Abstract

Solitons are the solutions of certain nonlinear partial differential equations, with interesting properties. Because of a balance between nonlinear and linear effects, the shape of soliton wave pulses does not change during propagation in a medium. In the following seminar, I present the general properties of solitons and the way these can be used in physical applications. Focusing on optical solitons, both temporal and spatial solitons are presented together with the physical effects that make them possible. The advantages and difficulties regarding soliton based optical communication are explained. Furthermore, the potential future application of solitons and soliton interactions for ultra fast optical logical devices is introduced.
# Contents

1 Solitons 2
   1.1 General properties ........................................ 2
   1.2 A brief history ............................................. 3

2 Optical solitons 3
   2.1 Temporal solitons ........................................... 4
       2.1.1 Derivation of the nonlinear Schrödinger equation .... 5
       2.1.2 Soliton solutions ...................................... 6
       2.1.3 Soliton stability ....................................... 8
       2.1.4 Fiber losses ........................................... 8
       2.1.5 Soliton amplification .................................. 9
   2.2 Spatial solitons ............................................. 10
   2.3 Spatio-temporal solitons ................................... 12
   2.4 Soliton interactions ....................................... 13
   2.5 Soliton logic gates ....................................... 13

3 Conclusion 15

4 References 15
1 Solitons

1.1 General properties

Although it is hard to find a precise definition of a soliton, the term is usually associated with any solution of a nonlinear equation which

1. Represents a wave of permanent form
2. Is also localized (though moving)
3. Can interact strongly with other solitons and retain its identity

[1] In other words, a soliton is a wave packet (a pulse) that maintains its shape while traveling at a constant speed. Solitons are caused by a cancellation of nonlinear and dispersive effects in the medium. [2] Dispersion is the phenomenon in which the phase velocity of a wave depends on its frequency. [3] Every wave packet can be thought of as consisting of plane waves of several different frequencies. Because of dispersion, waves with different frequencies will travel at different velocities and the shape of the pulse will therefore change over time. It is important to point out that dispersion does not add any new frequency components to the original specter of the pulse, it merely rearranges the phase relations between the existing ones. On the other hand, nonlinear effects (such as the Kerr effect in optics) may modify the phase shift across the pulse and thus create new frequency components in the specter of the pulse (in optics this effect is known as self-phase modulation). If the original pulse has the right shape, nonlinear effects may exactly cancel dispersion thus producing a pulse with a constant shape: a soliton. [2] The third above mentioned property of solitons, the fact that they can interact strongly and continue almost as if there had been no interaction at all, is what led N. J. Zabusky and M. D. Kruskal to coin the name soliton (in order to emphasise their particle-like behavior in collisions).

[1] While many nonlinear dispersive partial differential equations yield soliton solutions the most important ones (of those describing physical systems) are the Korteweg-de Vries equation (describing waves on shallow water surfaces) and the nonlinear Schrödinger equation (describing waves on deep water surfaces and more importantly light waves in optical fibers). [1] Examples of observable soliton solutions include solitary water waves (on the surface or undersea - internal waves) and also atmospheric solitons such as Morning Glory clouds (vast linear roll clouds produced by pressure solitons travelling in a temperature inversion). [4]

Figure 1: Morning Glory clouds in Queensland, Australia - these vast linear clouds are the product of pressure soliton waves. [4]
1.2 A brief history

In 1834, John Scott Russel, a Scottish naval architect was conducting experiments to determine the most efficient design for canal boats. During his experiments he discovered a phenomenon that he described as the wave of translation. He described his discovery in an article called *Report on waves* [5] which represents the first scientific account of solitons in history. The phenomenon was a water wave that formed in a narrow channel and displayed some strange properties. For one, the wave was stable, it neither flattened out or steepened like normal waves and Russel was able to follow it for a few kilometers. Furthermore, the wave would not merge with other waves, a small wave traveling faster would rather overtake a large slower one. [5] Through a series of measurements, Russel managed to determine an empirical formula for the velocity of such waves but was unable to find the correct equation of motion. [2]

![Figure 2: A modern recreation of Russel’s wave of translation - a soliton water wave.](image)

In 1895 Diederik Korteweg and Gustav de Vries derived a nonlinear partial differential equation (today known as the Korteweg-de Vries equation) that describes Russel’s solitary waves. Their work fell into obscurity until 1965 when Norman Zabusky and Martin Kruskal published their numerical solutions of the KdV equation (and invented the term soliton). In 1973 Akira Hasegawa was the first to suggest that solitons could exist in optical fibers, due to a balance between self-phase modulation and anomalous dispersion. In 1988 Linn Mollenauer and his team transmitted soliton pulses over 4000 kilometers using a phenomenon called the Raman effect (see chapter 2.1.5), to provide optical gain in the fiber. Only three years later, a Bell research team transmitted solitons error-free at 2.5 gigabits per second over more than 14000 kilometers, using erbium (rare earth) optical fiber amplifiers. [2]

2 Optical solitons

In optics, the term soliton is used to refer to any optical field that does not change during propagation because of a balance between nonlinear and dispersive effects in the medium. In both cases, the nonlinear part in equations is a consequence of the Kerr effect in media. [7] The Kerr effect is a phenomenon
where an applied electrical field induces a change in the refractive index of a material. What distinguishes the Kerr effect from others of this kind (e.g. the Pockels effect) is that the change in refractive index is directly proportional to the square of the electric field (or in other words, directly proportional to the intensity). The refractive index is then described by the following equation:

\[ n(I) = n + n_2 I \]  

(2.1)

where \( I \) is the field intensity and \( n_2 \) the second-order nonlinear refractive index. [3]

There are two main kinds of optical solitons: spatial and temporal solitons. Temporal solitons were discovered first and are often simply referred to as solitons in optics. We talk about temporal solitons when the electromagnetic field is already spatially confined (e.g. in optical fibers) and the pulse shape (in time) will not change because the nonlinear effects balance dispersion. Spatial solitons on the other hand may occur when the electromagnetic field is not spatially confined but the pulse shape in space does not change with propagation because of nonlinear effects (self-focusing) balancing out diffraction. [7]

2.1 Temporal solitons

The main problem that limits transmission bit rate in optical fibers is group velocity dispersion. Because of dispersion and the fact that impulses have a non-delta specter, the envelope of impulses widens for \( \Delta \tau \) after traveling the distance \( L \) in an optical fiber with the dispersion parameter \( D \), as shown in the following equation:

\[ \Delta \tau \approx DL \Delta \lambda \]  

(2.2)

where \( \Delta \lambda \) is the bandwidth of the impulse. The impulse widens whether we are in the region of normal \((D < 0)\) or anomalous \((D > 0)\) dispersion. However, soliton solutions, after adding the Kerr effect in the picture, are only possible in the region of anomalous dispersion, where higher frequency waves travel faster than low frequency ones. On the other hand self-phase modulation, acting in the Fourier domain, produces a chirped pulse but does not change the original envelope. [8]

Figure 3: Dispersive (top) and nonlinear effects (bottom) on a Gaussian pulse. [8]
2.1.1 Derivation of the nonlinear Schrödinger equation

An electric field propagating in a medium showing optical Kerr effect through a guiding structure (such as an optical fiber) that limits the power on the xy plane can be expressed in the following way:

\[ E(r, t) = A_m u(t, z) f(x, y) e^{i(k_0 z - w_0 t)} \]  

(2.3)

where \( A_m \) is the maximum amplitude of the field, \( u \) the envelope of the pulse normalized to 1, \( f \) the shape of the pulse in the x, y plane, determined by the optical fiber and \( k_0 \) the angular wave number for propagation in the z direction. The scalar representation of the electric field is appropriate because of the small biefringence of optical fibers. [2] To obtain the wave equation for the slowly varying amplitude \( E(r, t) \), it is more convenient to work in the Fourier domain.

\[ \tilde{E}(r, \omega - \omega_0) = \int_{-\infty}^{\infty} E(r, t) e^{-i(\omega - \omega_0) t} \]

Inserting (2.3) into the wave equation we obtain the Helmholtz equation where the refractive index depends on the frequency and also on the intensity:

\[ \nabla^2 \tilde{E} + n^2(\omega) k_0^2 \tilde{E} = 0 \]  

(2.5)

The phase change parameter \( \beta(w) = n(w) k_0 \) can be described as:

\[ \beta(w) = \beta_0 + \beta_1(w) + \beta_{nl} = \beta_0 + \Delta \beta(w) \]  

(2.6)

[8] We assume that both dispersive and nonlinear effects are small (which is true in optical fibers [2]): \( |\beta_0| \gg |\Delta \beta(w)| \). It is therefore appropriate to approximate the linear (dispersive) part with its Taylor expansion around some central frequency \( \omega_0 \):

\[ \beta(w) = \beta_0 + (\omega - \omega_0) \beta_1 + \frac{(\omega - \omega_0)^2}{2} \beta_2 + \beta_{nl} \]  

(2.7)

where \( \beta_u = \frac{d^2 \beta(w)}{dw^2} \). Inserting the electric field (2.3) into equation (2.5) we obtain:

\[ 2i \beta_0 \frac{\partial \tilde{u}}{\partial z} + [\beta^2(\omega) - \beta_0^2] \tilde{u} = 0 \]  

(2.8)

where we have neglected the second derivate of \( \tilde{u} \) because of the assumption that \( \tilde{u}(z, \omega) \) is a slowly varying function of \( z \): \( \frac{\partial^2 \tilde{u}}{\partial z^2} \ll |\beta_0 \frac{\partial \tilde{u}}{\partial z}| \). This is known as the slowly varying envelope approximation. Because of small dispersive and nonlinear effects we also make the following approximation:

\[ \beta^2(\omega) - \beta_0^2 \approx 2\beta_0 \Delta \beta(\omega) \]  

(2.9)

Transforming back into the time domain: \( \Delta \beta(\omega) \iff i \beta_1 \frac{\partial}{\partial t} - \frac{\beta_2}{2} \frac{\partial^2}{\partial t^2} + \beta_{nl} \) and expressing the nonlinear component in terms of the amplitude of the field:

\[ \beta_{nl} = k_0 n_2 I = k_0 n_2 \frac{|E_0|^2}{2\eta_0^2} = k_0 n_2 n^2 \frac{|A_m|^2}{2\eta_0^2} |u|^2 \] (where \( \eta_0 \) is the wave impedance of empty space), we obtain:

\[ i \frac{\partial u}{\partial z} + i \beta_1 \frac{\partial u}{\partial t} + \frac{\beta_2}{2} \frac{\partial^2 u}{\partial t^2} + \frac{1}{L_{nl}} |u|^2 u = 0 \]  

(2.10)

5
Since we know that the pulse propagates in the direction of the z axis with the group velocity \( \frac{1}{\beta_1} \), we make the substitution \( T = t - \beta_1 z \), in order to follow only the changing of the envelope of the pulse. Also, because we assume anomalous dispersion, we know that \( \beta_2 < 0 \). The equation is now finally:

\[
\frac{1}{2} \frac{\partial^2 u}{\partial \tau^2} + i \frac{\partial u}{\partial \zeta} + N^2 |u|^2 u = 0
\]

(2.11)

[8] In this equation we introduce the so called soliton units, so that the equation is in its general form. Equation (2.11) is the fundamental equation describing temporal (and as we shall see also spatial) solitons. It is called the nonlinear Schrödinger equation \(^1\). The variables in the equation are: \( \tau = \frac{T}{T_0} \) where \( T_0 \) is connected with the FWHM of the fundamental soliton solution \( (2\ln(1 + \sqrt{2})T_0 \approx 1.763 T_0) \), \( \zeta = \frac{z}{L_d} \), where \( L_d \) is a characteristic length for effects of the dispersive term, connected with the dispersion coefficient (similarly is \( L_{nl} \), a characteristic length for nonlinear effects, connected with the maximum amplitude) and \( N^2 = \frac{L_{nl}}{L_d} \). It is easy to identify the dispersive (first) and nonlinear (third) terms in the equation. \(^2\) The physical meaning of the parameter \( N \) is the following:

1. if \( N \ll 1 \), then we can neglect the nonlinear part of the equation. It means \( L_d \ll L_{nl} \) therefore the field will be affected by the linear effect (dispersion) much earlier than the nonlinear effect.
2. if \( N \gg 1 \), then the nonlinear effect will be more evident than dispersion and the pulse will spectrally broaden because of self-phase modulation.
3. if \( N \approx 1 \), then the two effects balance each other and soliton solutions are possible.

This of course holds when the length of the fiber\(^2\) \( L \) is \( L \gg L_{nl} \) and \( L \gg L_d \), otherwise neither nonlinear nor dispersive effects will play a significant role in pulse propagation. Normally, in the case when \( L < 50 \text{ km} \), nonlinear and dispersive effects may be neglected for pulses with \( T_0 > 100 \text{ ps} \) and \( P_0 = |A_m| < 1 \text{ mW} \). However, \( L_d \) and \( L_{nl} \) become smaller as pulses become shorter and more intense. For example, \( L_d \) and \( L_{nl} \) are \( \approx 100 \text{ m} \) for pulses with \( T_0 \approx 1 \text{ ps} \) and \( P_0 \approx 1 \text{ W} \). For such optical pulses, both the dispersive and nonlinear effects need to be included if fiber length exceeds a few meters. \(^8\)

2.1.2 Soliton solutions

For \( N = 1 \) the solution of the equation is simple and is known as the fundamental solution:

\[
u(\tau, \zeta) = \text{sech}(\tau) e^{i \zeta / 2}
\]

(2.12)

Since the phase term in the equation has no dependence on \( t \), the soliton is completely nondispersive. That is, its shape does not change with \( z \) either in the temporal domain or in the frequency domain. \(^2\) For soliton solutions, \( N \) must be an integer and is said to be the order of the soliton.

\(^1\)It is important to note that this equation got its name because of a similarity to the well known Schrödinger equation, but does not describe the evolution of quantum wave functions.

\(^2\)The described effects have almost no dependence on the other two dimensions of the fiber.
Figure 4: Dispersion of a pulse of initial sech\((T/\tau)\) profile in a medium without nonlinear effects. The pulse FWHM broadens from 16.5 fs to 91.5 fs, after traveling the distance of 5 \(Z_0\) (confocal dispersive distances). \[7\]

Figure 5: Temporal soliton pulse of initial sech\((T/\tau)\) profile in a medium with both dispersive and nonlinear effects. After propagating 5 \(Z_0\), the pulse position and duration are the same. \[7\]

Higher order solitons\(^3\) change shape during propagation but in a periodic manner with the period \(\zeta_0 = \frac{\pi}{2}\). To give an example, the typical soliton period for pulses with \(T_0 = 1\) ps is about 80 m and scales as \(T_0^2\) becoming 8 km when \(T_0 = 10\) ps. Also, the maximum amplitude needed to produce higher order solitons scales as \(N^2\) compared to the one needed for the fundamental soliton. Since in higher order solitons, the maximum amplitude rises dramatically during the changing of the envelope in every soliton period, it is possible to obtain amplitudes which render some of the proposed approximations obsolete, or even such that may damage the medium. \[8\]

\(^3\)Soliton solutions where \(N\) is an integer and \(N > 1\). \[8\]
Figure 6: The intensity pulse shape of a third order ($N = 3$) soliton during one soliton period. [8]

The evolution pattern of higher order solitons (as seen in Fig. 6) is the consequence of mutual dispersive and self-phase modulation effects. In the case of a fundamental soliton, these effects balance each other so that neither pulse shape nor spectrum change during propagation. In higher order solitons, self-phase modulation dominates initially, but dispersion soon catches up leading to a periodic behavior. [8]

2.1.3 Soliton stability

The natural question to ask is what happens if the initial pulse shape and/or maximum amplitude are not appropriate for a soliton solution. In other words, the scenario where $N$ is not an integer. It has been shown that the pulse adjusts its shape and width as it propagates along the fiber and evolves into a soliton. A part of the pulse energy is dispersed away in the process. This part is known as the continuum radiation. It separates from the soliton as $\zeta$ increases and its contribution to the soliton decays as $\zeta^{-\frac{1}{2}}$. For $\zeta \gg 1$ the pulse evolves asymptotically into a soliton whose order $\tilde{N}$ is an integer closest to the launched value of $N$ (of the initial pulse). This can be written as $N = \tilde{N} + \epsilon$, where $|\epsilon| < 0.5$. The initial pulse broadens for $\epsilon < 0$ and narrows for $\epsilon > 0$. For $N \leq \frac{1}{2}$ no solitons are formed. One may conclude that the exact shape of the input pulse used to launch a fundamental soliton is not critical. However, it is important to realize that, when input parameters deviate substantially from their ideal values, a part of the pulse energy is invariably shed away in the form of dispersive waves as the pulse evolves to form a fundamental soliton. Such dispersive waves are undesirable because they not only represent an energy loss but can also affect the performance of soliton communication systems [8].

2.1.4 Fiber losses

Because solitons result from a balance between the nonlinear and dispersive effects, the pulse must maintain its maximum amplitude if it has to preserve its soliton character. Fiber losses are detrimental simply because they reduce the maximum amplitude of solitons along the fiber length. As a result, the width of a fundamental soliton also increases with propagation because of power loss. [8]
If the peak power decreases exponentially $P = P_0 e^{-\alpha z} = C|A_m|^2$, the equation

$$1 = N^2 = \frac{T_0^2 |A_m|^2 n_2 n_k \eta_0}{2 \pi p_{0c} |\beta|^2}$$

shows that in order to keep the soliton, the width $T_0$ must exponentially increase during propagation. This is true for small distances where $\alpha z \ll 1$ since after a substantial broadening of the pulse, the nonlinear effects become negligible and the pulse continues to broaden in a linear fashion (as shown in Fig. 7). Even though optical fibers have a relatively low linear loss ($\approx 0.2 \text{ dB/km}$) this is a serious problem for temporal solitons propagating in fibers for several kilometers [2].

2.1.5 Soliton amplification

To overcome the effect of fiber losses, solitons need to be amplified periodically so that their energy is restored to its initial value. Two different approaches have been used for soliton amplification: the so called lumped and distributed amplification schemes. In the lumped scheme, an optical amplifier boosts the soliton energy to its input level after the soliton has propagated a certain distance. The soliton then readjusts its parameters to their input values. However, it also sheds a part of its energy as dispersive waves (continuum radiation) during this adjustment phase. The dispersive part is undesirable and can accumulate to significant levels over a large number of amplification stages. This problem may be solved by reducing the spacing $L_a$ of amplifiers so that $L_a \ll L_d$. For high bit-rates, requiring short solitons ($T_0 < 10 \text{ ps}$), $L_d$ becomes very short thus making this method impractical. [8]

The distributed-amplification scheme uses stimulated emission for amplification: either stimulated Raman scattering or erbium-doped fibers. Raman amplification is based on the stimulated Raman scattering (SRS) phenomenon, when a lower frequency input photon induces the inelastic scattering of a higher-frequency pump photon in an optical medium. As a result of this, another photon is produced, with the same phase, frequency, polarization, and direction as the input one while the surplus energy of the pump photon is resonantly passed to the vibrational states of the medium. For this method of amplification, a pump beam (up-shifted in frequency from the soliton carrier frequency by nearly $13 \text{ THz}$) is injected periodically into the fiber. The up-shift in frequency determines which frequencies will be most amplified$^4$ as shown in Fig. 8. [8]

$^4$Although the pulses are not monochromatic, their spectrum is nevertheless narrow enough, so that the Raman gain is roughly the same for all the represented frequencies. [8]
The Raman gain versus the frequency difference between input and pump photons. The figure demonstrates that frequencies down-shifted from the pump frequency for about 12 Thz will be maximally amplified. [8]

In the case of erbium-doped fibers, the fiber is doped with $\approx 0.1\%$ by weight of $Er^{3+}$ ions which have an emission and absorption cross-sections in the wavelength range 1525 nm – 1565 nm. This range coincides with the typical soliton carrier wavelengths making erbium an ideal stimulated emission amplifier. As before, energy is provided by means of a pump beam injected periodically in the fiber. [2]

In both cases, optical gain is, in contrast to lumped amplification, distributed over the entire fiber length. As a result, solitons can be amplified while maintaining $N \approx 1$, a feature that reduces the dispersive part almost entirely. [8] However, spontaneous Raman emissions from excited states also occur. These photons have a random phase, polarization and direction, and represent a noise with the spectrum approximately the same as the gain spectrum of the amplifier (e.g. for SRS Fig. 8). These photons may then be amplified by the process of stimulated emission in the same way as input photons and are called amplified spontaneous emission (ASE). ASE represents the main difficulty for long-distance optical communication with solitons. ASE photons randomly change the soliton parameters and degrade the signal-to-noise ratio. The main problem however is that ASE noise also produces random variations of the solitons central frequency. Because of dispersion, the pulses start to travel with a changed velocity, producing a jitter in pulse arriving times. This effect is known as the Gordon-Haus effect. Since the timing deviations of the pulses accumulate more and more with propagation, this is a serious difficulty for ultra long haul communication. Regularly spaced optical filters, or simply amplifiers with limited gain bandwidth are normally used to limit this effect. [2]

2.2 Spatial solitons

Spatial solitons result from the balance between linear diffraction and non-linear self-focusing. Self-focusing is possible in media with nonlinear effects (such as the Kerr effect). Because of the dependence of the refractive index on the intensity (in other words on the pulse shape), the refractive index thus depends on the position in space. The phase change with propagation will therefore be different for different parts of the pulse, thus creating an effect similar to the propagation through a thin lens. The result is self-focusing and depends on the shape of the wavefront. [7]
Figure 9: The focusing of a thin lens - the phase change depends on the position. The intensity pulse shape in a material exhibiting the Kerr effect changes the refractive index of the medium so that it acts as a thin lens. [3]

Coupled with linear diffraction, spatial solitons are possible.

Figure 10: The combined effects of linear diffraction and self-focusing. [7]

It has been shown that spatial solitons are stable in one transverse dimension, meaning that the balance between diffraction and nonlinear effects prevent small amplitude or phase perturbations from destroying the soliton. If a perturbation acts to widen the soliton, nonlinear self-focusing overpowers diffraction to restore balance and vice versa. This is in perfect analogy with the temporal case where non-soliton initial pulses would evolve into solitons of the nearest order. For a system confined in one spatial dimension, the electric field can be expressed in the following way:

\[
E(r, t) = A_m u(x, z) f(y) e^{i(k_0 z - w_0 t)}
\] (2.13)

where as before \(A_m\) is the maximum amplitude of the field, \(u\) the envelope of the pulse normalized to 1, \(f\) the shape of the pulse in the \(y\) plane, determined by the geometry of the medium (a planar waveguide) and \(k_0\) the angular wave number for propagation in the \(z\) direction. The equation describing spatial solitons is then the following:

\[
\frac{1}{2} \frac{\partial^2 u}{\partial \xi^2} + i \frac{\partial u}{\partial \zeta} + N^2 |u|^2 u = 0
\] (2.14)

This again is the nonlinear Schrödinger equation that governed the behaviour of temporal solitons\(^5\) (eq. 2.11), the only difference is that instead of a second

\(^5\)For a detailed step by step derivation of the nonlinear Schrödinger equation for spatial solitons see [7].
derivate of time, here is a second derivate of the transversal spatial dimension (since our pulse envelope is now \( u = u(x, z) \neq u(t) \)). The soliton units in this case are: \( \xi = \frac{x}{X_0} \), where \( X_0 \) is connected with the FWHM of the fundamental soliton solution \( (X_{FWHM} = 2 \ln(1 + \sqrt{2}) X_0 \approx 1.763 \ X_0) \), \( \zeta = \frac{z}{L_d} \) as before and \( N^2 = \frac{L_d}{L_{nl}} \) also. [7] Spatial and temporal solitons are governed by the same equation owing to the fact that the nonlinear Schrödinger equation describes all transmission lines with dispersive and nonlinear properties (under certain approximations\(^6\)), even outside the field of optics. [2] This means that soliton solutions and their properties as described in chapters 2.1.2 and 2.1.3 are the same for spatial solitons under the assumption of one confined spatial dimension (and the pulse envelope being a function of \( x \) and not \( t \)). In this case \( L_d = X_0 k_0 n \) stands for a characteristic diffraction length. [7]

### 2.3 Spatio-temporal solitons

A very interesting field of research are spatio-temporal solitons which are stationary in both space and time. Although it is known that 2D and 3D (in literature often noted as (1+1)D and (2+1)D to distinguish between temporal and spatial dimensions) solutions are unstable (pulses either diffract at low powers or collapse and break into filaments at high powers), research in the last two decades has shown that certain processes beyond linear dispersion/diffraction or the Kerr effect may well arrest the tendency of pulses to collapse. One of this processes is the quadratic nonlinearity (opposed to cubic which is the Kerr effect). There are a number of effects that may arrest the collapse of the pulse but the same effects will then also remove energy from the pulse precluding the formation of a spatio-temporal soliton. However, in quadratic nonlinear media a saturable nonlinearity is possible in a non-intuitive sense. The effect is a consequence of the interaction between the fundamental field and its second harmonic\(^7\). The nature of these effects is very complicated and I present it here only because the first spatio-temporal soliton was produced in this way (Fig. 11). [10]

![Figure 11: The first spatio-temporal soliton. Consisting of two fields, the fundamental (red) and its harmonic (blue), mutually trapped in a form of a stripe. Because of their lack of confinement along the horizontal spatial dimension, they collapse into a number of filaments (white dots) which propagate for a short while as fully confined STS (packets of radiation confined in time and all spatial dimensions), and then collapse. [9]](image)

Stable fully confined 3-dimensional STS, also called tight bullets have not been observed to date and remain an area of research. [9] Spatio-temporal solitons

---

\(^6\)To derive the nonlinear Schrödinger equation for spatial solitons one has to work in the paraxial approximation and also assume that the envelope varies slowly with \( x \). [7]

\(^7\)The following article is a good introduction and a list of references: [9].
are a promising field because of their possible use in ultrafast optical logic gates. [7]

2.4 Soliton interactions

Soliton interactions\(^8\) have been known since the first treatment of the nonlinear Schrödinger equation. The motivation for the study came from the effects of soliton interactions on the timing jitter in soliton communications systems. These interactions, which represent a difficulty for communications, can be useful for switching though. The most common interactions between solitons (Fig. 12) are: collision, attraction/repulsion and trapping/dragging. [7]

![Diagram of soliton interactions](image)

Figure 12: An illustration of common soliton interaction geometries. The dashed lines indicate the soliton paths in the case of no interaction. [7]

If two same-polarized solitons interact, the interaction is phase-sensitive. On the other hand, the interaction of two orthogonally-polarized solitons is phase-insensitive. Typically, the collision between two same-polarized solitons produces a spatial shift in the propagation of each soliton after collision. Since the interaction is a symmetric one, there is no net change in the propagation angle. The collision of two orthogonally-polarized solitons may on the other hand result in a angle change. Two same-polarized and in phase solitons will attract each other while two solitons with a relative phase \(\pi\) will repel (for other relative phases, more complex behavior is expected). Trapping and dragging are interactions where in the beginning two solitons overlap, the former being phase-sensitive and the latter phase-insensitive. [7]

2.5 Soliton logic gates

An optical logical device could be made if signals in the form of optical solitons could affect the propagation of another, potentially more energetic, soliton called the pump\(^9\), which is always present and is analogous to the power supply in electronic logic gates. The pump soliton propagates through the logical gate to the output, providing the high state of the device, unless affected by the interaction with a signal soliton. When present, a signal soliton interacts with the pump, altering the propagation of the pump by inducing a spatial (or temporal)

---

\(^8\) These interactions have of course nothing to do with the fundamental interactions, they are merely interpretations of the solutions of the NLS equation. [7]

\(^9\) This pump soliton has of course nothing to do with pump photons used in soliton amplification.
shift, angle (or frequency) change, rotation of polarization, etc. Since displacement is more easily detected than other changes, only spatial shifts and angle changes fall into the category of potential mechanisms. Furthermore, since the pump soliton continues to propagate without broadening after the interaction, angle changes result in arbitrarily large spatial shifts, dependent on the length of the gate (thus making angle changes more suitable than single spatial shifts).

Another issue is the nature of interaction. In a computing or switching system consisting of a large number of gates, a fixed relative phase may be difficult or even impossible to maintain, therefore it is essential to reduce or completely eliminate phase-dependence of the interaction. This is possible to achieve if the two interacting solitons are orthogonally-polarized.

Figure 13: The left hand plot shows the paths of two orthogonally-polarized solitons without interaction and the right hand plot their paths after the dragging interaction. The interaction changes the angles of both solitons (and thus the final positions) enough that the threshold contrast of such a gate is > 1000.

Figure 14: Simulation (left) and observation (right) of spatio-temporal soliton formation in a Y-junction geometry. (a) and (b) show individual STS formation with only the right- and left-going fields, respectively. (c) shows STS formation along the center with both fields present. A detector placed at the center of the frame sees a signal only if both fields are launched - a logical AND operation.
3 Conclusion

In this seminar I have presented the general properties of solitons as solutions of certain nonlinear partial differential equations. Solitons may exist if there is a balance between opposing linear and nonlinear effects. Their interesting and nonintuitive properties make them candidates for various applications in physics. The canceling of dispersive broadening of pulses makes temporal solitons suitable for ultra long haul high bit rate optical communication. On the other hand, the particle-like behavior in interactions, makes possible the use of solitons for ultra fast optical logical devices.

4 References


