Abstract

Various trends in the spacecraft industry are driving the development of low-thrust propulsion systems. In this seminar I will present the cold gas microthruster as a simple propulsion system for fine attitude and orientation controls. I will show how they are miniaturized for the use in a microspacecraft. I will also show the results of the calculation of the velocity flow in the microthruster nozzle calculated using finite volume method.
1 Introduction

In the last decade there has been a research and development initiative by the aerospace industry to reduce satellite life-cycle costs while still delivering a spacecraft with the capability of performing useful science or commercial service. The directives are: faster, better, cheaper. A wide range of resources from program management techniques to advanced technologies are being focused on achieving each of these directives. One of the means to this end is the development of microspacecraft in the 1-20 kg class. Such a satellite may contain only one instrument, but this reduction in complexity will lower costs by facilitating systems integration. In addition, the small sizes allow the selection of a smaller, less expensive launch vehicle or integration of multiple satellites per vehicle. Launch costs for a typical interplanetary mission may be as high as 30% of the overall mission cost, and these costs may be reduced significantly as a result of substantially reduced spacecraft masses. [1],[2]

2 Micropropulsion

Building microspacecraft in the 1-20 kg class necessitates the miniaturization of every subsystem in order to maintain the high degree of onboard capability required to ensure an acceptable scientific return for the mission. One of the sub-systems that is included in such a reduction in weight and size is propulsion. Although in the past many very small spacecraft have lacked propulsion systems altogether, future microspacecraft will likely require significant propulsion capability in order to provide a high degree of maneuverability.
In particular, interplanetary mission scenarios will require propulsion capability on microspacecraft for course corrections as well as attitude control to maintain precise orbits and accurately point the spacecraft for observation or communication. [1]

In order to meet microspacecraft propulsion requirements, the use of lightweight, small sized, low thrust and small impulse (change in momentum) bit systems will be needed. The $I_{bit}$ is the minimum impulse obtained once the thruster is given the command to fire. It is the integrated thrust $T$ over the fastest valve cycle time $t_{min}$.

$$I_{bit} = \int_{0}^{t_{min}} T(t) \, dt \quad (1)$$

This is achieved with the use of Microelectromechanical systems (MEMS) which attain a high degree of functionality at a much smaller scale. MEMS devices are sensors and actuators which are fabricated with the use of micromachining techniques developed by the microelectronics industry. Using MEMS several propulsion components, such as thrusters and valves, plus the required control electronics, may be integrated onto a single chip, or a 3D-stack of chips. [1],[2]

In 1966 G. Sutherland and M. Maes reviewed different propulsion architectures. They stress the need to generate not only low thrust, but also to minimize the impulse bit $I_{bit}$. They also showed that for the purpose of attitude control an improvement by decreasing $I_{bit}$ is more important than an improvement in increased specific impulse $I_{sp}$ (impulse per unit amount of propellant used). [2]

$$I_{sp} = \frac{I_{tot}}{m_p} \quad , \quad (2)$$

where $I_{tot}$ is the total impulse and $m_p$ is the propellant mass. The total impulse is the integrated thrust over the firing cycle time of the thruster

$$I_{tot} = \int_{0}^{t_b} T(t) \, dt \quad , \quad (3)$$

where $T$ is the thrust and $t_b$ is the thrust duration of a single firing. [2]

With the utility of micropropulsion established, the ideal architecture for meeting the spacecraft mission requirements can be assessed. London laid out the domains of applicability for different propulsion architectures. This is shown in Figure 1 which shows which architectures minimize mass and power consumption for a given set of mission requirements. A particular architecture is dominant for a given mission scenario because it results in the lowest mass and power system for the control authority desired. The metrics are $\Delta V$, which indicates the amount of usage a thruster will get, and thrust, which is a factor in how quickly the actuation will occur. [2]

Nowadays the best micropulsion system for the use in the microspacecrafts is the cold gas system which offers a low $I_{bat}$ and a small thrust for slewing requirements. [2]

### 3 Cold gas thruster

Cold gas thrusters represent the smallest rocket engine technology available today. The propulsion mechanism here is derived simply from the expansion of a pressurised gas from a reservoir through a nozzle. Cold gas systems are valued for their low system complexity, small $I_{bat}$ and thrust capability and the fact that, when using benign propellants (e.g.
N₂), they present no spacecraft contamination problems. Sometimes high reliability is also referred to as one of the advantages. However, valve leakage problems have resulted in repeated losses of spacecraft due to premature depletion of propellant through valve internal leaks. Another disadvantage of cold gas systems are their low specific impulse performances, unless very light gases (H₂, He) are used. Neither hydrogen or helium is commonly used because low density of these gases would invoke storage problems due to heavy and large tankage, and additional concerns with valve leakage. [1]

Table 1 gives a list of typical cold gas performance values. It shows molecular weight \( M_r \), density \( \rho \), melting temperature \( T_m \), boiling temperature \( T_b \), theoretical specific impulse \( I_{sp,t} \) and measured specific impulse \( I_{sp,m} \) for different gases.

<table>
<thead>
<tr>
<th>Propellant</th>
<th>( M_r ) (kg/kmol)</th>
<th>( \rho ) (241 bar) (g/cm³)</th>
<th>( T_m ) (°C)</th>
<th>( T_b ) (°C)</th>
<th>( I_{sp,t} ) (s)</th>
<th>( I_{sp,m} ) (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen - H₂</td>
<td>2.00</td>
<td>0.02</td>
<td>-259</td>
<td>-253</td>
<td>296</td>
<td>272</td>
</tr>
<tr>
<td>Helium - He</td>
<td>4.0</td>
<td>0.04</td>
<td>-272</td>
<td>-269</td>
<td>179</td>
<td>165</td>
</tr>
<tr>
<td>Neon - Ne</td>
<td>20.4</td>
<td>0.19</td>
<td>-249</td>
<td>-246</td>
<td>82</td>
<td>75</td>
</tr>
<tr>
<td>Nitrogen - N₂</td>
<td>28.0</td>
<td>0.28</td>
<td>-210</td>
<td>-196</td>
<td>80</td>
<td>73</td>
</tr>
<tr>
<td>Argon - Ar</td>
<td>39.9</td>
<td>0.44</td>
<td>-189</td>
<td>-186</td>
<td>57</td>
<td>52</td>
</tr>
<tr>
<td>Krypton - Kr</td>
<td>83.8</td>
<td>1.08</td>
<td>-157</td>
<td>-152</td>
<td>39</td>
<td>37</td>
</tr>
<tr>
<td>Xenon - Xe</td>
<td>131.3</td>
<td>2.74</td>
<td>-112</td>
<td>-108</td>
<td>31</td>
<td>28</td>
</tr>
<tr>
<td>Propane - C₃H₈</td>
<td>41.1</td>
<td>liquid</td>
<td>-78</td>
<td>-33</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Butane - C₄H₁₀</td>
<td>58.1</td>
<td>liquid</td>
<td>-138.3</td>
<td>-0.5</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Sulfur hexafluoride - SF₆</td>
<td>146.1</td>
<td>—</td>
<td>—</td>
<td>-64 (S)</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Carbon dioxide - CO₂</td>
<td>44</td>
<td>liquid</td>
<td>-78 (S)</td>
<td>67</td>
<td>61</td>
<td></td>
</tr>
<tr>
<td>Ammonia NH₃</td>
<td>17</td>
<td>liquid</td>
<td>-78</td>
<td>-33</td>
<td>105</td>
<td>96</td>
</tr>
</tbody>
</table>

Table 1: Cold gas propellant performances. [3]

For the expelled gas higher atom weight is desirable due to the third Newton law. Together with the contamination free constraint, the moderate low boiling and melting temperature are desirable features for a propellant gas from system point of view, where mass efficient storage of the gas is a major concern. [3]

Of the gases listed is nitrogen for now by far the most frequently used as a cold gas, due to a combination of reasonable propellant storage density, performance and lack of
Another problem for a cold gas thruster using high pressure gas is tank storage. Tank design completely dominates the spacecraft design layout and consumes a substantial portion of the overall spacecraft mass and volume. For example: for a microspacecraft with the mass of 10 kg the required attitude control propellant mass of nitrogen is 0.7 kg. At a storage density of 0.28 g/cm$^3$ for nitrogen and taking a 10% increase because of propellant leakage into account a tank volume of about 2750 cm$^3$ is required. Assuming a spherical tank, this translates into an inner tank diameter of roughly 37 cm. This tank size is slightly larger than the envelope assumed for a 10 kg spacecraft. [1]

An alternative to conventional cold gas propulsion using high pressure gas tanks is the use of ammonia as a propellant. As can be seen from Table 1, specific impulses obtainable with ammonia are higher than those achievable with nitrogen. The ammonia system would allow for liquid storage, reducing tank size and mass and propellant leakage concerns. It can be then heated up to a gas stage before use. [1]

For microspacecraft attitude control there is a need to further reduce impulse bits. This requires the development of either faster valves or smaller nozzle throat areas. Small nozzle throat diameters can be fabricated using MEMS technology. [1]

Figure 2 shows the scanning electron microscope picture of a silicon MEMS nozzle with width of about 200 µm and throat width of 10 µm. The wavy shaped heat exchange structure which is used to heat up the gas in the chamber is also visible.

![Figure 2: MEMS nozzle with 10 micron throath.][4]

### 4 Thruster miniaturization

As the scale of a component is reduced, if it is fabricated from a material of similar density, its mass is reduced. Smaller mass implies quicker dynamic and thermal response times. An object under loading will accelerate more quickly if it has a smaller mass. This implies fast acting microvalves are possible due to the low mass of its components. Heaters can come to temperature more quickly since the thermal mass is lower. This reduces the transient time of a given architecture, which in the context of a propulsion system will reduce the minimum impulse bit. For a cold gas thruster this is a function of the valve response time and the thrust of the nozzle.[2]

In traditional space propulsion applications, the combination of high speeds and moderate-to-large length scales result in very high Reynolds numbers, sufficiently large that inviscid
analyses are employed as a first approximation. However, the importance of viscous effects in micro flows has emerged as a result of the development of micro-scale propulsion systems. With the characteristic length scales considered for these new propulsion systems being of the order of microns to millimetres, the corresponding Reynolds numbers within the nozzles are \(10^{-3}\) and hence viscous effects can no longer be ignored. Aside from viscous forces, other important effects such as heat transfer and flow rarefaction may also be present on the micro-scale. The former becomes a concern as the thermal mass of the flow is reduced and the surface area-to-volume ratio increases on the micro-scale. The latter introduces additional complications since gas kinetic (non-continuum) effects begin to emerge as the characteristic length scales begin to approach molecular mean free paths. Taken together, the combination of viscous/thermal/rarefaction effects on the micro-scale can significantly impact the flow behaviour in micro-nozzles. Nozzles based on macro-scale designs will exhibit performance degradations which are not predicted from traditional analyses. These degradations are especially significant for micro/nano satellite propulsion scenarios where fuel supply is inherently limited. From an engineering perspective the accounting for these micro-scale effects is essential in the design of efficient micro-nozzles. [2]

If the nozzle scale is reduced, lower thrust (which is required for these applications) is achieved. By shrinking the scale, higher chamber pressures can be used to achieve the same low thrust. The frictional losses are governed by the Reynolds number which is defined as

\[
Re = \frac{\rho v_1 D_t}{\mu_t} = \frac{\Phi_m}{\mu h_0}
\]  

(4)

where \(\rho\) is the chamber density (proportional to chamber pressure), \(v\) is the velocity, \(D\) is the throat width, and \(\mu\) is the viscosity, all defined at the nozzle throat. This can be defined in terms of the mass flow rate, and the nozzle height \(h_0\) (for an extruded nozzle). [2]

The thrust is proportional to the mass flow rate multiplied by the exit velocity. The exit velocity is ideally set by the expansion ratio of the nozzle. But, as viscous effects become important, it also decreases with Reynolds number. A component of the thrust is the exit pressure, which is neglected since it is also only a function of the nozzle expansion ratio. The thrust \(T\), for an extruded nozzle, can then be approximated in terms of the Reynolds number as

\[
T \propto Re h_0 \mu_t v_{exit}
\]  

(5)

Thus, for a constant thrust, the Reynolds number increases as scale decreases. This will result in lower frictional losses and higher delivered specific impulse \(I_{sp}\). Though the tank mass will not directly reduce with scale, the amount of propellant required is less due to the higher \(I_{sp}\), and the engine mass is also lower. Thus, the thrust to propulsion system mass can be significantly increased with a reduction in scale. [2]

Just as there are direct benefits to pursuing a reduction in scale, there are direct drawbacks to small features. As the scale is reduced, the residence time of a particle in a flow channel is reduced. This leads to incomplete equilibration of the flow energy. If the residence time is less than the relaxation time, the thermal energy of the stored gas will not fully convert into kinetic energy and performance will suffer. These are known as frozen losses. Frozen losses may also represent the energy that is used to break molecular bonds, which is not converted into fluid kinetic energy because the molecules do not have time to recombine while in the nozzle. Thus, the flow is a fixed (or frozen) composition at a lower energy state throughout the nozzle. These losses are a function of Reynolds number since collision frequency is a function of gas density and residence time or scale. Another direct drawback of the microscale is the limitation of fabrication technology. This leads to surface roughness that is a larger fraction of the local nozzle width than for macro devices. [2]
5 Cold gas microthruster fabrication

Microfabrication is the prime tool used in the development of MEMS systems. It is the geometrical or chemical alteration of a base material through etching or diffusion to create structures that enable a desired electrical or mechanical behavior. These processes were initially developed by the microelectronics industry to facilitate the batch-fabrication of integrated circuits on the most popular semiconductor, silicon. Silicon is an excellent working material in its single crystal form. It has a very high fracture strength and high thermal conductivity. [2]

The scale of these devices is typically limited with the size of the silicon wafer on one side and the resolution of the lithography used to pattern features of interest on the other side. For the processing 150 mm diameter wafers ranging anywhere from 300 microns to 1mm thick (500 microns is standard thickness) are used. The lithography step can resolve features as small as 2 microns when operated with a standard ultraviolet source with a contact mask. These limits are guidelines for the fabrication of devices that are primarily created through the techniques of surface micromachining (deposition and selective removal of thin films) and bulk micromachining (selective removal of the silicon substrate). Surface and bulk micromachining can be augmented with the ability to bond wafers together in order to convert etched trenches into sealed cavities. [2]

Figure 3 shows two etched nozzles with different throat widths. In Figure 3.a the width is 34 microns and in Figure 3.b the width is 18 microns.

6 Nozzle design and physical model

The cold gas micro-propulsion system performance depends strongly on the use of gas flow control. The complete gas handling system of four independent thrusters is integrated in the assembly of four structured silicon wafers. Each independent thruster contains a proportional valve, sensors for pressure, temperature, and thrust feedback and a converging-diverging micro-nozzle. The mass of the system is below 60 g. In total, this will provide the spacecraft with a safe, clean, low powered, redundant and flexible system for three-axis stabilization and attitude control until it runs out of gas in the tanks. [2]
6.1 Design estimates

Figure 4 shows the 2D geometry of the nozzle of a cold gas microthruster. Subsonic gas entering a converging zone increases its velocity, while supersonic gas increases its velocity in a diverging zone. Thus, if the gas reaches a sonic speed at the nozzle throat, it will continue its velocity increase to supersonic values beyond the throat in the nozzle expansion area. [4]

6.1.1 De Laval nozzle

The microthruster nozzle is shaped as a de Laval nozzle (also called convergent-divergent nozzle). As such the nozzle increases flow from a subsonic to a supersonic flow. This can be derived using 1D isentropic flow theory [5]. We start with the Euler equation where we take into account that flow is stationary and in one dimension: \( \vec{v} = (v(x), 0, 0) \). [5]

\[
\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} = -\frac{\nabla p}{\rho} \quad \rightarrow \quad vdv = -\frac{1}{\rho} dp \tag{6}
\]

From the conservation of mass \( \rho Av = \text{konst} \), where \( A \) is the cross section surface, the continuity equation is derived. [5]

\[
\frac{dp}{\rho} + \frac{dA}{A} + \frac{dv}{v} = 0 \tag{7}
\]

Using the simplified Euler equation (6), continuity equation (7) and the equation for the speed of sound

\[
c^2 = \left( \frac{dp}{d\rho} \right)_s = \frac{1}{\rho \chi s} = \frac{\kappa RT}{M} \tag{8}
\]

the equation for the derivative \( dv/\text{d}A \) is derived

\[
\frac{dv}{\text{d}A} = \frac{v}{A} \left( \frac{v^2}{c^2} - 1 \right)^{-1} \quad M = \frac{v}{c} \tag{9}
\]

\( M \) is the Mach number. [5]

From the equation (9) we see that for \( M > 1 \) (supersonic flow) is \( dv/\text{d}A > 0 \). The flow is slowing down \( (dv < 0) \) for a convergent channel \( (dA < 0) \) and increasing for a divergent channel \( (dA > 0) \). If we have \( M < 1 \) (subsonic flow) then \( dv/\text{d}A < 0 \) and the flow speed increases for a convergent channel and decreases for a divergent channel. At the point where convergent and divergent channels meet is \( dv/\text{d}A = 0 \) and the Mach number is one. [5]
Inserting the equation for adiabatic gas flow $pV^\kappa = \text{konst}$ and equation for ideal gas $p/\rho = RT/M$ into equation (6) gives us the relation between Mach number and temperature

$$\frac{T_0}{T} = 1 + \frac{\kappa - 1}{2} M^2,$$

where $\kappa = C_p/C_v$ is the heat capacity ratio and $T_0$ is the reference temperature condition for $v_0 = 0$. \[5\]

For the adiabatic process the pressure and density dependences from Mach number can be derived from equation (10):

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\kappa - 1}{2M^2}} = \left(1 + \frac{\kappa - 1}{2} M^2\right)^{\frac{\kappa - 1}{2M^2}}$$

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{1}{\kappa - 1}} = \left(1 + \frac{\kappa - 1}{2} M^2\right)^{\frac{1}{\kappa - 1}}.$$ 

Figure 5.a shows a simple diagram of speed, temperature and pressure inside different parts of the de Laval nozzle. Speed is increasing through the nozzle while temperature and pressure are decreasing.

For ideal flows in the de Laval nozzle we derived the speed dependence above in this section. But on the scale of a micronozzle the viscous effects must also be considered. Figure 5.b shows Mach number ($M = v/c$) dependence of the distance from the inlet of the nozzle. The characteristic length $L$ is the distance between the inlet and the nozzle throat. The point $x/L = 1$ on the graph represents nozzle throat and the point $x/L = 3$ represents nozzle outlet. The outlet Mach number decreases with the decrease of the nozzle scale. With the decrease in the size of the nozzle, the position of the sonic point moves away from the throat to the outlet of the micronozzle. \[6\] The full physical model with viscosity will be treated in section 6.2.

### Figure 5: Effect of the nozzle scale on the flow speed.

#### (a) Diagram of speed, temperature and pressure in an ideal de Laval nozzle. \[7\]

#### (b) Distribution of centerline Mach number in micro nozzles with different scales. Throat width is shown in the legend. $L$ is the characteristic length of the nozzle - distance from the inlet to the throat. \[6\]

#### 6.1.2 Microthruster thrust

The ultimate deliverable for a nozzle in a propulsion application is the production of thrust and impulse. A control volume analysis of the nozzle indicates that the thrust...
produced at any moment in time can be well approximated by the momentum flux at the nozzle exit $T$:

$$
T = \int_{A_{exit}} \rho \mathbf{u} (\mathbf{u} \cdot \mathbf{n}) dA \approx \Phi_m \frac{1}{A_{exit}} \int_{A_{exit}} \mathbf{u} d\mathbf{x} = \Phi_m \mathbf{v}_{exit},
$$

(13)

where $\rho$ is density, $\mathbf{u}$ is the velocity vector, $\mathbf{n}$ is exit surface normal vector, $A$ is surface, $\Phi_m$ is mass flow rate and $\mathbf{v}_{exit}$ is averaged velocity at the nozzle exit plane. [8]

From the isentropic 1D flow theory [5] (where it is assumed, that the variables are uniform over the cross-section at each point) the mass flow rate can be estimated. Mass flow rate defined at the throat is $\Phi_m = \rho v A_{throat}$. Density and speed can be expressed from equations (10)-(12). The mass flow rate is

$$
\Phi_m = A_{throat} \sqrt{\frac{\kappa p_0 \rho_0 \left( \frac{2}{\kappa + 1} \right)^{\frac{\kappa+1}{\gamma}}}{\frac{\gamma - 1}{\gamma}}}, \quad \kappa = \frac{C_p}{C_v},
$$

(14)

where $C_p$ is heat capacity at constant pressure, $C_v$ is heat capacity at constant volume, $p_0$ and $\rho_0$ are determined by the inlet conditions.

One common measure of the efficiency of a nozzle design can be obtained by normalising the specific thrust (13) by the corresponding value obtained from isentropic quasi 1D flow theory of nozzle flow [5]. For a given operating configuration, the nozzle efficiency $\mu_T$ is given by:

$$
\mu_T = \frac{|T|}{\Phi_m \sqrt{\frac{2 \rho_0 p_0 \left( \frac{\gamma - 1}{\gamma} \right)}{p_e / p_0}}},
$$

(15)

where $p_e / p_0$ is the ratio between the exit pressure and the reservoir pressure at the nozzle inlet and $\Phi_m$ is determined from equation (14). [8]

6.2 Physical model

The micro-nozzle flow field can be described with the Navier-Stokes-Fourier equations which govern the conservation of mass, momentum and energy. The Navier-Stokes-Fourier give us the time evolution of density, velocity and temperature. The sistem of equations is

$$
\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0, 
$$

(16)

$$
\partial_t (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) + \nabla p = \nabla \cdot \mathbf{T},
$$

(17)

$$
\partial_t (\rho s) + \nabla \cdot (\rho s \mathbf{u}) + \nabla \cdot \left( \frac{\mathbf{q}}{T} \right) = \sigma,
$$

(18)

$$
\frac{d}{dt} \int_{\Omega} \left( \frac{1}{2} \rho |\mathbf{u}|^2 + \rho e \right) d\mathbf{x} = 0,
$$

(19)

where $\rho$ is density, $\mathbf{u}$ is velocity field, $T$ is temperature, $\partial_t$ is time derivative, $\partial_x$ is spatial derivative, $p$ is pressure, $\mathbf{q}$ is heat flux, $\mathbf{T}$ is viscous stress tensor, $s$ is specific entropy, $e$ is specific energy and $\sigma$ is the rate of entropy production. In the system above equation (16) represents conservation of mass (continuity equation), equation (17) represents conservation of momentum, equation (18) and equation (19) represents conservation of energy. [8]

The system of equations (16)-(19) is closed with the Gibbs’s relation, Fourier law and ideal gas equation of state:

$$
Tds = de + pd \left( \frac{1}{p} \right), \quad \mathbf{q} = -\kappa \nabla_x T, \quad p = \rho RT.
$$

(20)
We assume that fluid doesn’t slip at the micronozzle wall. This leads to boundary condition

\[ u = 0, \]  

(21)

where \( u \) is fluid velocity on the micronozzle walls which is zero in all directions. [8]

7 Numerical calculation of the fluid velocity in the micronozzle

We will take a look at the numerical calculation of gas velocity in the micronozzle. The method used for temporal and spatial evolution of gas velocity field is finite volume method.

7.1 Finite volume method

The finite volume method uses the integral form of the conservation equations as its starting point. The solution domain is subdivided into a finite number of contiguous control volumes, and the conservation equations are applied to each control volume. At the centroid of each control volume lies a computational node at which the variable values are to be calculated. Interpolation is used to express variable values at the control volume surface in terms of the nodal (control volume center) values. Surface and volume integrals are approximated using suitable quadrature formulae. As a result, one obtains an algebraic equation for each control volume, in which a number of neighbor nodal values appear. [9]

The usual approach is to define control volumes by a suitable grid and assign the computational node to the control volume center. Figure 6 shows a 2D and a 3D grid of control volumes with nodes at the center. [9]

Surface integrals in finite volume method are approximated so that the net flux through the control volume boundary is the sum of integrals over the four (in 2D) or six (in 3D) control volume faces:

\[ \int_S f dS = \sum_k \int_{S_k} f dS \]  

(22)

Volume integrals are approximated as the product of the mean value of the integrand and the control volume volume:

\[ Q_p = \int_\Omega q d\Omega = \bar{q}\Delta\Omega \approx q_P\Delta\omega \]  

(23)
The finite volume method can accommodate any type of grid, so it is suitable for complex geometries. The grid defines only the control volume boundaries and need not be related to a coordinate system. The method is conservative by construction, so long as surface integrals (which represent convective and diffusive fluxes) are the same for the control volumes sharing the boundary. [9]

The disadvantage of finite volume method compared to finite difference (differential equation are approximated by replacing the partial derivatives by approximations in terms of the nodal values of the functions) schemes is that higher order methods for interpolation, differentiation, and integration are more difficult to develop in 3D. [9]

7.2 Micronozzle mesh

To use finite volume method to calculate the velocity flow in the thruster micronozzle, a 3D geometry of the nozzle has to be created and used for mesh generation.

Geometry and mesh are made using Gmsh, an open source 3D mesher. Gmsh creates a mesh which is composed from tetrahedrons. They act as the control volumes in finite volume method.

Mesh density is very important for the quality of solution. When fixed mesh is used the user should foresee areas that demand higher mesh density. These typically include narrow sections, sharp bends, etc. Number of tetrahedron should be kept low in order to shorten simulation times.

Figure 7 shows an example of mesh that can be used in calculations. For better visibility on the picture the tetrahedron sizes are larger then the actual used for calculations, however we can still see more tetrahedrons in the narrower part of the micronozzle.

![Figure 7: Micronozzle mesh made from tetrahedrons.](image)

7.3 Numerical calculations

For calculating the velocity flow in the micronozzle we used open source project OpenFOAM, which has many solvers for calculating flows of compressible fluids. We used the sonicFoam solver - transient solver for tans-sonic/supersonic, laminar flow of a compressible gas. Before running the program for a specific mesh we must define boundary conditions for the mesh walls.

We divide the geometry in three different parts: inlet, outlet and fixed walls. Inlet is the surface of the mesh where the gas enters the nozzle, outlet is the surface where gas exits the nozzle and fixed walls are all boundary surfaces of the mesh. The velocity boundary condition is constant mass flow rate through the inlet surface and zero velocity.
in all directions on fixed walls. We fix the temperature of all three different surfaces to 300 K. Pressure boundary conditions need to be defined next. For inlet and fixed walls we define zeroGradient i.e. the normal gradient of pressure is zero. For outlet we define wave-Transmissive boundary condition, which means that there are no pressure waves reflected from this surface. WaveTransmissive boundary condition requires the use of inletOutlet boundary condition for velocity and temperature on the outlet surface.

With mesh created and boundary conditions defined, we can run the solver which after some time (it can be very long depending on initial conditions and mesh complexity) gives us temporal evolution of velocity flow in the nozzle. We ran the solver till the flow reached a stationary state.

### 7.4 Results

Let us take a look at the results of the numerical calculations. Figures 8 through 12 show the stationary state of the gas velocity in the nozzle for different nozzle throat widths. The boundary conditions for the simulation were: zero outlet pressure and fixed pressure and mass flow on inlet.

It is convenient to normalize all dimensions to inlet width $h$. The outlet width is $1.5h$, the length from the inlet to the nozzle throat is $3.75h$, the length from the throat to the outlet is $2h$ and the thickness of thruster is $0.5h$. The nozzle throat widths will be: $0.175h$, $0.25h$, $0.375h$, $0.5h$ and $0.625h$.

Velocity is normalized to the maximum velocity, which was reached in the nozzle with the narrowest throat. Legend on figure 8 applies to all images.

![Figure 8: Velocity profile of the micronozzle for the throat width $0.175h$.](image)

![Figure 9: Velocity profile of the micronozzle for the throat width $0.25h$.](image)
We can see a distinctive jet after the gas flow goes past the nozzle throat. Both maximum flow velocity and jet collimation rise with decreasing throat width.

Figure 13 shows the maximum velocity as a function of throat width. Throat widths and maximum velocities are taken from pictures above.
the mass flow rate through the inlet is prescribed and constant in all cases, the velocity increases linearly as we change the width of the throat.

The offset in the slope at throat width $0.175h$ indicates that velocity is approaching a maximum limit where viscous effects hinder further velocity increase. We can see that after throat width $0.175h$ there is a drop in the maximum velocity. It is to expect that maximum velocity would reach 0 at throat width equal to 0 (closed nozzle). Further smaller throat widths were not explored due to unstable behavior of numerical solution.

8 Conclusion

For attitude control functions, currently available cold gas systems approach in performance the requirements imposed by microspacecraft designs with respect to minimum thrust and impulse bit values. Impulse bits, however, will have to be lowered even further beyond the values obtainable with today’s smallest thrusters for nanospacecraft (mass less then 10 kg). In addition, significant leakage concerns exist for cold gas systems and the required high-pressure storage tanks will completely dominate microspacecraft design with respect to both size and mass, even for relatively benign attitude control requirements. Ammonia cold gas thrusters or hydrazine warm gas systems may provide near-term solutions to the propellant storage and leakage issues. [1]

We calculated the velocity profile in the stationary state for a microthruster and we showed that the maximal velocity inside the microthruster is linearly dependent of microthruster throat width.

References


