Active noise control

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Abstract

Active noise control is a method of reducing unwanted sound in the environment by using destructive interference between sound fields generated by primary sources of the original noise and secondary sources which can be controlled and are usually moving coil loudspeakers controlled by an electrical signal. It works best at low frequencies of sound, where the distance between these sources is small compared to acoustical wavelength. There are two basic control strategies called feedforward and feedback control, the basic difference being that feedback control only measures resulting error field in the so called zone of quiet around monitor microphone and feedforward control uses an reference signal of the noise before it reaches this zone. Some examples of system where active noise has proven to be effective are used in reducing noise in ducts, engine exhausts, mains transformers, protective headsets, car and aircraft interiors.
1 Introduction

Noise control is the field of acoustical engineering that deals with reducing unwanted sound in the environment. Three basic principles are: reducing the energy of noise near a listener, creating a sound barrier between the source of noise and a listener and reducing the power of a noise at its source. The energy density of sound waves decreases as they spread out so the trivial solution is to increase the distance from source, but that is not always an option. Conventional passive methods include sound insulation, silencers, vibration mounts, damping and absorptive treatments and mufflers. They work best at middle and high frequencies but are not very effective at low frequencies because size and mass of passive treatment usually depend on acoustical wavelength. In air under normal conditions a sound wave with frequency 100 Hz will have a wavelength of about 3.4 m. This leads to some very impractical solutions. Many important noise problems are dominated by contributions from low frequencies and therefore methods of active noise control were developed. They work on the principle of destructive interference between sound fields generated by primary sources of the original noise and other secondary sources which can be controlled. Secondary source is usually a moving coil loudspeaker controlled by an electrical signal.

The basic ideas of active noise control were first described by Paul Lueg in a patent in United States in 1936 [1]. He described the principle of measuring the sound field of plane waves traveling in a duct with a microphone and then feeding an appropriate signal to secondary loudspeaker so that the superposition of both waves results in destructive interference. This strategy is using an upstream microphone can be characterized as feedforward control. Another paper was published in 1953 by Harry Olson and Everet May [1] with description of a system they called electronic sound absorber. Their ideas of application in an airplane or automobile were quite visionary for that time. They also measured the extent of zone of quiet around single microphone. By contrast with Lueg’s earlier work, they concentrated on using no prior knowledge of the sound field so their system was characterized as feedback control. In some cases it is not necessary to detect the primary waveform with a microphone to generate reference signal as it was shown in 1956 by William Conover [1] who was working on reduction of acoustic noise from large mains transformers. Noise from these transformers is principally at the even harmonics of the line frequency, so any signal with the same frequency components as the original transformer noise will be an adequate reference.
However methods of active noise control at that time were not practical because the technological limitations of analogue electronic control systems. So the practical step forward was with first applications of digital signal processing techniques and devices in this field in the 1970s.

2 Acoustical principles

2.1 Description of sound waves

Sound waves are any disturbance that is propagated in elastic medium. In fluids, that is liquids and gases, sound waves can only be longitudinal which is associated with compression and decompression of fluid in the direction in which wave travels. In solids however we also get transverse waves, due to shear deformation of elastic medium perpendicular to the direction of travel. In this seminar we only deal with sound waves in air which is a compressible fluid.

A variation in pressure above and below atmospheric pressure is called sound pressure measured in Pascals [Pa]. A person with normal hearing can detect sound pressure as audible in the frequency range from 15 Hz to 16 kHz and can detect pressures as low as about 20 \( \mu \text{Pa} \) at frequencies between 3000 and 6000 Hz where the ear is most sensitive [2].

The sound wave equation for pressure field \( p(r, t) \) is written as

\[
\frac{\partial^2 p}{\partial t^2} - c^2 \nabla^2 p = 0,
\]

where \( c \) is the speed of sound which can be written with thermodynamic definition of compressibility \( \chi_S \) as

\[
c = \sqrt{\left( \frac{\partial p}{\partial \rho} \right)_S} = \sqrt{\frac{p_0}{\chi_S}} ,
\]

where \( p_0 \) is density of air. At 20\(^\circ\)C the speed of sound is 331.5 m/s [2].

A monochromatic plane sound wave can be represented by the equation for sound pressure

\[
p(r, t) = \text{Re}[p_0 \exp(i(kr - \omega t))] ,
\]

where \( p_0 \) is the maximum amplitude, \( \nu \) is the frequency so that \( \omega = 2\pi \nu \) and \( k \) is the wave vector defined as \( k = (\omega/c)n \), where \( n \) is the unit vector in direction of propagation. The time from \( t = 0 \) to \( t = 1/\nu = T \) is known as the period. Propagation of acoustic wave is a linear process and the principle of superposition applies. Therefore monochromatic waves are important because a sound wave may be viewed as a combination of harmonically related and unrelated single monochromatic waves with various frequencies and wave vectors.

We define root-mean-square or effective value of amplitude as

\[
p_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T p^2 \, dt} .
\]

For non-periodic waves such as noise \( T \) is not the period of the wave and we must calculate the limit of integral in (eq:prms) as \( T \to \infty \).

Because the range of pressure amplitudes and intensities can span over many orders of magnitude we use logarithmic measures called levels with units dB. The most used is sound pressure level defined as

\[
\text{SPL} = 20 \log(p_{\text{rms}}/p_\text{ref}) \text{ dB} ,
\]

where the reference pressure is \( p_\text{ref} = 20 \, \mu \text{ Pa} \).
A sound spectrum is a plot of $p_{\text{rms}}^2$ versus frequency. If the number of combinations are small (finite) we have a line spectrum spectrum otherwise we have continuous spectrum of sound. For example a random noise has a continuous spectrum.

If sound wave travels through boundary between two mediums with different propagation speeds it undergoes refraction and reflection. In first medium we get a combination of reflected and incident wave, in the second medium we get only refracted wave. The relation between these waves depends on boundary condition at the surface of separation, where the pressure and normal velocities of fluid of these waves must be equal.

2.2 Basic principles

The basic idea of active noise control is illustrated in figure 1 where have two acoustic sources placed a distance $L$ from one another. One is called disturbance source and is the source of unwanted noise. The other, called control source, is the loudspeaker with which we want to create sound that will be out of phase with the sound from disturbance source in order to cancel it as much as possible due to destructive interference between two waves. We can observe the basic limitation of active noise control. Cancellation of the sound radiation in all directions is only possible if the distance between control and disturbance is small compared to acoustic wavelength, because the phases of the two sound fields surrounding the two sources are nearly the same at every location in space [2] which is shown in figure 2.

When a monitor microphone which measures primary source at some point and a secondary source loudspeaker are positioned close together they will be well coupled and only a modest drive voltage is required to achieve cancellation at these point so that sound pressure at other
points further away will not be significantly affected. We get a zone of quiet around the monitor microphone where we have reductions in the primary sound level greater than 10 dB. Dimensions of the zone are approximately one tenth of a wavelength. This is useful at low frequencies. A frequency of 100 Hz with $\lambda = 3.4$ m in air, translates to a zone of quiet with dimensions of 0.34 m. However, the full three-dimensional shape of the zone of quiet is much more complicated and is discussed in literature [1].

2.3 Sound field of two monopole sources

Let’s consider primary and secondary monopole sound wave sources at distance $L = 2h$ with same frequency $\omega$. The configuration is shown in figure 3. We set the amplitude of secondary source to be the same but out of phase with the primary source. Both sources are small and spherical - they radiate sound in all directions.

![Figure 3: Configuration of primary and secondary monopole sound source](image)

Because the superposition principle, we can write sound field at point $T$ as

$$p(T) = \frac{A}{r_1} \exp(i(\omega t - kr_1)) - \frac{A}{r_2} \exp(i(\omega t - kr_2)).$$

(6)

We assume that $T$ is in the farfield and therefore $r \gg L$ and $r_1$, $r_2$ and $r$ are approximately parallel

$$r_1 \approx r - h \cos \vartheta, \quad r_2 \approx r + h \cos \vartheta.$$  

(7)

We use these expressions in the exponents or phase factors in equation (6). The terms $\pm h \cos \vartheta$ are small compared to $r$, however they can be significant because of their effect on the phases of two signals. For example at given point they determine that if two signals arrive at that point in phase and thus sum together or if they arrive out of phase and cancel each other out. In amplitude factor of equation (6) we can simplify $1/r_1 \approx 1/r_2 \approx 1/r$, because the extra distance has little effect on the amplitude. Using these approximations we can write equation (6) as

$$p = \frac{A}{r} \exp(i(\omega t - kr)) \left( \exp(ikh \cos \vartheta) - \exp(-ikh \cos \vartheta) \right).$$

(8)

$$= \frac{A}{r} \exp(i(\omega t - kr)) \left( 2i \sin(kh \cos \vartheta) \right),$$

(9)

where we can see that the first expression before the brackets with sine represents the pressure of a single monopole $p_1$ without any additional monopole sources.
For calculating the intensity of power that is radiated by these two sources we use the following equation [3], that holds for outgoing spherical waves in free field

\[ I = \frac{p_{\text{rms}}^2}{\rho_0 c}, \]  

(10)

where \( \rho_0 \) is the density of a medium and \( c \) the speed of sound in that medium, which is air in our case. The quantity \( Z = \rho_0 c \) is also known as specific acoustic impedance of a spherical wave in free field. If we define the intensity of a single monopole source as \( I_1 = \frac{p_{\text{rms1}}^2}{\rho_0 c} = \frac{A^2}{2r^2 \rho_0 c} \)
we can write the ratio of intensities of two sources versus one source as

\[ \frac{I}{I_1} = 4 \sin^2(kh \cos \vartheta). \]  

(11)

It is common in acoustics that we define the logarithmic ratio in dB as

\[ \text{Ratio} = 10 \log\left(\frac{I}{I_1}\right) \, \text{[dB]} \]  

(12)

In figure 4 we can see the change of this ratio from equation (12) as a function of angle \( \theta \) and the product of acoustical wave number and separation of the two sources \( k_0 L \). If \( k_0 L = 0.1 \) we get reduction of 20 dB or more for all angles. However for higher values of \( k_0 L = 1 \) we get reasonable cancellation only at certain angles or even for large value of \( k_0 L = 10 \) we get almost no cancellation at all.

![Figure 4: Change in the ratio of controlled to uncontrolled sound pressure level as a function of frequency and angle [2]](image)

To calculate the total radiated power at the distance \( r \) from the two sources, we integrate the intensity over a sphere centered at origin

\[ P = \int I \, dS, \]  

(13)

using surface element \( dS = 2\pi r^2 \sin \vartheta \, d\vartheta \). Defining total radiated power of single source as \( P_1 = 4\pi r^2 I_1 \) we can write

\[ P = 2P_1 \left(1 - \frac{\sin(kL)}{kL}\right). \]  

(14)
Note that $L = 2h$. Again we can define the logarithmic ratio in dB as $10 \log(P/P_1)$. This result is shown in figure 5 as dashed curve where we can see that for $r < 0.3\lambda$ the total power of two sources is less than that of primary source alone. However, for $r > \lambda$ the total power of both sources will be approximately double that of primary source alone.

![Figure 5: Change in the total radiated output power as a function of the separation between sources for unoptimized secondary source $P_{td}$ and optimized secondary source $P_{to}$.

We can minimize the total power output $P_{td}$ by adjusting the secondary source strength in respect to $kL$. Let us denote with $A_p$ the strength of primary source and $A_s$ the strength. For small values of $kL$ we can use the same configuration as we already mentioned before, that is two sources of equal strength but opposite phase, that is $A_s = -A_p = -A$. This is also known as dipole configuration. For large values of $kL$ we want the strength of secondary source to be $A_s = 0$. Obviously then the total radiated power at least won’t be greater than the power of a single source. The optimal solution that is discussed in literature [1] but it’s exact derivation is beyond the scope of this seminar so we will only write the result for optimum secondary source strength as

$$A_{s\text{opt}} = -A_p \frac{\sin(kL)}{kL}.$$  \hspace{1cm} (15)

Using this optimal source strength gives an expression for minimum total power output which can be achieved [1]

$$P_{to} = P_1 \left( 1 - \left( \frac{\sin(kL)}{kL} \right)^2 \right).$$  \hspace{1cm} (16)

As it is clear from the solid curve in figure 5 this is never larger than the radiated power from just a single monopole source $P_1$.

Thus with observing this simple model of two monopole sources in free space, we have a way of reducing total radiated power of primary source for small values of $kL$. Small values of $kL$ mean either low frequencies as $k = \omega/c$ or small separation $L$ between primary and secondary source. Usually loudspeakers can be modeled as simple spherical sources and therefore can be used as this controlled secondary source.
3 Control System Design

The basic elements of active noise control systems are sensors for example microphones, controllers like digital filters and control sources that are usually loudspeakers in acoustic applications.

The active element of a control source might be used to move a speaker cone, modulate airflow or even deform a it’s structure. A sufficient control authority, that is sufficient space and power must be available in order to have an adequate performance of such a system. For simple example the diameter of a speaker cone must not be small compared to the wavelength of sound it produces. The propagation of an acoustic wave is a very nearly linear process unless it’s amplitude is corresponding to extremely loud noise [1]. Most nonlinearity is usually due to problems with loudspeaker design. For example, when producing a low-frequency the cone of the speaker might undergo considerable excursions which can generate higher frequency harmonics. These harmonics will not be canceled and may become audible.

In signal processing theory we describe system response with transfer function $H(s)$, that are defined as the ratio of Laplace transformation of input $X(s)$ and output $Y(s)$ signal

$$H(s) = \frac{X(s)}{Y(s)} = \frac{L[x(t)]}{L[y(t)]}.$$  \hspace{1cm} (17)

Active noise controllers are basically filters. It’s inputs are signals from sensor microphones and it’s outputs are the drive signals to the control surfaces. When required magnitude and phase responses of the filters are relatively simple function of frequency we can use analog controllers. However in most cases, especially when characteristic of these filters are required to change over time and are made adaptive, digital controllers offer many advantages over analog controllers in terms of flexibility, accuracy and cost.

Let’s look at two basic control architectures.

3.1 Feedback control

A very basic configuration of a feedback control approach is illustrated in figure 6 on the left where we have just a sensor microphone connected to a amplifier that drives the control speaker. On the right we have the block diagram of signal path in such setup. There we can see that we basically have a simple feedback loop where we drive the signal measured with microphone denoted as residual $e$ through control filer $W$ and plant $P$ back to the disturbance signal of the original noise $d$. Both signals are summed together at the point where we have our microphone. With the transfer function of the filter $W$ we describe what happens with the signal as it travels through electrical path of the system. With transfer function of the plant $P$ we describe the physical path of the signal, that is response of the loudspeaker that generates the control sound signal and any effects on sound wave as it travels from the speaker to the microphone. We can write the transfer function of such system as ratio between disturbance and measured error [2]

$$\frac{E(s)}{D(s)} = \frac{1}{1 + P(s)W(s)}.$$ \hspace{1cm} (18)

For periodic signals we can use operator $s = i\omega$.

In the ideal case where the frequency response of the plant $P$ and electronics $W$ would be relatively flat and free from phase shift we could use a simple inverting amplifier $W(i\omega) = A$ with $A \rightarrow \infty$ and therefore causing the overall transfer function (18) to become very small. In the real case this cannot be done, because the electro-acoustical response of a moving coil speaker induces considerable phase shift near its mechanical resonance frequency [1]. Some delay is also
inevitably introduced due to acoustic propagation time between loudspeaker and microphone. When the phase shift approaches $\pi$ the negative feedback becomes positive and the system can become unstable, which means that the amplitude of the output signal grows beyond all means, thus amplifying the noise.

We can look at filter system as a regulator part $H(i\omega)$ with added compensating filters $G(i\omega) \approx P(i\omega)^{-1}$ that are designed to approximate the plant inverse and compensate for its amplitude and phase shifts. We can write $W(i\omega) = G(i\omega)H(i\omega)$. Entering this into equation (18) we can see that the transfer functions simplifies to $1/H(i\omega) \ll 1$ when $|H(i\omega)| \gg 1$. When $|H(i\omega)| \ll 1$ equation (18) is equal to 1 which means no gain or reduction of input signal. This means that we can design a regulation filter with gain that is large for frequency range where control is desired. For frequencies that are not controlled, regulator gain remains small and we do not amplify any noise outside regulation band.

Stability is described with Nyquist criterion, which in this particular case states that the phase of $H(i\omega)$ should not exceed $\pi$ until its magnitude is less than 1. For more detailed and formal Nyquist stability requirements we must draw a Nyquist plot. That is a curve in the complex plane with the real part of loop gain on the x axis and imaginary part on the y axis that is traced out as the frequency changes. The radius from origin to the point on the curve represents the magnitude of the loop gain at particular frequency. The angle from x axis to the radius represents phase angle or phase shift. The analysis of system stability and the optimal design of such filters is somewhat beyond the scope of this seminar [1].

The most successful application of active noise control using feedback system are active noise cancellation headsets mainly used by aircraft pilots. A schematic example is illustrated in figure 7, where we can see the shell of one side of the headset in which we have a small speaker and a microphone placed very close together. Here we have a special case because the speaker not only generates the control signal to reduce noise coming through the shell from outside, but it also generates whatever signal, for example from communications tower, pilots need to hear. Both signals are picked up by microphone and we must design filter $W$ so it attenuates the unwanted noise but keeps the desired signal relatively unchanged. Because microphone and speaker are placed so close together, the upper frequency limit of 1 kHz is mostly because the electronics stability requirement. In very loud environments the performance of such headset is also limited because the small speaker can only generate limited sound level within the shell.
3.2 Feedforward control

An example of basic feedforward control system would be a system for controlling sound in a duct that is shown in figure 8 on the left. We have a reference microphone with which we measure unwanted noise somewhere in a duct. The signal is driven through appropriate electronics to create a signal that drives a loudspeaker some distance downstream from the microphone. If everything is done right the sound wave from the speaker cancels out with unwanted noise.

If we look at the signal path diagram in the figure 8 on the right we again have an electronic control filter $W$ and physical plant $P$, through which the reference signal $r$ from microphone is driven to the point where it adds up with uncontrolled disturbance signal $d$. In this particular case we also have a transfer function $B$ that describes how the noise reference signal travels as physical sound waves through the air in duct from the point where it was measured with the reference microphone to the point when it reaches the control speaker. No information about residual error signal $e$ is driven back to the system and therefore we have no feedback loop. The transfer function of such system is thus

$$\frac{E(i\omega)}{D(i\omega)} = 1 - \frac{P(i\omega)W(i\omega)}{B(i\omega)}. \quad (19)$$
The main difference with feedback control is that now we have a separate reference signal \( x \) that is used with electrical controller \( W \) to drive the secondary source [1]. Reference signal must be well correlated with the primary source. It provides advance information of primary noise before it reaches the monitor microphone. In special cases of so called harmonic control the electrical reference can be obtained directly from the mechanical operation of primary source.

We could for example use the signal from engine’s tachometer as a reference in active exhaust muffler. In these cases we control only certain frequency ranges as contrast to broadband noise control.

Sometimes it is useful to use both approaches. That means that in addition of using a reference microphone we also use a microphone to measure the residual signal and we drive it back in a feedback loop. A schematic example of such feedforward control in a car is illustrated in figure 9. Somewhat similar principle can be used in the control of aircraft interior as illustrated in figure 10. In addition to control speakers, other actuators such as piezoelectric patches on fuselage panels and inertial shakers on the ribs and strings have also been tried, effectively turning the fuselage into a loudspeaker [2].

Figure 9: Schematic of an active automobile interior noise suppression system. [2]

Figure 10: Schematic of an active aircraft interior noise control system. [2]
3.3 Practical example

Let’s look at a particular example [2] of a diesel-electric locomotive that generates significant noise when operated at full power. This can have a significant adverse impact on the quality of life of people living near major railroad lines. We can see in figure 11 that the primary source of locomotive noise come from engine exhaust and the cooling fans.

![Figure 11: Noise sources on an SD40-2 diesel electric locomotive measured at 30.48 m at full throttle and at full load. [2]](image)

Active noise control concentrated on reducing noise from the exhaust and the basic concept is illustrated in figure 12 where we can see a plan top view of locomotive hood with number of loudspeakers surrounding the exhaust and a number of microphones located near the edges that act as residual sensors. Due to tonal characteristic of this noise a feedforward architecture was selected. It used a tachometer on the locomotive diesel engine as the reference. This example used active system was set with performance goal of reducing the noise by 10 dBA. Here we use a A-weighted sound pressure level defined as

\[
L_A = 10 \log \left( \frac{p_A(t)}{p_{ref}} \right)^2 \text{ dBA},
\]

where \( p_A \) is the instantaneous sound pressure measured using the standard A-frequency weighting [2].

An active system provides 10dB reduction of noise below 250 Hz and a passive silencer provides 5 dB of broadband noise reduction from 250 to 500 Hz and 15 dB reduction from 500 to 5500 Hz. The system is illustrated in figure 13.

The performance of such system can be observed in figure 14 which shows the reduction of tonal noise at microphone on the roof due to the use of active system. Locomotive was operating at half the full throttle. We can see significant reduction of all the important tones (peaks in spectrum) with some amplification of low-amplitude tones. The reduction of the overall sound level below 250 Hz is in excess of 12 dB.
Figure 12: Plan view of the locomotive hood with basic system concept. [2]

Figure 13: Control speaker arrangement in the locomotive. [2]

Figure 14: Noise reduction performance of the active system at half throttle loaded as measured at a microphone on the roof of the locomotive. [2]
4 Conclusions

Active noise control can be effective way of reducing unwanted noise in the environment if we consider it's limitations. It works best for sound fields that are spatially simple for example low frequency sound waves traveling trough a duct which is an one dimensional problem. Active control system are more locally oriented as often reducing noise in some local region causes increase in the noise elsewhere. It does not reduce noise globally unless sound fields are very simple and the primary mechanism is impedance coupling. Controlling spatially complex field such as surroundings of a house is somewhat hopelessly complex as much more so called actuators, microphones and loudspeakers are required. Active noise control also works well in enclosed spaces such as various vehicle cabins.

As we saw it is most effective at reducing noise in systems where the acoustic wavelength is large compared to dimensions of the system. At higher frequencies passive methods are usually much better. Therefore active noise control systems are usually not used alone since a combination of passive and active systems can cover a larger range of frequencies.

Another important factor in active noise control is whether or not the disturbance can be measured before it reaches the area where noise reduction is desired. This so called feedforward control is sometimes not possible and the control signal can only be calculated from error sensor measurement. This feedback control can be very unstable under some circumstances and is usually even less effective at higher frequencies as feedforward control.

Noise that contains wide range of frequencies, so called broad band noise, is clearly much harder to control than narrow band noise. For example, in aircraft cabin it is difficult to control the broadband noise of wind flowing over an aircraft fuselage, but it is much easier to control the tonal noise caused by propellers which move with somewhat constant rotational speed.

Special case of active noise control is adaptive noise control where controller usually employs a mathematical model of the plant dynamics and if possible one of the sensors and actuators. Sometimes the plant changes too much over time because changes in temperature and other operation conditions and the performance suffers. Good controller is one that monitors the plant continuously and updates its internal model of dynamics.

Active noise control is closely related to the field of active vibration control where we control the unwanted vibrations of solid mechanical systems. Sometimes it is possible to reduce the noise of a system by reducing it’s vibration which may cause unwanted noises. For example when dealing with unwanted noise in rooms and buildings, the source of outside noise for a observer in a room are actually the walls of the room. If we reduce vibrations of these walls that are caused by outside noise sources we can reduce the transfer of noise from outside into a room. Here we can control vibrations with actuators placed over the whole surface of the walls.
References


