and thus can be neglected. Considering the topology of space-time ($M^4 \times S^1$) we can see that $S^{m5} = 0$ and also the second term vanishes. From (2.12) we can also immediately see that the electric charge and mass should be,

$$\begin{aligned} q &= \frac{p_5}{\sqrt{\alpha}} \\ m &= p_5. \end{aligned} \quad (2.15)$$

Both are obviously connected with motion in extra dimension. Finally, integrating over extra dimension x^5, the effective 4-dimensional Dirac action is obtained,

$$S_{D_{eff}} = 2\pi R \int d^4x(\frac{1}{2}\Psi^\dagger\gamma^0(\gamma^m(p_m - qA_m) + \gamma^5 m)\Psi + h.c.).$$

Kaluza-Klein theory also provides the explanation for quantization of electric charge. For this purpose we have to consider the periodicity of the compactified extra dimension. We have already mentioned that 5th dimension is rolled-up in a circle, thus $x^5 = x^5 + 2\pi R$. The Fourier expansion of spinor Ψ, periodic in x^5 with the coefficients that depend upon x^μ, can be made,

$$\Psi = \sum_n \Psi_n(x^\mu) Y_n(x^5),$$

where Y_n are orthonormal eigenfunctions of the operator $-\partial_5^2$

$$\begin{aligned} Y_n(x^5) &= \frac{1}{\sqrt{2\pi R}} e^{-inx^5/R} \\ -\partial_5^2 Y_n &= \frac{n^2}{R^2} Y_n. \end{aligned} \quad (2.16)$$

Now we can place the Fourier expansion of spinor into action (2.13), differentiate with respect to x^5 where necessary and integrate over x^5, which yields

$$S_{D_{eff}} = 2\pi R \sum_n \int d^4x(\frac{1}{2}\Psi_n^\dagger\gamma^0(\gamma^m(p_m - \frac{n}{R\sqrt{\alpha}}A_m) + \gamma^5 \frac{n}{R})\Psi_n + h.c.).$$

Quantized electric charge and mass are therefore [5]

$$q_n = \frac{n}{R\sqrt{\alpha}},$$

$$m_n = \frac{n}{R}. \tag{2.17}$$

Knowing the experimantal data for electron mass and electric charge, we can verify the validity of results (2.17). If we set $q = e_0$ and calculate the electron mass, we get the result $m \approx 3 \times 10^{30} MeV$ instead of experimantally known $m = 0.51 MeV$. Thus Kaluza-Klein theory encounter some serious problems obtaining the right electron charge and mass ratio among the other problems, mentioned in section 3.

2.2 Higher dimensions

Once understanding the 5-dimensional Kaluza-Klein theory we can extend the theory to higher dimensions. What we have to determine is the number of dimensions that would describe all the known interactions. The $(1+(3+D))$-dimensional space-time has the topology $M^4 \times B^D$, where B^D must contain $SU(3) \times SU(2) \times U(1)$ as a symmetry subgroups. The symmetry group of electromagnetism is $U(1)$, of weak interaction is $SU(2)$ and of strong interaction is $SU(3)$. One of the choices for B^D is $CP^2 \times S^2 \times S^1$, where CP^2 is a 4-dimensional complex projective space with symmetry $SU(3)$, S^2 is a 2-dimensional sphere with symmetry $SU(2)$ and as we already know, S^1 is a 1-dimensional circle with symmetry group $U(1)$. We can simply count that this space has 7 dimensions. Although this is not the only space with required symmetry, only the 7 or more dimensional spaces contain this symmetry. This implies that the number of dimensions of realistic Kaluza-Klein theory has to be at least $d = (1 + 10)$ [5].

3 Prospectives of Kaluza-Klein theory

As mentioned earlier in section 2.1.2, Kaluza-Klein theory encounter some difficulties. One of them is already described deviation of electron mass and electric charge ratio from experimantal data. Besides that, E. Witten has proved the so-called *Witten no-go theorem*, telling that the Kaluza-Klein(like) theories have severe difficulties obtaining massless fermions chirally coupled to the Kaluza-Klein-type gauge fields in $(1+3)$ dimension, as required by the standard model. This can be solved by putting extra gauge fields by hand in addition to gravity in higher dimensions, but this loses the elegance and is no longer the pure Kaluza-Klein(like) theory. Since the assumption that the extra dimensions are compactified is used in the Witten no-go theorem, there are also the attempts to achieve masslessness by appropriate choices of vielbeins and spin connectionn fields in noncompact (almost compact) spaces [9].

The theory can also be extended to explain the origin of families in addition to origin of charges. For this, at least $d = (1+13)$ dimensional space-time is needed, as can be seen in *spin-charge-family theory* [10]. The author is claiming that her theory is promissing in explaining the assumptions of the standard model. She starts derivation with only vielbeins and two kinds of the spin connection fields, connected with the two kinds of the Dirac gamma matrices, γ^a and $\tilde{\gamma}^a$, ending up at low energies effectively with the standard model.

Although the original $(1 + 4)$-dimensional Kaluza-Klein theory failed to provide the realistic description of nature, its basic ideas lead to many new unified field theories. For example, connecting Kaluza-Klein theory with supergravity resulted in improved supersymmetric Kaluza-Klein theory. Many theories, united under the common name *Multidimensional Unified Theories*, are using the idea of Kaluza and Klein to postulate extra (compactified) space dimensions. One of these theories is also perhaps the most widely known string theory.

4 Conclusion

The original Kaluza-Klein theory provided very advanced starting idea which enable to show the elegant way to unify gravity and all the other gauge fields. Although it turned out that it still needs a lot of proofs and justifications that it might be the right next step beyond present theories and current understanding of the laws of nature, it looks very promissing because of the elegance and simplicity. The theory also set the solid background for many other unified filed theories. By now no proven consistent theory has been found, but more and more physicists start to accept that postulating the extra space dimensions is the right way in obtaining the unification.

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