Seminar

Ultracold atoms in optical lattice

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Abstract

In the seminar, physical principles underlying behavior of atoms in optical lattices are presented. Optical lattice is formed by the interference of counter-propagating laser beams, which creates effective potential that traps ultracold atoms. Special emphasis is given to usage of such systems to simulate many-body quantum systems. Two most prominent physical models that can be simulated in optical lattice are introduced, namely Bose-Hubbard and Hubbard model, together with basic theoretical background for each model. For given models, appropriate mapping to system of atoms in optical lattice is also given, and recent experimental results of simulations are presented.
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1 Introduction

An optical lattice is formed by the interference of counter-propagating laser beams. Resulting interference pattern creates effective periodic potential that may trap neutral atoms if they are sufficiently cooled. Such optical lattice with trapped atoms resembles real crystals (fig. 1a), where neutral atoms play a role of electrons in real crystal. Yet while in real crystals typical lattice dimensions are tiny, with atoms spaced around nanometre apart, optical lattice has typical dimensions about 1.000 times larger, thus providing enlarged version of real crystal. This analogy makes atoms in optical lattices very intriguing for solid-state physics. Real crystals are incredibly complex as many competing interactions, disorder and lattice vibrations must be taken into account. Therefore it is very hard to account for various experimentally observed phenomena. On the other hand, optical lattice provides an idealized version of crystal, where interactions can be controlled and finely tuned, thus providing a testbed for solid-state physics, where various theoretical models can be probed[1].

One might ask why not simply simulate given theoretical model on classical computer. It is the nature of quantum mechanics that makes simulations of many-body quantum systems intractable on classical computers due to exponential growth of the required space and time resources with the number of particles in a system. Nevertheless, ability to simulate such systems could help solving many open problems in various branches of physics, most notably condensed-matter physics. High-temperature superconductivity is an example of such problem. It has been intensively investigated in last decades, yet its theoretical explanation is still missing and is still subjected to many debates. There are many microscopic models that are suspected to be able to explain the phenomenon, though this can not be confirmed at present due to nonexistent analytical or numerical solutions of underlying models in thermodynamic limit, where large number of particles must be taken into account.

It was the simulation of many-body quantum system that gave rise to the idea of quantum computing. On quantum computer, such many-body problems could be efficiently simulated. Yet currently, practical quantum computer is very far from being reality. Therefore, alternative schemes for quantum simulations are also being investigated, which need not be as universal as quantum computation, but nevertheless enable simulation of specific models that are of theoretical interest. One way of simulating physical model is by mapping it to some alternative physical system, which can be easily manipulated and controlled, yet it is described by same mathematics. From this alternative system, properties of the original model can be extracted and its parameters can be finely tuned, so problem is essentially simulated. As we have already seen for the case of real crystal, cold atoms in optical lattice enable such simulation. Many important theoretical models can be mapped to cold atoms in optical lattice[2], which makes them one of the most promising quantum simulators.

In following sections, physics of cold atoms in optical lattices will be discussed first. Latter, few most prominent models that can be simulated in optical lattices will be presented, as well as recent experimental achievements.
Figure 1: Ultracold atoms in an optical lattice can for example simulate condensed-matter phenomena that usually occur only in the 'electron gas' of solid-state crystal. In an optical lattice (a), atoms are trapped in a sinusoidal potential well (gray) created by a standing-wave laser beam. The atoms’ wavefunctions (blue) correspond to those of valence electrons in a real crystal. In real crystal, (b), the periodic potential is caused by the attractive electrostatic force between the electrons and the ions. The motion and interaction of the particles, whether ultracold atoms or electrons, determine the physics of the material. Thus, for example, superfluidity in a gas of ultracold atoms corresponds to superconductivity in electron gas[1].

2 Optical lattice

2.1 Trapping potential

Atoms are trapped in optical lattice by means of the interaction between an induced electric dipole moment in an atom and electric light field from a laser. Electron in an atom in the presence of oscillating electric field of a laser attains a time dependent electric dipole moment. Given dipole moment interacts with the same electric field to produce an effective potential that traps atom in optical lattice. In the present section, details will be given on the simplified model of oscillator in classical radiation field [3].

When an atom is placed into laser light, the oscillating electric field $E$ induces an atomic dipole moment $p$ that oscillates at the driving frequency $\omega$. Electric field of the standing light wave can be expressed as a product of static spatial and oscillating time-dependent part, $E(r, t) = \hat{e}E(r)\exp(-i\omega t)$, while the dipole moment is $p(r, t) = \hat{e}p(r)\exp(-i\omega t)$, where $E(r)$ and $p(r)$ are amplitudes and $\hat{e}$ is a unit polarization vector. Amplitude of the dipole moment $p$ is simply related to the field amplitude $E$ by

$$p(r) = \alpha(\omega)E(r),$$

(1)

where $\alpha(\omega)$ is the complex polarizability, which depends on the driving frequency $\omega$. Higher-order moments (e.g. quadrupole) can be neglected.

The interaction potential of the induced dipole moment $p$ and the driving field $E$ is given by

$$V_{\text{dip}}(r) = -\frac{1}{2\varepsilon_0 c}\text{Re}(\alpha)I(r).$$

(2)

Field intensity is related to amplitude of electric field as $I = \varepsilon_0 c|E|^2$. Atoms will thus be attracted to minimas or maximas of light field intensity, depending on the sign of real part.
of complex polarizability. If field intensity $I(\mathbf{r})$ is periodic, there will be number of such minimas or maximas, therefore providing lattice potential for atoms.

Interaction between electric field and induced dipole moment also leads to absorption of electromagnetic energy by dipole. This is undesirable effect as it leads to heating of atoms in the lattice, which should be avoided. Atoms must be kept at ultra cold temperatures for them to be localized in weak interaction potential. Scattering rate for absorption of laser photons by dipole is related to field intensity by imaginary part of complex polarizability,

$$\Gamma_{sc} = \frac{1}{\hbar \epsilon_0 c} \text{Im}(\alpha) I(\mathbf{r}). \quad (3)$$

We have expressed two quantities of main interest for our discussion of optical lattices, effective potential $V_{dip}$ and scattering rate $\Gamma_{sc}$, in terms of light field intensity $I(\mathbf{r})$ and complex polarizability $\alpha(\omega)$. Field intensity can be easily calculated for a given configuration of laser beams. What remains is the calculation of polarizability of atom in lattice. It is in general very complex function of frequency, depending on internal electronic structure of atoms and requires quantum treatment. However, for driving frequencies that are far from transition frequencies of atom, it can be well described by classical Lorentz oscillator model, which consist of classical electron in a harmonic potential with frequency $\omega_0$ (relevant transition frequency of atom), driven by an external electric field of frequency $\omega$. Using given model to express $\alpha(\omega)$, effective potential and scattering rate can be expressed as

$$V_{dip}(\mathbf{r}) = \frac{3\pi c^2}{2\omega_0^3} \left( \frac{\Gamma}{\Delta} \right) I(\mathbf{r}) \quad (4)$$

$$\Gamma_{sc}(\mathbf{r}) = \frac{3\pi c^2}{2\hbar \omega_0^3} \left( \frac{\Gamma}{\Delta} \right)^2 I(\mathbf{r}), \quad (5)$$

where $\Delta = \omega - \omega_0$ denotes laser detuning - difference between laser frequency and frequency of relevant atomic transition. Value of $\Gamma$ is specific to relevant atomic transition.

From these simplified relations, following observations can be made that are of major importance for practical usage of optical lattices:

- **Sign of detuning**: If frequency of laser light is bellow atomic resonance frequency ("red detuning", $\Delta < 0$), dipole potential $V_{dip}$ is negative. Atoms are attracted to potential minima, that is region with maximum electric field intensity $I(\mathbf{r})$. For other case ("blue detuning", $\Delta > 0$), situation is opposite, and atoms are attracted to regions with minimal field intensity.

- **Scaling with intensity and detuning**: Usually, we would like to achieve dipole potential as large as possible. On the other hand, scattering of photons should be minimized as possible, as it introduces random noise into system and is also one of the main processes that dissipates heat into system. Dipole potential scales as $I/\Delta$, whereas the scattering rate scales as $I/\Delta^2$. Therefore, lasers with high intensity and large detuning from resonant atomic frequencies are usually used.
The presented classical treatment of an atom in optical lattice gives good quantitative explanation of phenomenon, and often also gives good estimates to experimental results. For higher precision, quantum mechanical treatment is required, which is out of scope of this seminar.

2.2 Lattice geometries

With different configurations of laser light sources, various geometries of trapping potential can be realized, which is very useful feature for simulations of condensed-matter systems. Interference of two laser beams, orientated in opposite directions, produces standing wave with period $\lambda_L/2$. By interfering more laser beams one can obtain 1D, 2D or 3D periodic potential. In each dimension, various geometries of lattice can be achieved by interfering lasers at different angles.

Arrangement of laser beams that correspond to a given physical lattice can be derived by writing periodic potential in the reciprocal lattice}

$$V(r) \propto I(r) = \sum_{K_j} \tilde{I}_{K_j} e^{iK_j \cdot r}, \tag{6}$$

where binding potential was taken to be proportional to the light intensity. The set of reciprocal vectors $K_j$ are defined as usual, according to the condition $R_i \cdot K_j = 2\pi\delta_{ij}m$, where $R_i$ are primitive vectors of a given lattice. On the other hand, if we write intensity of electric field as a sum of plane waves with equal frequencies, we get

$$I(r) = \frac{1}{4} \sum_{j,j'} E^*_j E_j (\epsilon^*_j \cdot \epsilon_j) e^{i(\phi_j - \phi_{j'})} e^{i(k_j - k_{j'}) \cdot r} + c.c. \tag{7}$$

If we take $k_j - k_{j'}$ as a reciprocal vector of the lattice, equation (7) has the same form as (6). Therefore, with properly arranged lasers with wavevectors, whose difference equals to reciprocal vectors $K_j$, approximation to given geometry of lattice can be attained. Relative phase difference between beams $\phi_j - \phi_{j'}$ leads only to overall translation of the lattice.

When considering experiments in optical lattices, one must also take into account that the lattice potential is not strictly periodic as given in eq. (7). It is superimposed by a harmonic potential due to the gaussian profile of laser beams (see fig. 2a). This additional harmonic confinement of atoms is usually weak (oscillation frequencies around 10-200 Hz), compared to confinement in each lattice site (oscillation frequencies around 10-40 kHz). However, when comparing experimental results to theoretical predictions obtained for ideal periodic lattice, this inhomogeneity must be accounted for.

2.3 Atoms in optical lattice

Almost any kind of atoms can be trapped into optical lattice, but alkali atoms are mostly used due to single valence electron, which simplifies description of their behavior in optical
Figure 2: Example of basic potential lattices that can be created by interference of laser beams in various dimensions. (a) Single laser beam creates potential, that is proportional to intensity of a beam. (b) Two intersecting laser beams create sinusoidal standing wave in one dimension. Ultracold atoms can be trapped in the potential minima. (c) With four lasers in two dimensional orthogonal setup, 2D lattice can be created. (d) With additional lasers, 3D lattices can also be created. [1]

lattice. In fact, their interaction with lattice is adequately described by previously given classical derivation of effective lattice potential.

For behavior of atomic gas in lattice, it is most crucial whether atoms are fermions or bosons. Fermions can not occupy the same quantum state due to Pauli exclusion principle, whereas bosons can. Specially at temperatures as low as required for trapping in optical lattice, this leads to very different statistical properties of trapped atoms. Whether particular atomic isotope is bosonic or fermionic depends on the number of its constituents: protons, neutrons and electrons. If its number is even, total spin of atom is integer, and atom is boson; if it is odd, total spin is half integer and atom is fermion. As bosons following isotopes are most commonly used: $^{87}\text{Rb}$, $^{23}\text{Na}$, $^{39}\text{K}$, $^{133}\text{Cs}$; and as fermions: $^{40}\text{K}$, $^{6}\text{Li}$, $^{87}\text{Sr}$.

For atomic gas to be bound to optical potential, it must be cooled to very low temperature. Its density also has to be very low. Those requirements come from two accounts. Atoms must be kept at temperatures and densities so low that no chemical binding can occur among atoms. Interaction between atoms in such regime is well described by van der
Walls force. On the other hand, thermal energies of atoms must be much lower than binding energy of lattice site. Typical temperatures that have to be attained are of sub-mK range, and densities of gas are between $10^{12}$ and $10^{15}$ particles per cm$^3$. To attain such low temperatures, sophisticated cooling techniques have to be used\[3\], laser cooling being the most prominent of them. Sufficiently low temperatures are much more difficult to attain in a gas of fermionic atoms than for bosons, mainly due to Pauli exclusion principle.

Trapping of atoms is usually done by cooling free atoms to a temperature where Bose-Einstein phase emerges if atoms are bosonic, or an equivalent cold gas of fermions. Then the intensity of laser light is slowly increased, and as result, atoms rearrange to adapt to given periodic potential.

Noninteracting atoms in optical lattice can be reasonably well described as being in an infinite periodic potential, for which solution techniques are well known from condensed-matter physics. For periodic potential $V_p(r + R) = V_p(r)$, exact eigenstates for single particle are given by Bloch functions, which are of the form $\psi_{n,k} = e^{ik \cdot r} u_{n,k}$. States are characterized by a discrete band index $n$ and a quasi-momentum $k$ within the first Brillouin zone of the reciprocal lattice. Band structure for a given potential can be calculated by standard methods, such as tight-binding method \[4\].

Structure of one-particle solutions is utilized in measurement techniques for atoms in optical lattice[2]. As an example, two techniques will be presented that enable direct measurement of momentum distribution and band population. Both are based on the ability to control the trapping potential by modifying the intensity of laser light.

- **Sudden release** If the lattice potential is turned off abruptly, a given Bloch state with quasi-momentum $k$ will expand in space as a superposition of plane waves with momentum $p_K = \hbar (k + K)$ where $K$ is reciprocal lattice vector. This is evident as Bloch state can be written as

$$\psi_{n,k}(r) = e^{ik \cdot r} u_{n,k}(r) = e^{ik \cdot r} \sum_K e^{iK \cdot r} \tilde{u}_{n,k}(K) = \sum_K \tilde{u}_{n,k}(K)e^{i(k+K) \cdot r}, \quad (8)$$

where we have used the Fourier transform of $u_{n,k}$. After certain time, density distribution of released particles can be measured using standard absorption imaging methods (fig. [4]). It can be shown that the measured density distribution is nothing but the momentum distribution of the particles trapped in the lattice[2].

- **Adiabatic mapping** This method also relies on releasing atoms from a lattice, but the potential in not turned off instantly. Intensity of lasers is adiabatically lowered, so that particles remain in eigenstates of this modified potential. Eventually lasers are turned off, and atoms are released into free space. Under adiabatic transformation of lattice depth, the quasi-momentum $k$ is preserved and Bloch wave in the $n$th energy band is eventually mapped onto corresponding free particle momentum $p$ in $n$th Brillouin zone(fig. [5]). Because in this method each Bloch wave is mapped onto a specific free-particle momentum state, it can be used to efficiently probe the distribution of the particles over Bloch states in different energy bands.
Figure 3: Simultaneous cooling of bosonic and fermionic quantum gas of $^7$Li and $^6$Li to quantum degeneracy. In the case of Fermi gas, the Fermi pressure prohibits the atom could to shrink further in space as quantum degeneracy is approached. In the case of Fermi gas, the Fermi pressure prohibits the atom could to shrink further in space as quantum degeneracy is approached [5].

Figure 4: (a) Schematic setup for absorption imaging after time-of-flight period. (b) Absorption image for Bose-Einstein condensate released from harmonic trap. (c) Absorption image for Bose-Einstein condensate released from a shallow optical lattice. The interference peaks occur as all particles are in state with $k = 0$ [2].

To give the feeling of parameters that are used in an experimental setup for probing ultracold gases in the optical lattice, we will give a quick review of the ground breaking experiment by Greiner et al. in 2002, which demonstrated transition from superfluid to insulating state in cold gas of bosonic atoms [7]. In experiment, spin-polarized samples of laser-cooled $^{87}$Rb atoms in the ($F = 2, m_f = 2$) state were used, where $F$ denotes the total angular momentum and $m_f$ the magnetic quantum number of the atom, which correspond to the electronic state $5S_{1/2}$. For cooling and trapping, optical transition $5S_{1/2} \rightarrow 5P_{3/2}$ is relevant, with frequency $\omega_0$ corresponding to a laser wavelength of 780nm. Sample of $2 \times 10^5$ atoms was cooled to temperatures bellow 20nK, which is well below temperatures where Bose-Einstein condensate emerges. Magnetic trap was used to localize Bose-Einstein condensate to spherical distribution with diameter of 26µm. Optical lattice was created by "red detuned" ($\Delta > 0$) laser diodes operating at wavelength of $\lambda = 852$nm. Effective potential of optical lattice attained in experiment can be described by simple cubic type geometry,

$$V(x, y, z) = V_0(\sin^2(2kx) + \sin^2(2ky) + \sin^2(2kz)),$$

(9)

where $k$ denotes the wavevector of laser light. Depth of potential $V_0$ is conveniently measured in units of the recoil energy $E_r = h^2k^2/2m$, where $m$ is mass of trapped atom. Maximal intensity of laser light attained in the experiment gave rise to potential depth of $22E_r$. Intensity was ramped up to its maximal value over a period of 80ms, which ensured that atoms remained in ground state of trapping potential. After raising the lattice potential, the condensate has been distributed over more than 150,000 lattice sites ($\sim 65$ lattice sites in single direction), with an average atom number of up to 2.5 atoms per lattice site.
Figure 5: Adiabatic mapping of crystal momentum onto free-space momentum of an atom. (a) Bloch bands for different potential depths. During an adiabatic ramp-down the crystal momentum is conserved. (b) A Bloch wave with crystal momentum $q$ in the $n$th energy band is mapped onto a free particle with momentum $p$ in the $n$th Brillouin zone of the lattice. (c) 2D Brillouin zone scheme for a 2D simple cubic lattice configuration. (d) Adiabatic mapping of statistical mixture of Bloch states within lowest energy band leads to the observation of box-shaped expanding atom cloud, corresponding to homogeneously filled central Brillouin zone. (e) When the higher energy bands are occupied, populations of higher Brillouin zones becomes visible. [6]

3 Simulations with optical lattices

We have reviewed basic physical principles governing behavior of atoms in optical lattice, so we can now focus to simulations of many-body quantum systems. Two models which are of great theoretical and historical importance will be introduced in this section. They are also distinguished by very direct mapping to physics of ultracold gas in optical lattice. Both models are used to describe weakly interacting particles in periodic potential. In case of the so called Bose-Hubbard model, particles are bosons, and in case of Hubbard model, particles are fermions.
Figure 6: In a 2D Bose-Hubbard model two distinct ground states exist. (a) For weak interaction strengths, the ground state is a superfluid. The atoms have a common macroscopic wavefunction, with particles fully delocalized throughout the available space. After the atoms are released from the lattice, the state is characterized by the interference pattern in the measured particle density. (b) When interaction becomes stronger, system transitions to the Mott insulator state, in which each particle becomes localized to one specific lattice site. Interference pattern in the particle density disappears\cite{6}.

3.1 Bose-Hubbard model

Bose-Hubbard model provides good description of cold bosonic atoms in the limit of sufficiently deep optical lattice. Model is essentially an extension of tight binding approximation, also taking interparticle interaction into account. If interparticle interactions become increasingly stronger relative the kinetic energy of particles, system can no longer be described by simple matter wave, as in the case of noninteracting particles. The resulting strongly correlated quantum states are difficult to handle theoretically as they can not be reduced to the effective one-particle theory.

If one considers only occupations of the lowest Bloch band (which is true if thermal and mean interaction energies are much lower than the separation to the first excited band), Bose-Hubbard hamiltonian is given as

\[
H = -J \sum_{\langle ij \rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1). \tag{10}
\]

The first term is the kinetic energy, describing the tunneling coupling of strength $J$ between neighboring lattice sites $\langle i, j \rangle$, $\hat{a}_i^\dagger$ ($\hat{a}_i$) are bosonic creation/destruction operators that create (destroy) a particle on lattice site $i$, taking appropriate symmetry of bosons into account. The second term describes on-site interaction between the particles, with $\hat{n}_i$ counting the number of particles on site $i$. Only interaction between atoms on the same site on lattice

10
is taken into account, giving rise to the interaction energy $U$. Such interaction term works well for ultracold neutral atoms in periodic potential, as their interactions are short-ranged and no long-range Coulomb forces exist between particles.

Bose-Hubbard model exhibits two prominent ground states. For weak on-site interaction relative to the kinetic energy, $U/J \ll 1$, the system exhibits Bose-Einstein condensation, which is a superfluid state of matter as particles can move without friction and are delocalized over the entire lattice. Superfluid state is accompanied by an interference pattern in the measured particle density after sudden release of atoms from optical lattice. When the lattice ‘depth’ is increased (intensity of the effective lattice potential), the role of on-site interaction between atoms compared to kinetic energy of tunneling between sites becomes decisive ($U/J \gg 1$) and the system transitions into what is known as a Mott insulator state. Particles can not move freely along the lattice anymore because each particle is effectively localized to one lattice site. Interference pattern that characterized superfluid phase is absent in the Mott insulator state.

Initially, it was the Bose-Hubbard model that was most extensively studied with optical lattices, mainly due to relative ease of practical implementation. Compared to other models of strongly interacting many-body physics, theoretical aspects of Bose-Hubbard model are also well known, so experimental results could be compared to numerical predictions. Transition in Bose-Hubbard system from superfluid phase to Mott insulator state was indeed observed in optical lattices, as predicted by numerical calculations (fig. 6).

In the case of Bose-Hubbard model, optical lattice was used to confirm existing numerical and theoretical predictions, so simulation with optical lattice did not lead to any new results. Situation is different as soon as we introduce extensions to given model, which are not so well understood theoretically. One of such extension is Bose-Hubbard model for atoms with multiple spin states. As different types of interactions compete, each favoring a different type of magnetic ordering, exciting new states such as ‘spin liquids’ are expected to arise. For the simplest case of spin-1 atom, hamiltonian is given as

$$H = -J \sum_{\langle ij \rangle, \sigma} \hat{a}_{i,\sigma}^\dagger \hat{a}_{j,\sigma} + \frac{1}{2} U_0 \sum_{i} \hat{n}_i (\hat{n}_i - 1) + U_2 \sum_{i} \hat{S}_i^2, \quad \hat{n}_i = \sum_{\sigma=-1}^{1} \hat{n}_{i,\sigma},$$  \(11\)

where $S_i^2$ denotes square of total spin operator at $i$th site and $\hat{a}_{i,\sigma}^\dagger (\hat{a}_{i,\sigma})$ are creation(destruction) operators for particle with spin $\sigma$ at $i$th lattice site.

There is also a range of models that can be mapped to certain limits of the Bose-Hubbard model. One of the simplest mappings exist for $XY$ model of spin chain in transverse field, which is defined as

$$H = -t \sum_{\langle ij \rangle} \left[ \hat{S}_i^x \hat{S}_j^x + \hat{S}_i^y \hat{S}_j^y \right] - \mu \sum_{i} (\hat{S}_i^z + 1).$$  \(12\)

$S^x$, $S^y$ and $S^z$ are usual spin 1/2 operators, and $\langle ij \rangle$ denotes summing along nearest neighbors. In the limit of high on-site interaction in the Bose-Hubbard model ($U \gg t$), lattice site can have at most one boson per site. We can therefore encode the spin 1/2 states as a
presence (↑) or absence (↓) of boson at the site \[\mathcal{V}\]. Spin models are of great importance in condensed-matter physics as they are believed to explain many anomalies in thermodynamic and transport properties of specific solids, so efficient simulations would provide valuable insight.

### 3.2 Hubbard model

![Figure 7: Fermi surfaces of fermionic \(^{40}\text{K}\) atoms in 3D optical lattice, recorded by adiabatic ramp down of the optical potential. Density of atoms in optical lattice is increased until lowest Bloch band is completely filled and shape of Fermi surface is revealed. Number of \(^{40}\text{K}\) atoms in trap is in range of \(\sim 10^5\), and temperature \(T = 52\) nK, which is well below Fermi temperature \(T_F\) of free fermionic gas. [9]](image)

Hubbard model is a fermionic counterpart of Bose-Hubbard model for bosons. It was originally proposed in 1963 to describe electrons in solids. With the discovery of high-temperature superconductivity, it stood out as one of the simplest and promising models that could explain this phase transition, which takes place at temperatures much higher that relatively well understood ordinary superconductivity[10]. Hubbard model was exactly solved in 1D case, where a transition from insulating to conducting state occurs. On the other hand, solutions for 2D and 3D case are not known, as analytical techniques used in 1D case can not be extended to higher dimensions while numerical simulations gets bogged down by the already mentioned issues of many-body simulations. Importance of the model in condensed-matter physics makes it specially appealing for simulation with ultracold atoms in optical lattice.

If atoms are confined to the lowest energy band and there are only two possible spin states \(|\uparrow\rangle, |\downarrow\rangle\) for the fermionic particles, Hubbard hamiltonian reads:

\[
H = -J \sum_{\langle ij \rangle, \sigma} \hat{c}_{i,\sigma}^\dagger \hat{c}_{j,\sigma} + U \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow},
\]

where \(\hat{c}_{i,\sigma}^\dagger(\hat{c}_{i,\sigma})\) are fermionic creation(destruction) operators for spin state \(\sigma\) at site \(i\). First term is due to the tunneling of atoms to neighboring sites while the second term is due to on-site interaction. Analogy to the Bose-Hubbard model is obvious, yet due to Pauli exclusion principle for fermions, this model exhibits vastly different behavior than the bosonic counterpart.

Due to the fermionic nature of particles, experimental realization of optical lattice is more cumbersome, as cooling of fermionic atoms to temperatures in the range of sub-mK proves...
to be much more difficult as for bosonic atoms. Nevertheless, progress is being made at a rapid pace. Just recently, a sample of ultracold Fermi gas has been successfully loaded into optical lattice, thus realizing a good approximation of the 3D Hubbard model[11]. These first experiments have revealed tantalizing evidence of the metal-insulator transition when varying interaction between particles, analogous to the one observed in Bose-Hubbard model. Many theoretical proposals are also being made to extend the range of parameters for which Hubbard model can be probed, and experimental results are expected to follow in near future.

4 Conclusion and outlook

Ultracold atoms in optical lattices have already proved to be very versatile system with which we can simulate various many-body problems. We have presented only a fraction of models that can be simulated in optical lattice. They can be as exotic as mixture of bosons and fermions in lattice, which are expected to exhibit various new phenomena. An intense line of research is also to introduce long-range interactions into optical lattices by trapping molecules with a dipolar ground state instead of atoms that interact through electric dipole-dipole forces. Many efforts are also directed into improving experimental techniques for cooling and measuring atomic gas in optical lattice.

In addition to usage of optical lattices as simulators, there are also many other applications. Because as few as one or two atoms can be isolated in each of the small traps, they can be also used to probe atomic interaction with very high precision. Even formation of molecules from atoms in separate lattices sites can be highly controlled, giving valuable tool to field of ‘micro-chemistry’. Last but not least, atoms in optical lattice are considered as one of more promising systems for practical implementation of a quantum computer. For this goal to be achieved though, great improvements must be made in experimental methods because quantum computation is very sensitive to even smallest disturbances from environment.

Due to wide range of applications and very promising recent experimental results, future research in optical lattices is expected to provide many breakthroughs in various fields of physics. As such it is expected to be very active field of research in years to come.

References


