Abstract

A wineglass can be made to “sing” by gently rubbing its rim with a moist finger. The friction between the finger and the rim of the wineglass causes the wineglass to oscillate, producing a loud, pure tone. In this paper I will investigate natural resonant frequency of wineglass by the means of the energy method and how the frequency of the oscillations depends on the volume of liquid inside the wineglass - adding the liquid lowers the frequency.
1 Introduction

Musical glasses have been appreciated for centuries because of the "ethereal sound" they produce. These pure tones are commonly produced by rubbing a moistened finger around the rim of a wineglass. The pitch of the sound can be tuned by adding some liquid inside the glass. It can be easily shown that pouring some water into the glass lowers the pitch of the note. This technique is commonly used in musical glass instruments.

The word "glass harmonica" (also glassharmonica, glass armonica, Armonica de verre in French, Glasharmonika in German) refers to any instrument played by rubbing glass or crystal goblets or bowls.

The Irish musician Richard Puckeridge is typically credited as the first to play an instrument composed of glass vessels by rubbing his fingers around the rims. Beginning in the 1740s, he performed in London on a set of upright goblets filled with varying amounts of water. During the same decade, Christoph Willibald Gluck also attracted attention playing a similar instrument in England. This instrument is called a glass harp, also called musical glasses, singing glasses, angelic organ, verrilion or ghost fiddle (Figure 1) [1].
When Benjamin Franklin invented his mechanical version of the instrument in 1761, he called it the armonica, based on the Italian word "armonia", which means "harmony" (Figure 2). In Franklin's treadle-operated version, 37 bowls were mounted horizontally on an iron spindle. The whole spindle turned by means of a foot pedal. The sound was produced by touching the rims of the bowls with moistened fingers. Rims were painted different colours according to the pitch of the note. A is were dark blue, B is purple, C is red, D is orange, E is yellow, F is green, G is blue, and accidentals white. With the Franklin design it is possible to play ten glasses simultaneously if desired, a technique that is very difficult if not impossible to execute using upright goblets. Franklin also advocated the use of a small amount of powdered chalk on the fingers which helped produce a clear tone in the same way rosin is applied to the bows of string instruments. Some attempted improvements on the armonica included adding keyboards, placing pads between the bowls to reduce vibration, and using violin bows. These variations never caught on because they did not sound as pleasant [1].
Mozart, Bach, Beethoven, Donizetti, Strauss, and more than 100 composers composed works for the glass harmonica; some pieces survived in the repertoire in transcriptions for more conventional instruments. Since it was rediscovered during the 1980s composers write again for it. The instrument popularity did not last far beyond the 18th century. Some claims this was due to strange rumours that using the instrument caused both musicians and their listeners to go mad. Later it was discovered that players were marking the notes with lead paint, and they were licking their fingers to play the instrument, and therefore eating lots of poisonous lead paint, which was probably more of the problem [1].

Music for glass harmonica and glass harp was all-but-unknown from 1820 until the 1930s, when German virtuoso Bruno Hoffmann began reanimating interest in the glass harp repertoire with his stunning performances. Playing a standard “glass harp” (with real wine glasses in a box), he mastered almost all of the literature written for the instrument, and commissioned contemporary composers to write new pieces for it [1].

2 How does a wineglass make a sound?

In order to make a wineglass sing, you rub the rim with your finger. The ridges in your finger set the glass into vibration at its natural frequency. Your fingertip needs to be wet in order to remove oils from between your finger’s ridges. This helps produce just the right amount of friction between your fingertip and glass. As your finger moves along the rim, it is alternately slipping and sticking to the rim. This is called ‘slip-stick’ motion. The motion of your hand sets up a wave of vibration travelling through the glass.

When a glass makes sound, it is vibrating much like a bell with its rim and sides moving in and out very quickly - several hundred times each second. We call this the vibration’s "frequency" in cycles per second. Just as a swing likes to swing back and forth at its own special frequency, vibrating objects like bells and glasses have their own special "natural frequencies" at which they like to vibrate. When you tap the rim of a glass gently with a spoon, the glass vibrates at its special natural frequency and sings out a note of the corresponding musical pitch. Whether the vibration frequency and note pitch are high or low depends on the glass’s size and shape — how tall it is, how fat it is, how thick its walls are, of what kind material it is made of. Although good crystal glasses may work best, less expensive glasses can also work well.
3 Oscillation of a wineglass

A wineglass typically has a strong, rigid supporting stem, and a bow that is rather thick at the base and becomes progressively thinner toward the upper rim. Vibrations of this system occur most easily under conditions that leave the circumferential length of the rim unchanged, because glass is highly resistant to extension or compression. This condition is satisfied, to a high approximation, if the rim deforms from a circle into an ellipse, and back again through the circle into another ellipse with its major axis at right angles to the first (Figure 3). All other horizontal sections of the wineglass will go through similar motions, but with amplitudes that decrease as one goes down from the level of the rim [4].

![Figure 3 Vibration pattern of a singing glass.](image)

The rim and sides of the glass vibrate together in the pattern shown in the diagram (Figure 3), as viewed looking down on the glass from above it. The rim’s shape changes repeatedly and rapidly, several hundred cycles each second, between the two elliptical or egg shapes shown. When points A and B move inward, points C and D move outward, and vice-versa half a cycle later. The amount of vibration of an actual glass is much less than the diagram shows, but this at least gives you some idea of what is happening to the glass. As the sides of the glass vibrate, they push air back and forth creating sound vibrations in the air that travel to your ears.

As your finger moves along the rim, it is alternately slipping and sticking to the rim, just like a violin bow slips and sticks as it moves across a violin string. This is called "slip-stick" motion. It’s the reason for squeaking of door hinges or the squealing of chalk being dragged across a blackboard. You can demonstrate slip-stick motion by dragging your fingers across the surface of an inflated balloon. This makes a rather funny sound as the balloon vibrates. But there the vibration’s pattern and frequency depend mostly on how you drag your fingers across the balloon - how fast they move and how hard you press. When you rub your finger along the rim of a glass, you produce the vibration pattern with its own special natural frequency for that glass. Changing the speed or pressure you use with your finger can make the sound louder or softer and make the
glass vibrate and sing more easily or not, but it doesn’t change the vibration frequency. Wetting your finger prepares its surface and the rim of glass to make the slip-stick motion easier to produce, just like rubbing rosin on a violin bow prepares the bow and string to slip and stick and make their sounds more easily.

Although the greatest movement of the glass’s rim is in and out at points A, B, C, and D, there are smaller movements at other points that can be sideways along the rim. The movements of several of these points (E, F, G, and H), which are in-between points A, B, C, and D, are shown with smaller arrows. The slip-stick vibration of your moving finger can get the glass’s vibration going by pushing the vibration along the rim at these in-between points. But your finger doesn’t stay in one place, so the vibration pattern has to follow your finger around the rim. A neat way to see the vibration pattern is to fill a glass almost full of water and then rub your finger around the rim. Now when the glass sings its note, you can see ripples in the water near the sides of the glass. These ripples are largest near points A, B, C, and D in the vibration pattern, and the whole pattern rotates around the glass to follow your finger [2, 3].

On a simplified view, we can represent the wineglass as a thin-walled cylinder, attached to a rigid circular base (Figure 4). A vertical section trough this cylinder looks rather like a tuning fork, and the complete glass is generated by rotating this section about the vertical axis of symmetry. In oscillation, the condition of constant rim perimeter forces a definite phase relationship, such that the “tuning fork” having maximum outward displacement of its prongs at a particular instant is 90° away in azimuth from the fork having maximum inward displacement.

![Diagram of a wineglass](image)

**Figure 4** Vertical section of vibrating wineglass trough section AB.
Given the above form of the oscillation, its actual magnitude can be characterized by the amplitude of oscillation of any one chosen point. For convenience, let this be an antinodal point of the rim at the top of the glass. Its displacement at any instant can then be written

$$\Delta(t) = \Delta_0 \cos \omega t.$$  \hfill (1)

The displacement of any other point is related to this by a time-independent geometrical factor that depends on the azimuth $\theta$ and the vertical coordinate $z$. For the form motion that we are assuming, the horizontal radial displacement $x$ of any arbitrary point of the wall of the wineglass can be written

$$x(z, \theta, t) = \Delta_0 f(z) \cos 2\theta \cos \omega t,$$  \hfill (2)

where $f(z)$ rises from zero at the bottom of the wineglass to unity at the top rim. The $\cos 2\theta$ factor describes the elliptical distortion from the static circular shape, giving maximum amplitudes of oscillation along the perpendicular axes through $\theta = 0$ and $\pi/2$, and diagonal nodal lines through $\theta = \pi/4$ and $3\pi/4$, as indicated in Figure 4a. The form of this motion, with its nodes and antinodes can be made visible if chalk dust or other fine powder is sprinkled on the liquid surface of a partially filled wineglass that is being excited into oscillation [4].

When glass vibrates in higher modes, radial displacement $x$ of any arbitrary point of the wall of the wineglass can be written as

$$x(z, \theta, t) = \Delta_0 f(z) \cos n\theta \cos \omega t.$$  \hfill (3)

In higher modes $n > 2$. In the first higher normal mode of vibration where $n = 3$, the vibration pattern has 6 nodes (Figure 5).

![Figure 5 Vibration pattern for $n = 3$.](image-url)
4 Kinetic and potential energies

Assuming linear elastic restoring forces, the total elastic potential energy of the vibrating wineglass is proportional to \( \Delta^2 \), and its total kinetic energy is proportional to \((d\Delta/dt)^2\). Hence the total energy of the system can be written as

\[
E = A \left( \frac{d\Delta}{dt} \right)^2 + B\Delta^2,
\]

where \( A \) and \( B \) are constants. Ignoring damping, this total energy is constant. If we substitute \( \Delta = \Delta_0 \cos \omega t \) from Eq. (1), it can be seen that requirement \( E = \text{const.} \) leads to the result

\[
\omega^2 = \frac{B}{A}.
\]

Thus the calculation of the natural frequency of vibration reduces to a matter of evaluating the total kinetic and potential energies of the system in terms of the rim displacement \( \Delta \).

Suppose for simplicity that the wineglass is modelled, as in Figure 4b, as a vertical cylinder of radius \( R \) and height \( H \), with a rigid base and side walls of uniform thickness \( a \). The mass of an element of the wall, lying between \( z \) and \( z + dz \) in height, and between \( \theta \) and \( \theta + d\theta \) in azimuth, is equal to \( \rho_g aR \, d\theta \, dz \), where \( \rho_g \) is the density of the glass. Its instantaneous radial velocity \( dx/dt \) is given, according to Eq. (4), by

\[
\frac{dx}{dt} = -\omega \Delta_0 f(z) \cos 2\theta \sin \omega t.
\]

In addition to this radial motion, however, the condition of fixed perimeter implies tangential displacement also. It is clear from symmetry that there is no such displacement for \( \theta = 0 \), but in general a point that initially has the coordinates \( R, \theta \) will, when displaced, move to \( R + x, \theta' \). The transverse velocity is thus given by

\[
\frac{ds}{dt} = \frac{1}{2} \omega \Delta_0 f(z) \sin 2\theta \sin \omega t.
\]

Hence the total kinetic energy \( K \) of the vibrating glass at time \( t \) after the integration over \( \theta \) simplifies to

\[
K = \frac{5\pi}{8} \rho_g aR \omega^2 \Delta_0^2 \sin^2 \omega t \int_{0}^{H} [f(z)]^2 \, dz.
\]

Because \( f(z) \) increases from zero at \( z = 0 \) to a maximum at \( z = H \), the major contribution to \( K \) will come from the motion of the upper parts of the wineglass [4].
The calculation of the potential energy is more complicated. It represents the energy of flexure of the glass in both horizontal and vertical planes. As a basis for the calculation, consider a curved segment of material which, when undeformed, has mean radius \( r_0 \) and radial thickness \( a \) (Figure 6a). Let its thickness perpendicular to the plane of the diagram be \( b \). Suppose that the segment has its mean radius of curvature changed to \( r_c \) (Figure 6b). We assume that the length \( l_0 \) of the center line of the segment remains unchanged. For a filament a distance \( y \) from this center line, we have

\[
\text{initial length } = \frac{r_0 + y}{r_0} l_0, \quad \text{deformed length } = \frac{r_c + y}{r_c} l_0, \quad \text{change of length } \delta l = l_0 y \left( \frac{1}{r_c} - \frac{1}{r_0} \right). \tag{9}
\]

![Figure 6](image)

*Figure 6*  
(a) Curved section of material in stress-free state.  
(b) Same section under stress.

Radial thickness of the filament is \( dy \), the Young’s modulus is denoted \( Y \), the force of tension or compression along the filament is given by \( F = b \, dy \, Y \, \delta l/l_0 \) and the stored energy is \( dU = 1/2 F \delta l \). Using all the equations above and integrating \( dU \) between the limits \( y = \pm a/2 \), we have

\[
\Delta U = \frac{l_0 Y a^3 b}{24} \left( \frac{1}{r_c} - \frac{1}{r_0} \right)^2. \tag{10}
\]

For the flexure in the horizontal plane of a curved segment lying between \( z \) and \( z + dz \) and between \( \theta \) and \( \theta + d\theta \), we have \( l_0 = R \, d \, \theta, \, b = dz \) and \( r_0 = R \). The radius of curvature \( r_c \) in the deformed state varies with the azimuth. All things considered and integrating over \( \theta \) and \( z \), the elastic energy stored in a horizontal circular segment is given by

\[
U_1 = \frac{3\pi Y a^3}{8 R^3} \Delta_0^2 \cos^2 \omega t \int_0^H [f(z)]^2 dz. \tag{11a}
\]

In the vertical plane, we can consider a segment of a length \( l_0 = dz \) and width \( b = R \, d \, \theta \). Under the assumption of small deformations and \( r_0 = \infty \), we have

\[
U_2 = \frac{\pi Y a^3}{24} R \Delta_0^2 \cos^2 \omega t \int_0^H [f''(z)]^2 dz. \tag{11b}
\]

Combining those two equations, we get the following expression for the total elastic potential energy at a given moment [4]:

9
\[ U = \frac{\pi Ya^3}{24R^3} \Delta_0^2 \cos^2 \omega t \left( 9 \int_0^H [f(z)]^2 dz + \int_0^H [f''(z)]^2 dz \right). \tag{12} \]

If each vertical strip of the wineglass can be considered as a uniform bar, free at top and clamped at the bottom end, then the form of \( f(z) \) is given by

\[ f(z) = A (\cosh \beta z - \cos \beta z) + B (\sinh \beta z - \sin \beta z), \tag{13} \]

where \( A, B \) and \( \beta \) are constants. For the lowest mode, \( \beta \approx 1.875/H \) and Eq. (11) can conveniently be written, to a very good approximation, in the following simplified form:

\[ U \approx \frac{3\pi Ya^3}{8R^3} \Delta_0^2 \cos^2 \omega t \left[ 1 + \frac{4}{3} \left( \frac{R}{H} \right)^4 \right] \int_0^H [f(z)]^2 dz. \tag{14} \]

## 5 Frequencies

Using the values of total kinetic energy from Eq. (7) and total potential energy from Eq. (13) we deduce that the values of the constants \( A \) and \( B \) in Eq. (2) are

\[ A = \frac{5\pi}{8} \rho g aR \int_0^H [f(z)]^2 dz, \tag{15a} \]
\[ B = \frac{3\pi Ya^3}{8R^3} \left[ 1 + \frac{4}{3} \left( \frac{R}{H} \right)^4 \right] \int_0^H [f(z)]^2 dz. \tag{15b} \]

It follows that the natural angular frequency of oscillation in the assumed (lowest) mode of vibration is given by

\[ \omega_0^2 = \frac{B}{A} = \frac{3}{5} \frac{Ya^3}{\rho g R^4} \left[ 1 + \frac{4}{3} \left( \frac{R}{H} \right)^4 \right]. \tag{16} \]

The fundamental frequency \( \nu_0 = \omega_0 / 2\pi \) in hertz is thus given by [4]

\[ \nu_0 = \frac{1}{2\pi} \left( \frac{3\nu}{5\rho g} \right)^{1/2} \frac{a}{R^2} \left[ 1 + \frac{4}{3} \left( \frac{R}{H} \right)^4 \right]^{1/2}. \tag{17} \]

Through this seminar no mention has been made of the possibility of simple torsional oscillation of the wineglass about its vertical axis. The process of rubbing around the rim of the glass to excite its oscillations would be expected to be particularly favourable to the excitation of this type of vibration. The frequency of such a torsional vibration would, however, be far higher than the mode we have discussed. For the basic elliptical mode frequency of torsional vibration would be greater by a factor of at least 10.

To sum up, in the case of the wineglass, a finger slides and sticks along the surface of the glass as you rub the rim. The rubbing imparts energy to the glass molecules and
causes them to resonate. The motion of your hand sets up a wave of vibration travelling through the glass. The vibrating glass causes air molecules to vibrate at the same frequency. The vibrating molecules are the sound waves that you hear. The frequency or pitch of the sound wave is the same as the resonant frequency of the glass.

So, how does the liquid change the pitch of the singing wineglass? The added liquid is forced to participate in the vibration mode, so the total kinetic energy for a given motion of a glass itself is increased. The potential energy of elastic deformation, however, remains unchanged. Thus, in Eq. (2) the constant $A$ increases, the constant $B$ remains the same, and the value of $\omega^2$ in Eq. (3) consequently goes down. For the liquid at rest of height $h$, we obtain

$$\left(\frac{v_2}{v_h}\right)^2 = 1 + \frac{4A\rho_l R}{5\rho_0 a} \left(\frac{h}{H}\right)^4.$$  

The pitch decreases as the liquid quantity increases. The term $h/H$ to the power of 4 is related to the larger contribution of the liquid as it gets closer to the rim of the glass. In other words, the closer the liquid gets to the rim, the more it moves, and consequently the more it contributes to the pitch being lowered. It is noteworthy that the power of the term $h/H$ is directly related to the shape of the shell [4]. How the frequency depends on liquid level you can see in figure 7.

![Figure 7](image)

Figure 7 Graph of how frequency depends on liquid (in this case water) level inside [5].

## 6 Conclusion

As you rub the glass with wet a finger, you will hear the singing sound of the glass. A glass vibrates with its natural frequency, called a resonant frequency. If you impart enough energy to the glass at its resonant frequency, you can cause the glass to shatter [7]. However, this takes more energy than you can provide by rubbing the rim. Some singers can sing a note equal to the resonant frequency of a wineglass and cause it to shatter. To shatter the glass, the singer’s voice has to be able to match that frequency. Also the singer has to sing very loudly. A singer has to produce a note at volumes at least 100 decibels. If the frequency is right, volume is a key player in the glass shattering
game, because the loudness of a sound is directly related to the extent it displaces air molecules. And the singer also needs to hold that note for at least two to three seconds for the vibration to build up enough to cause the glass to shatter. It helps if we have the right kind of glass. In 2005 the Discovery Channel television show Myth Busters tackled the question, recruiting rock singer and vocal coach Jamie Vendera and he done it. For the first time, proof that voice can indeed shatter glass was captured on video. He hold a world glass shattering record [8].

Resonance has also been shown to cause bridges to collapse. Marching troops of soldiers will often break cadence when crossing a bridge to prevent resonance collapse. The most famous example of resonance was the Tacoma Narrows Bridge in Washington State (also called Galloping Gertie). In 1940, just months after its completion, winds in the Tacoma Narrows matched the bridge's resonant frequency and caused the suspension bridge to sway uncontrollably. Within hours, the bridge collapsed (Figure 8).

![Figure 8 Tacoma Narrows Bridge in 1940 [1].](image)

References