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FACULTY OF MATHEMATICS AND PHYSICS
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Seminar II

Violin and the Wolf

(The Wolf Tone On Violin Family Instruments)

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Abstract

Players of bowed string instruments are troubled by spots in the playing range of their instruments in which it is more or less impossible to produce a steady tone of a good quality, rather the tone tends to vary strongly and harshly at pulsating frequency of approximately 5 Hz.

In the article I will first explain the basics of violin acoustics needed for understanding the production of the wolf tone. Then I will describe the nature of the wolf tone and finally I will look at the suggestions and players' solutions to avoid the wolf tone or at least suppress it as much as possible.

Contents

1 Introduction

The wolf tone is a particular effect to which bowed string instruments, particulary cellos, are known to be very susceptible. It is an unpopular phenomenon among musicians, since it gives rise to harsh and beating-like sound. It makes proper musical execution extremely difficult at a specific note - there is usually one such problematic spot, sometimes even more - and it is quite unpleasant for the listener. On the other hand, the wolf tone attracts a lot of attention of acoustics experts. As different as the explanations may be, the fact is that the wolf tone arises at the consonance of the fundamental frequency of the string with a very poorly damped natural frequency of the body of the instrument [1, 2].

After Helmholtz's discovery of the stick-slip motion of the bowed string at the end of 19th century, Raman was one of the first to explain this phenomenon in his article "On the wolf-note in bowed string instruments" in 1916, [3]. In 1963 Schelleng [4] applied an elementary electrical circuit ideas to bowed-string instruments. The two provide the basic knowledge about the wolf tone. Being referred to many times, only small changes and contributions have been added to their picture up to now, e.g. [5].

Firstly, let us briefly look how the bowing works, where we can expect strong body resonance and define the mechanical impedance, which will give us basic grounds for wolf tone explanation. I will mostly talk about the violin, since it is the main representative of the bowed family.

2 Violin as a Vibrating System

2.1 Construction of Violin

The essential parts of a violin are shown in the exploded view in fig. 1.

To produce sound, the violinist draws the bow across the violin strings in order to set them into vibration. Energy from the vibrating string is transferred to the main body of the instrument, composed of the top plate, back plate and ribs. The vibrations of main plates and the air within produce most of the sound. Four strings are supported by the bridge, which acts as a mechanical transformer. It converts the motion of the strings into the vibration of the body, fig. 1. Top and back plate are connected by sound post, which is mounted near the right foot of the bridge in order to efficiently transmit the vibrations to the back plate. The string fundamental frequencies are about 200, 300, 440 and 660 Hz - which corresponds to the notes G, D, A and E.

2.2 Bowing the String

The transverse vibration of the stretched string is usually used as the first and simplest example of a continuous vibratory system having an infinite series of normal modes of vibrations and corresponding natural frequencies:

$$f_n = \frac{c}{\lambda_n} = n\frac{c}{2L}, \qquad (n = 1, 2, ...), \tag{2.1}$$

where L is string length and c the transverse wave velocity on the string. It is defined as $c = \sqrt{T/\mu}$, where T is tension and μ is mass per unit length.

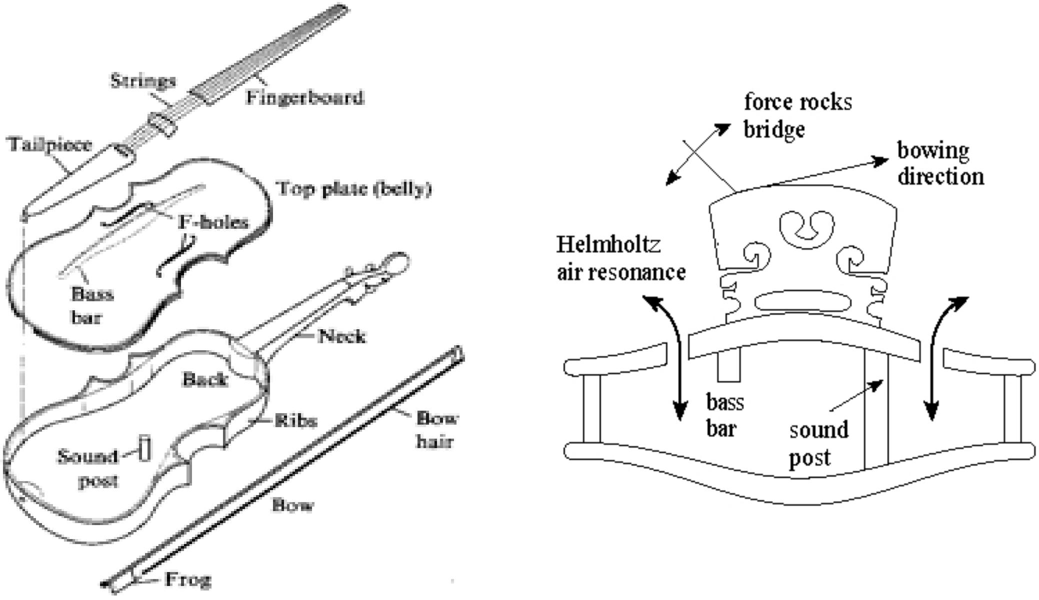

Figure 1: Parts of the violin and a bow on the left, [2], and shematic vibration transfer on the right, [6]: vibrating string, excited by the bow, exerts force on the bridge, which converts the transverse forces of the strings into driving forces applied by its two feet to the top plate of the instrument. Vibrating top plate is connected to back plate by sound post.

In the case of the violin string the shape of waves is somewhat more complex. Helmholtz (1877) showed that the string when bowed forms more nearly two straight lines with a sharp bend at the point of intersection rather than normal sinusoidal shape, [2, 7]. This bend travels around the curved path that we see, making one roundtrip each period of the vibration. This so-called Helmholtz motion is shown in fig. 2. The action of the bow on the string is often described as a stick and slip action. The bow drags the string - stick regime - until the bend arrives and triggers the slipping action of the string until it is picked up by the bow once again. The envelope (the dashed curve in fig. 2) is composed of two parabolas with a maximum amplitude that is proportional, within limits, to the bow velocity. The displacement $y(x,t)$ of the string and corresponding displacement velocity $v(x,t)$ at the bowing point is shown on the right in figure 2. The basics of this simple kind of slipping lie in the fact that the static friction is little greater than dynamic friction allowing the string sufficient slip. The detailed explanation of the sticking and slipping dynamics is far more complex (also due to fast and significant temperature changes in applied rosin) and it is the main subject of contemporary research.

To establish the Helmholtz motion the appropriate bow force is needed. Actually there exists a condition for a minimum and maximum bowing force limit within which a steady Helmholtz motion is possible. The calculated dependencies on bow position l (bow-to-bridge distance) and bow velocity v_b for force limits give, [5]:

$$F_{MAX} \propto \frac{v_b}{l} \qquad and \qquad F_{MIN} \propto \frac{v_b}{l^2}, \tag{2.2}$$

which is sumarized for fixed v_b in figure 3.

If applied bow force is below F_{MIN} the string slips before the bend arrives to trigger the slipping, thus giving way to vibration with some other frequency than the fundamental

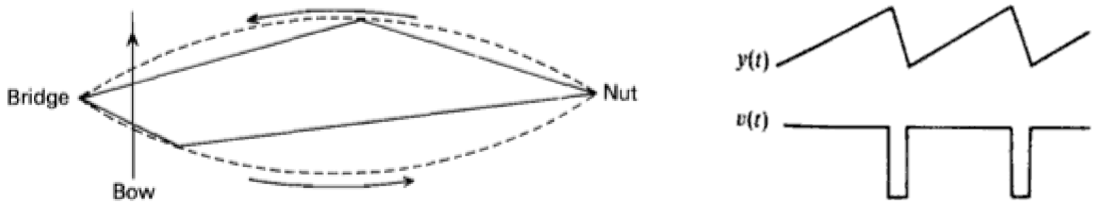

Figure 2: The simplest shematic ”stick & slip” Helmholtz motion of a bowed string. Sharp bend is traveling around an envelope when bowing the string, [8]. On the right: displacement and velocity of the string as seen at the bowing point at $l = L/4$, [2].

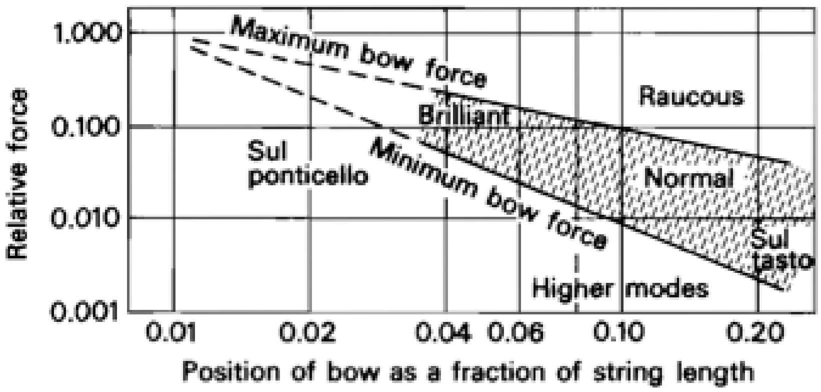

Figure 3: Bowing force dependence on bow-to-bridge distance l for a given bow velocity v_b and a given note, i.e. frequency, [8]. Bowing in the area within force limits produces steady Helmholtz motion and therefore familiar normal violin sound.

(actually with the second or even higher harmonics). If the force is above the F_{MAX} the bend is unable to trigger the slipping and the Helmholtz motion degenerates into rather chaotic vibrations and musically unacceptable sound.

2.3 Vibrational Response of the Violin Body

To determine the vibrational behavior of the violin body we usually rely on modal analysis of every vibrational part [2]. The normal modes of vibration or eigenmodes of the assembled violin are determined mainly by the coupled motions of the top plate, back plate and enclosed air. For our purpose it is sufficient to know only the response of the whole body at the string notch on the bridge - ”body response felt by strings”. We can then treat the vibrations of the violin as two oscillators, the body and the string which effects the body oscillations. Measured input response is shown in fig. 4.

2.4 Mechanical Impedance

In oscillating systems with sinusoidal external driving force we can define a *mechanical impedance* Z as a ratio between a driving force written in a complex form $F = F_0 e^{i\omega t}$ and a complex velocity v - the derivative of the responded displacement $x = x_0 e^{i\omega t}$:

$$Z = \frac{F}{v} \,. \tag{2.3}$$

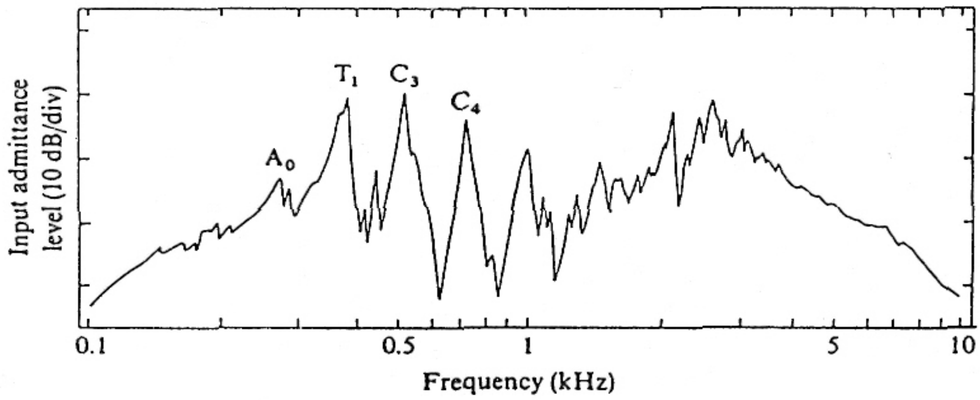

Figure 4: Response of the violin body at the string notch on the bridge - i.e. input admittance (see Ch. 2.4 for a definition), [2]. The modes designated A_0, T_1, C_3 and C_4 appear to be the important low-frequency resonances in a violin. They can be identified on the measured admittance curve of almost every violin body.

Similarly we define the *mechanical admittance* Y as a reciprocal of mechanical impedance $Y = 1/Z$, [2]. Mechanical impedance is analogue to electrical impedance, allowing us to apply the electrical circuit concepts for mechanical vibrations. Impedance of vibrational system tells us how much energy a system can store and how much resistance it introduces to the driving source.

Let us look at a model of a simple damped oscillator driven with sinusoidal external force shown in fig. 5 and its basic equation, eq. 2.4:

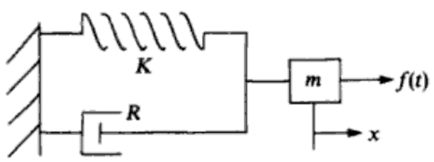

Figure 5: Simple mechanical model of a driven damped oscillator, [2].

$$F_0 e^{i\omega t} = m\ddot{x} + R\dot{x} + Kx, \tag{2.4}$$

with mass m, spring stiffness K in damping R. By solving differental equation 2.4 we derive the so called input mechanical impedance of such oscillator, [2]:

$$Z(\omega) = \frac{F}{v} = R + i(\omega m - K/\omega) = R + iX_m(\omega), \tag{2.5}$$

where we define X_m as *mechanical reactance.* The real part of the impedance - the damped term R - is independent of the frequency and it actually indicates the rate of energy transfer from the driving force to the system, fig. 6.

Figure 6 is actually equivalent to the resonance response curve (note that we derived the displacement $x(\omega)$ and put it in denominator, eq. 2.5). Similar is true for the admittance curve ($Y = 1/Z$).

In the resonance analysis we frequently use the *quality factor* Q, which is defined as the ratio of the eigen frequency ω_0 and the resonance width $\Delta\omega$ at the $1/\sqrt{2}$ height of the

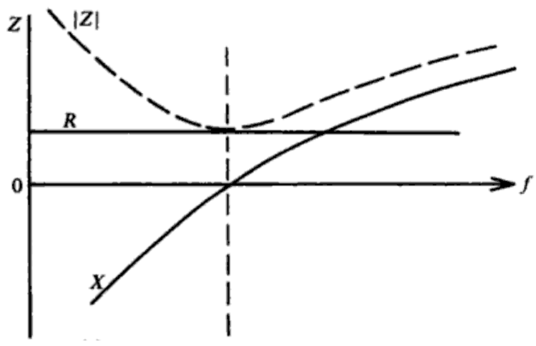

Figure 6: Mechanical impedance $|Z|$, eq. 2.4, its real R and imaginary X_m part of a simple oscillator depended on frequency f, [2].

resonance response curve, eq. 2.6. The resonance width is expressed in terms of equation 2.4 as $\Delta\omega = R/m$ and eigen frequency as $\omega_0 = \sqrt{K/m}$.

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{\sqrt{Km}}{R}. \tag{2.6}$$

Quality factor Q tells us the resonance strength. The smaller the damping R is, the higher the resonance peak - that is Q.

String has its own *characteristic impedance* defined as, [2]:

$$Z_0 = \frac{T}{c} = \mu c. \tag{2.7}$$

At the end, we have to keep in mind that we assume, while deriving the mechanical impedance, that the vibrating system is the linear one. In fact, musical instruments generally consist of nearly linear resonators, such as string and whole violin body system. But the energy source, such as bowing the string, has quite nonlinear behavior, [2].

3 Wolf Tone

3.1 The Nature of the Wolf Tone

As we previously mentioned, the wolf tone phenomenon may occur near one of the body resonances [1] that can be identified from the measured body response, fig. 4. Most often it occurs at the first one (designated A_0, that is approx. 275 Hz on the violin). The first measured oscillogram of the wolf tone is shown in figure 7, where we can clearly see that vibrations of body and string suggest some kind of beating and coupled oscillators.

Further measurements of a string velocity during the wolf tone clearly show the emergence of the second slip of the string. The normal Helmholtz motion degenerates into a *double slip* motion, fig. 8. The same can be assumed from figure 7, where we can see that the double slip action take place on the string approximately at the maximum of the body oscillation.

To explain beating some kind of linear analysis of oscillations is needed, while the alternation between the normal and double slip motion can be analysed in terms of exceeding the minimum bow force limit. Let us look at the latter first.

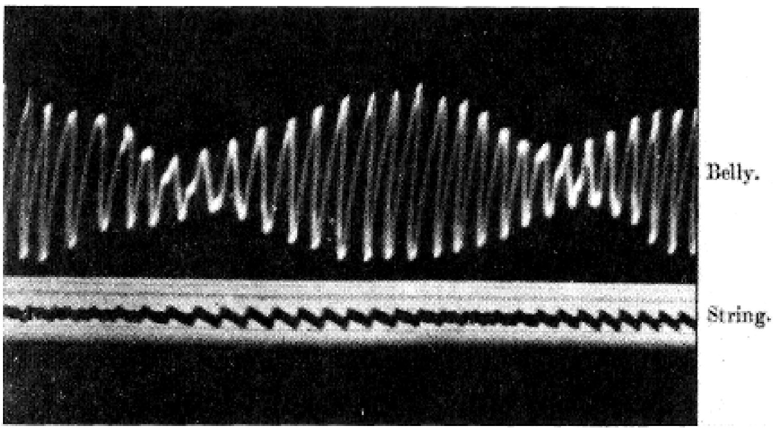

Figure 7: The classic oscillogram by Raman [3] shows the alternation between oscilllation of the string and of the top plate of the instrument body, measured at the bowing point. The typical Helmholtz motion of the string can be seen, when established. The oscillation of the top plate reaches its maximum amplitude when the fundamental vibration of the string collapses.

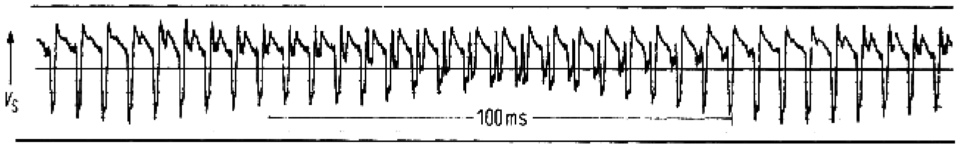

Figure 8: String velocity v_s at the bowing point during the wolf tone measured by Firth, [1]. Note the developement of the second slip (compare measures to the velocity of the normal Helmholtz motion, fig. 2).

3.2 Qualitative Explanation

A rather complete phenomenological description is quite straightforward. The minimum bowing force is actually needed to cover energy losses through string end supports, mainly through the bridge feeding the body vibrations. The more energy is lost, the higher the minimum bow force limit.

At starting the note sitting on the top of the strong body resonance, first the normal Helmholtz motion is established, while the body vibration takes a while to build up (the higher the Q of the body resonance, the longer it takes). As the losses increase along with the oscillation of the body, the minimum bow force limit is rising. At some point the minimum bow force treshold can exceed the actual bowing force we are using and the Helmholtz motion gives way to a double slipping motion, [3, 9].

The new, second slip is more or less in the mid-way between the original "Helmholtz" slips (see also fig. 9). It grows in size and the original one gets smaller. The new one is in the opposite phase to the old one, thus just in the right phase to suck energy back out of the body vibration. The body vibration more or less stops for a moment - all its energy has gone back into the string and the minimum bow force limit has gone down again. The original slip has entirely gone and the second slip becomes a new Helmholtz motion, [4, 9].

The cycle then repeats, giving the warbling sound of the wolf.

3.3 Time Domain Versus Frequency Domain

It seems from the previous section that the change between the fundamental frequency and the one octave higher (corresponding to two string slips per original cycle) is essential for the wolf tone. It is probably this "register change" that gave the name *wolf* to that phenomenon.

It was not clear at the beginning, if this essentially time domain picture suggests the beating at all, [3]. Namely, the beating should occur at the fundamental frequency - requiring a frequency pair with approximately the same amplitude. We can expect the pair of frequencies at coupled resonators, analysing the string and body coupling. The nonlinear process of bowing (although still periodic driving force) at one end (string) and the free radiation of energy on the other (body) could worsen the situation.

Anyway, the measured frequency spectrum of the wolf tone drives the doubts away, fig. 9. We can see the presence of the fundamental frequency pair with relatively the same amplitude and smaller higher harmonics. This suggests that the linear analysis of the beating to some extent is possible. This includes the comparison of the mechanical impedances of the according vibrating systems.

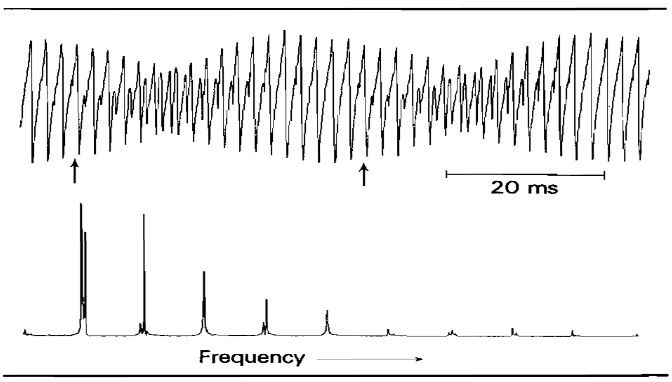

Figure 9: String velocity at the bowing point and corresponding frequency spectrum of the violin wolf tone measured byMcIntyre and Woodhouse, [8]. We can also clearly see the development of the second slip.

We can also paraphrase to some extent the previous section explanation in terms of mechanical impedances. Normally, the characteristic impedance of the string Z_0 is about 1/10 of the impedance of the body Z_B, [4]. This impedance mismatch is more than enough to provide the strong reflection needed for oscillations to build up on the string. But in the nearness of a strong body resonance with weak damping (high Q, see eq. 2.6 and fig. 6) the body impedance is significantly reduced, and may be equal or even lower than that of the string. In this case oscillations at a fundamental string frequency could not be fully developed, [1].

As might be expected in a strongly nonlinear problem, some features are easier to understand from one point of view and *vice versa*, but the two approaches are consistent, [8]. From time domain picture, section 3.2, it is obvious that pressing harder with the bow can suppress the wolf, since the minimum bow force treshold may not reach the actual bowing force. Whereas from the frequency domain point of view, developed in further sections, we will be able to show, for example, why cellos are more prone to the wolf tone problem.

4 Frequency analysis

4.1 Vibrating String Coupled to the Body

Let us consider the simple example of the string coupled to the body. The body is modelled as a simple oscillator having only one resonance at $\omega_0 = \sqrt{K/M}$, fig. 10. We neglect the energy dissipation in the string and in the body - let be $R = 0$ for now, [1, 2, 10]. This model actually corresponds to a system of string and body oscillating by itself without damping, for example as we were to pluck a string and leave it vibrating.

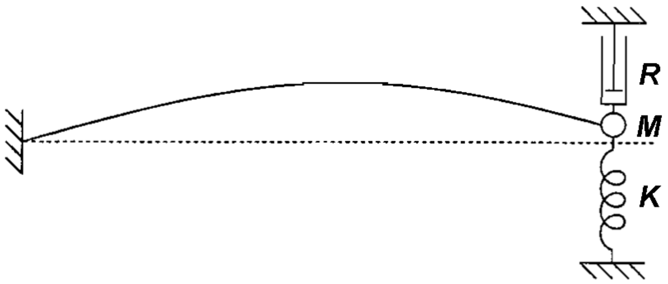

Figure 10: String coupled to the simple model of the body with one resonance, [5]. We actually determine resonance defining parameters M, K, R by measuring the shape of the body resonance, [4].

We consider a string fixed at $x = 0$ and terminated at $x = L$ by a support (a bridge and a body) characterised by a corresponding mass M and stiffness K. The boundary condition at the coupled end $x = L$ is written as:

$$-T(\partial y/\partial x)\mid_{x=L} = M\partial^2 y/\partial t^2 + Ky\mid_{x=L} . \tag{4.1}$$

where T is string tension and $y(x)$ the string displacement. The left term is the "string force" at $x = L$ driving the oscillator on the right. Applying the boundary condition at $x = 0$ to a general traveling harmonic waves solution $y(x,t) = Ae^{i(\omega t - kx)} + Be^{i(\omega t + kx)}$ gives the harmonic solution:

$$y(x,t) = A \sin kx\, e^{i\omega t}. \tag{4.2}$$

Substituting eq. 4.2 into eq. 4.1 gives:

$$-kTA\cos kL\, e^{i\omega t} = (-\omega^2 M + K)A \sin kL\, e^{i\omega t}. \tag{4.3}$$

Taking into account formulas for string wave velocity $c = \sqrt{T/\mu}$, wave number $k = \omega/c$, signing m as total mass of string $m = \mu L$ and defining koefficient K/c^2M as ${k_0}^2$, we get the transcendental equation, which can be solved graphically:

$$\cot kL = \frac{M}{m}\left(kL - \frac{(k_0L)^2}{kL}\right). \tag{4.4}$$

Figure 11 shows the graphic solution of equation 4.4 for the case in which the frequency of the free oscillations of the body matches the fundamental frequency of the string, $\omega_0 \simeq 2\pi f_1$. In this case, $k_0 L = \pi$, using the eq. 2.1.

The fundamental frequency and its higher harmonics of the string stretched between rigid supports are at $kL = \pi\,(2\pi, 3\pi, etc.)$. It can be seen that two intersections occur

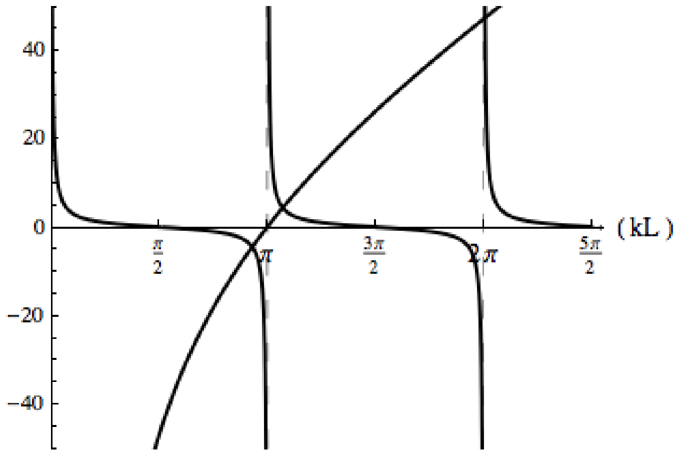

Figure 11: Graphical construction of the solution of eq. 4.4: the natural frequencies of the system composed of a string terminated in a simple oscillator. We can see that the new pair of frequencies occur in the place of the fundamental frequency of the uncoupled string or body ($kL = \pi$). The value 10 is chosen for M/m - the ratio of body to string mass.

near $kL = \pi$ - the frequency pair, which is responsible for "the beating at the fundamental frequency". Also the intersections corresponding to higher modes occur very near the $kL = n\pi\,(n = 2, 3, 4, etc.)$ and are just slightly altered. Note also that the frequency difference is smaller with larger mass ratio M/m suggesting that lighter string reduces the coupling.

4.2 Coupled oscillators with damping

The creation of two new modes, both of which are combinations of a string mode and a body mode, is similar to the behavior of the coupled vibrating systems. It can be seen from the fig. 11 that the greater the mass ratio M/m, the smaller the frequency splitting. Note that M is not the actual body mass, but the "effective mass" defining the resonance of the body.

Further discusion should take into account the damping and constant energy supplying, but let us just borrow the solution from Gough, [11], where the reader can find the detailed explanation on coupling. There exists the condition defining the coupling limits by comparing the mass ratio M/m to the quality factor Q_B of the body resonance.

The *weak coupling* occurs when the mass ratio is greater then a square of Q_B:

$$\frac{M}{m} \gg \frac{4{Q_B}^2}{\pi^2}. \tag{4.5}$$

In this case the coupling does not perturb the frequencies of the two normal modes (when the unperturbed frequencies of string and body coincide).

In the *strong coupling* the mass ratio is smaller then a square of Q_B:

$$\frac{M}{m} \ll \frac{4{Q_B}^2}{\pi^2}. \tag{4.6}$$

The strong coupling splits the resonance frequencies of the normal modes symmetrically about the unperturbed frequencies. In this case both modes have the same Q value of $2Q_B$.

However, this result confirms the solution in the previous section, suggesting that lighter string reduces the coupling and therefore the frequency splitting. This is also in accordance with reducing the string impedance, eq. 2.7, which can help to reduce the wolf tone susceptibility.

4.3 Impedance at the bowing point

Schelleng, [4], goes even further calculating impedance all the way up to the bowing point, arguing that impedance presented to the bow is the most informative. He applies the electrical circuit analogy to the vibrating system. The distributed nature of the mass and stiffness (actually the compliance, to be precise) of the string is considered as a transmission line, thus applying the standard computational methods for transmission lines. The model is shown in figure 12.

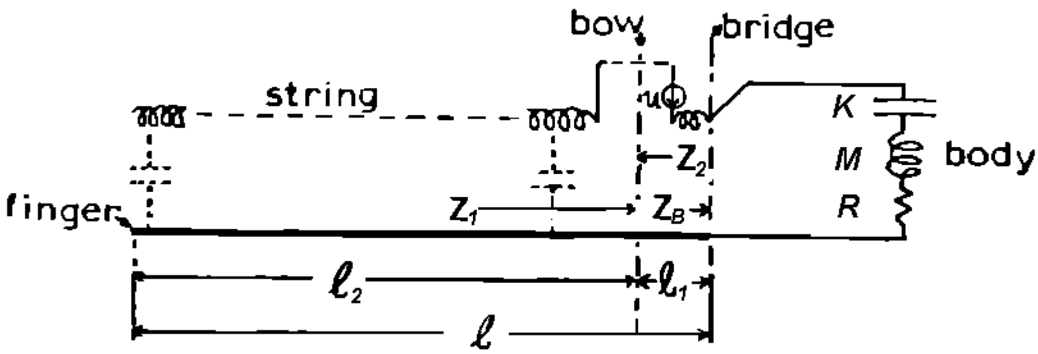

Figure 12: The vibrating system model with bowing point at distance l_1 away from the bridge and body on the right side, [4]. Vibrating string is considered as a transmission line and body as a line's load at right end. Body impedance is the same as in previous sections, i.e. body is modelled as a one resonance oscillator, fig. 10.

The impedance of string from the bowing point to finger Z_2 is calculated as a transmission line of length l_2 ended with "opened load" ($Z_{load} = \infty$). The impedance presented to the bow on the bridge side Z_1 is considered as a line with length l_1 ended in the body as a load with impedance $Z_{load} = Z_B$. Resistance in the string is neglected. Equations from the lossless transmission line theory for Z_1 and Z_2 are:

$$Z_1(l_1) = Z_0 \frac{Z_B + iZ_0 \tan(kl_1)}{Z_0 + iZ_B \tan(kl_1)}, \tag{4.7}$$

$$Z_2(l_2) = -iZ_0 \cot(kl_2), \tag{4.8}$$

where Z_0 is the characteristic string impedance, Z_B the body impedance and $k = \omega/c$ wave number. In order to get the informative picture let us sign the body resonance frequency as $\omega_B = \sqrt{K/M}$ and string fundamental frequency at length l as $\omega_S = \pi c/l$ (eq. 2.1). Let the relative frequency be $\Omega = \omega/\omega_B$ and the ratio of eigen frequencies $\Omega_S = \omega_S/\omega_B$. We rewrite the body impedance, eq. 2.5 and 2.6, in terms of K, M and Q and relatively to $\sqrt{KM}$:

$$Z_B' = \frac{Z_B}{\sqrt{KM}} = \frac{1}{Q} + i\left(\Omega - \frac{1}{\Omega}\right) \tag{4.9}$$

In the same way we assign the $Z_0' = Z_0/\sqrt{KM} = \gamma$. Factor γ will be important in further discussion. Relative bow to bridge position is $x = l_1/l$.

Equations 4.7 and 4.8 divided by Z_0 can be rewritten as:

$$Z_1' = \frac{(Z_B'/\gamma) + i\tan(\pi x\Omega/\Omega_S)}{1 + i(Z_B'/\gamma)\tan(\pi x\Omega/\Omega_S)}, \tag{4.10}$$

$$Z_2' = -i\cot(\pi(1-x)\Omega/\Omega_S), \tag{4.11}$$

The total impedance presented to the bow at the bowing point is $Z = Z_1 + Z_2$. In fig.13 we can see what is going on with the real and imaginary part of the impedance Z when the fundamental string frequency is in the vicinity of the body one, $\Omega_S \sim 1$.

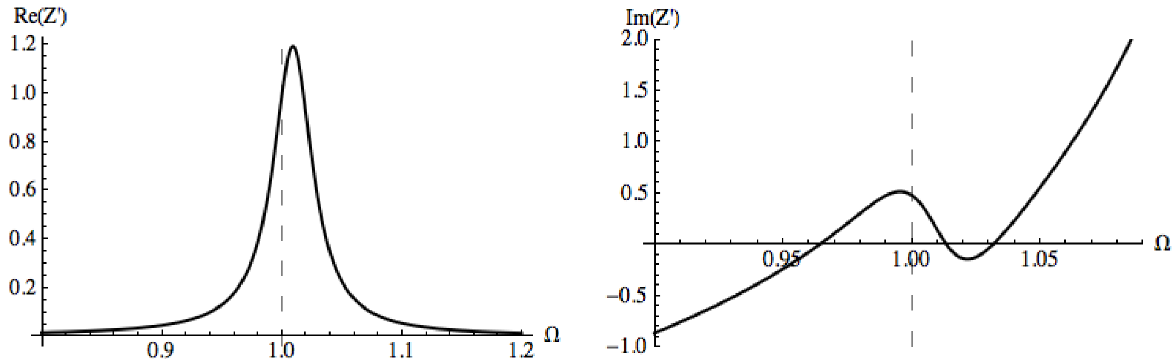

Figure 13: On the left is the resistant part of the impedance Z' and on the right is the reactance. Resistance has a sharp peak and reactance three zeros rather than one. Values used in graphs are: $\gamma = 10^{-2}$, $Q = 25$, $\Omega_S = 1$ and $x = 0.15$.

We can see that reactance has three zeros rather than one, meaning that steady oscillations can occur at any of these frequencies. But due to high resistance peak approximately in the middle, it is more likely that the oscillations occur at the two outside points. We can say that if the bow force is insufficient to cope the high resistance at the middle point, it still may be enough for the outside pair because of their much lower resistance. This is actually in accordance with the explanation presented in section 3.2.

A lot of parameters in fig. 13 need a comment. γ and x give the same picture for quite a range, the typical values for these have been chosen. Let us check the form of impedance dependant on Q and Ω_S. Figures 14 and 15 show few examples of reactance (resistance keeps the same form for these small changes). Through the analysis suggested in these figures we can define the frequency difference limit, fig. 14. From fig. 15 we conclude that for sufficiently low Q (highly damped body resonance) there is no tendency for the wolf tone to occur, which confirms our prepositions, that for the wolf tone the strong body resonance is needed.

Wolf tone criterion: final thing to do is to scan the parameter space to find out where the reactance curve changes its shape from flat to "S-shape", thus providing the conditions for the wolf tone to occur. Let us mention that the relative bow position x does not effect the situation much, except for some certain values x, but discusion on that

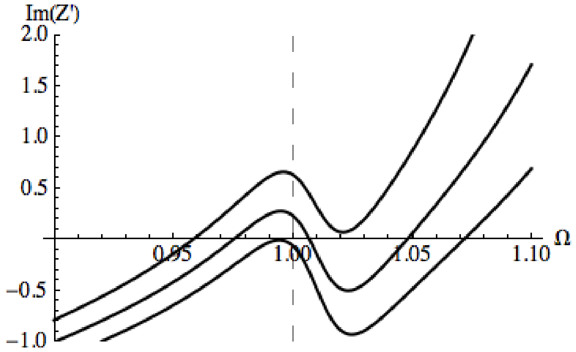

Figure 14: Reactance for $Q = 25$ and for three values for Ω_S from upper to lower curve: Ω_S: 0.99, 1.02, 1.05. For greater eigen frequency differences the three zeros in reactance curve disappear.

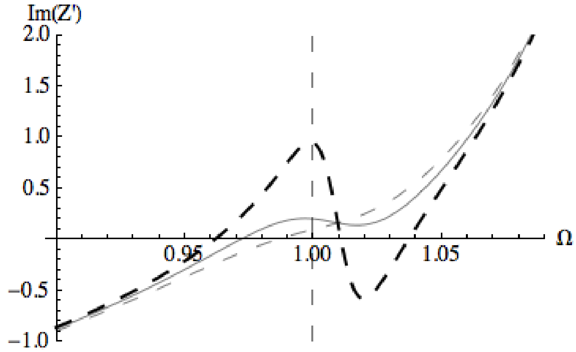

Figure 15: Reactance for $\Omega_S = 1$ and for three values for Q. Bold dashed curve: $Q = 40$, thin continuous one: $Q = 15$ and thin dashed one: $Q = 10$. In the last one the inflection point disappears.

exceeds our scope, further reading may be found in [5]. The good choice is the parameters $\gamma = Z_0/\sqrt{KM}$ and Q, [4]. A diagram in figure 16 is displayed in the original form, [4], since some specific measurements are included.

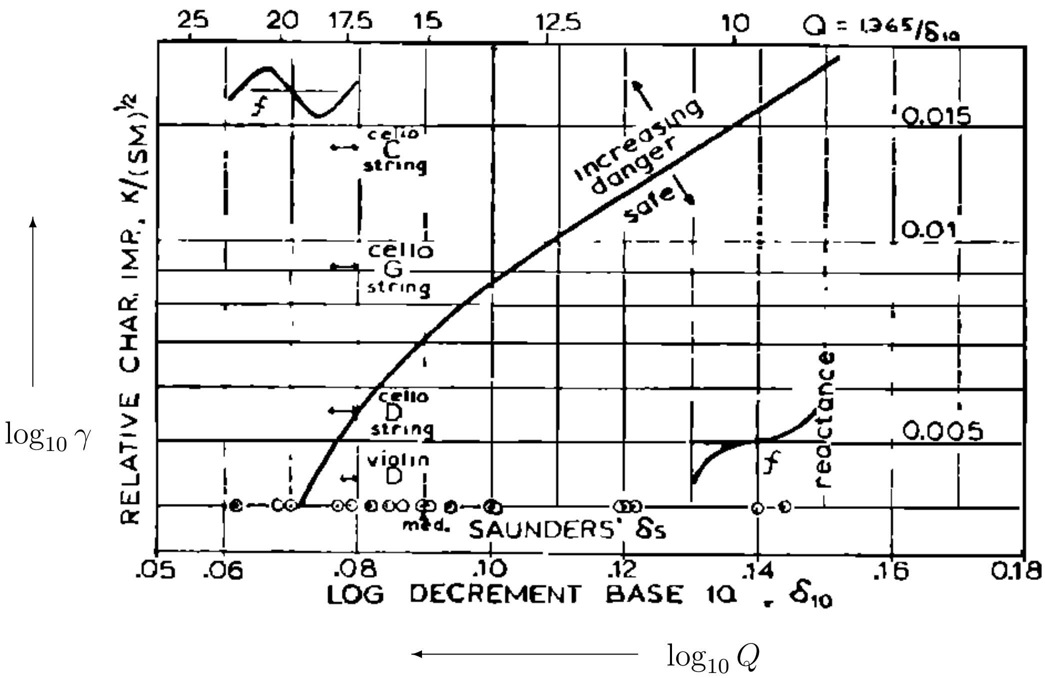

Figure 16: Wolf tone criterion, [4]. In case of greater γ, which means that the string impedance Z_0 is large compared to body parameters K and M, or in the case of strong body resonance - greater Q, the specific string may fall on the left upper side of the limit curve displayed in the diagram. On this side the reactance curve expresses the S-shaped form. On the limit curve the reactance is horizontal in the inflection point. On the bottom right side of the diagram the reactance is monotone - having one zero, thus safe from the wolf tone.

4.4 Summarizing the criterions

Woodhouse, [5], added in diagram 16 the results from the ”minimum bow force” picture, which is essentially time-domain picture, and find out that the two points of view are actually quite consistent (see [5] for further details). At the end of frequency analysis the comment on nonlinear generator (driving force) is needed. Curiously enough the whole linear picture actually requires the energy source to be slightly nonlinear. In the case of linear generator it is possible for oscillation to occur at one frequency of the new fundamental pair alone. The principle of superposition gives them complete independence. In the nonlinear case of so called self-excited vibrations (bowing), the two frequencies can coexist, [4].

Because of nonlinearity of the bowing, also the new frequencies are possible. An interesting explanation may be found in Benade's analysis, [12], where new frequencies even strengthen the wolf tone response.

5 Why is Cello More Susceptible to Wolf Tone

The question in terms of figure 16 is why cello has greater factor γ, i.e. smaller string to body impedance mismach. The quality factors Q of body resonances are more or less in the same range. We have to compare the sizes of instruments and find out how the parameters, such as eigen frequencies, resonance factors and string impedance factors are dependent on the size. It is done by the *dimensional scaling.* In general the complete scaling is not practicable because of the impossible demands that it places on strength or size of the player, for example. The three basic ratios are considered when going from *violin* to *cello*:

- the ratio of the lowest frequency: $\epsilon = 3$; violin frequency is divided by 3,
- seeing the instrument shape from above, horizontal dimensions are multiplied by: $\sigma = 2.1$,
- the pattern of body resonances is to remain the same logarithmically, thus second resonance frequency is divided by $\eta = 2.66$.

From these prescriptions we first calculate the thickness of plates in order to have apropriate eigen frequencies, and then the ratios for each body parameter. The detailed derivation can be found in [4], and more on eigen modes of plates and other resonators in [2]. Let us just borrow the result for cello, comparing the γ parameters:

$$\gamma_{cello}/\gamma_{violin} = \epsilon/\sigma \sim 1.5 \tag{5.1}$$

It places the cello higher in the diagram in fig. 16. It means that the cello is smaller than completely scaled one. Or, in other words, the cello strings are heavier compared to the those equally scaled with body, since the lower frequencies are needed.

In previous discussions we considered the bridge as a part of the body. Actually the bridge has its own vibrational modes, first one is at $3kHz$ for violin bridge and $1kHz$ for cello bridge. While the violin bridge does not affect the lower range of body response, fig. 4, the cello bridge might have an affect. Actually much taller bridge of the cello, which rocks back and forth in response to the vibration of the top plate, acts as a longer lever thus reducing the impedance seen by the string. It has been estimated that this additionally decreases the body impedance of cello by a factor 2.4, [1].

6 Suppressing the Wolf Tone

The only question for the player is how to suppress the wolf tone. Experienced players are familiar with a number of handy tricks which can suppress the wolf tone during playing, some of these are: increasing the bowing force, changing the point of bowing, applying more vibrato to playing, or even squeezing cello with the knees to prevent shaking of the body. They also know that the emergence of wolf tone highly depends on atmosfere conditions, such as humidity and temperature. The not so obvious fact is that the better-sounding instruments tend to have stronger wolf tones, so do well tuned instruments as well, since for a good sound strong resonances are needed.

Especially on the cello tricks are often not enough. There have been a number of suggestions how to suppress wolf tones. The basic idea is quite common in vibrational engineering of other systems (such as tall buildings), known as a *tunned mass damper*, [13]. An extra resonator is added to the system, tuned to exactly the same frequency as the problem mode, and designed to have fairly high damping. This selectively adds damping to that specific mode without making much difference to all others. The wolf is suppressed without affecting the sound of the instrument much, [9].

A convenient way to do this is to attach an appropriate mass to the extension of the string between the bridge and tailpiece, [1, 4]. This part of the string has the same tension but is much shorter than the part which is bowed, so its fundamental frequency is normally much higher. It is easy to choose the mass and its position to tune the frequency to the wolf one. The desired damping is achieved by using the material with high internal damping, such as plasticine or modeling clay. The modern cello damper, also called *wolf tone eliminator* is shown in fig. 17.

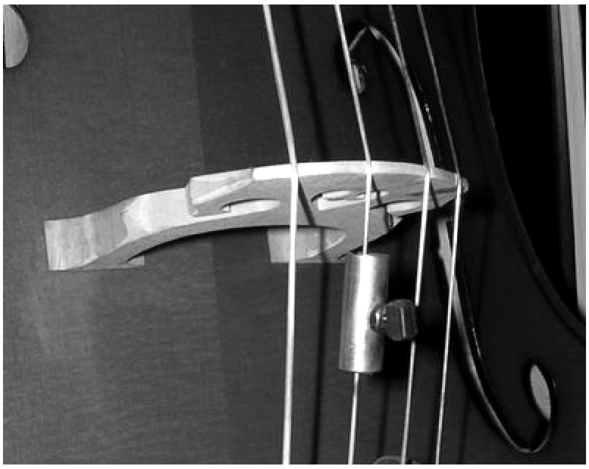

Figure 17: Wolf damper on cello. A metal tube and a mounting screw with an interior rubber sleeve fitted around the same problematic string and below the bridge. By choosing suitable position along the string it suppresses the frequency at which the wolf occurs, [14].

To a first approximation a damper of this kind can be modelled as a point mass on a massless string, [9]. The oscillation of a mass m_0 at the relative position x on a string is derived from the static force balance with string tension T at the displacement y, as shown in fig. 18. From equations 6.1 and 6.2 we get the dependence of the tuning frequency f on the position x, eq. 6.3, which is displayed in fig. 19.

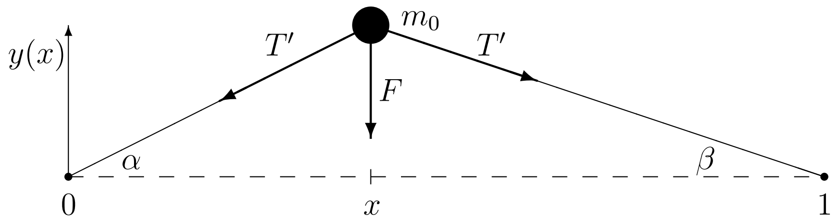

Figure 18: Model of a damper: point mass m_0 at relative position x on a light string.

$$F = \frac{T}{L}\Big(\frac{y}{x} + \frac{y}{1-x}\Big), \tag{6.1}$$

$$\ddot{y} = -\frac{T}{m_0 L}\frac{1}{x(1-x)} \cdot y, \tag{6.2}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{T}{m_0 L}} \cdot \big[x(1-x)\big]^{-1/2}. \tag{6.3}$$

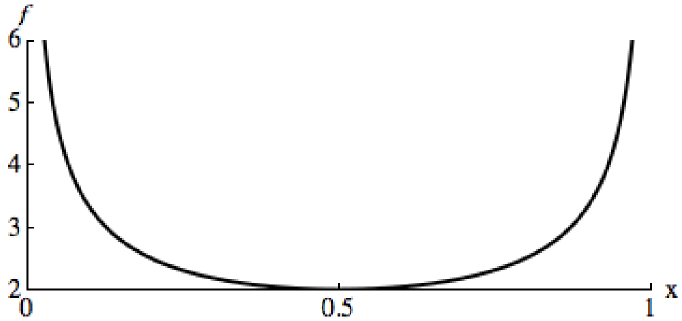

Figure 19: The dependence of the tuning frequency f on position of a point mass on a string, eq. 6.3.

7 Conclusions

From the players' point of view the wolf problem is more or less solved, since they already have at hand good enough solutions to perform music without worrying to much about the emergence of the wolf tone.

Due to strong nonlinearity of the bowing processes, the explanation present is probably the furthest we can get with the linear analysis. Schelleng's picture gave us a sufficient insight into the basic principles. Actually nowadays the wolf tone can be easily modelled by computer simulations. This corner of violin physics is quite well understood. Over the last few decades the extensive work is put into developing new bowing models and bowing robots to have a better insight into the complex frictional processes, [15]. The transient details of the nonlinear response to a given bow gestures are explored. This will also provide more information and prescriptions how to handle the wolf tone. The basics of computer modelling are described in Cremer's book, [1]. Up-to-date information can be obtained from contemporary researchers such as Jim Woodhouse, see [15].

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