Abstract: When looking at the properties of an electromagnetic medium under a coordinate transformation an alternative interpretation presents itself. One can simulate the electromagnetic behavior in non-flat space by choosing a material with different electromagnetic properties in Cartesian space. The material can therefore simulate behavior in spaces with holes resulting in electromagnetic invisibility. I will explain the theory behind this mechanism and describe the experiments that tried to build an invisibility device.
1 Introduction

The bending of light in media can create various optical illusions. Some can appear in nature, like a mirage in the desert. Here light rays are bended, so they are reflected back to the sky instead of going straight to the ground. The ground therefore seems invisible from a certain angle. But to make a perfect invisibility device one needs more than that. Because of the wave nature of light it is possible to hide object that are smaller than the wavelength, but that is not our goal. To make the perfect invisibility device, light would have to be guided around the object, so it would seem to the viewer that the object is not there.

The first proposal of such a mechanism was made only a few years ago [1], [2]. It is based on the fact that the Maxwell equations have the same form in all coordinate systems. A coordinate transformation therefore only changes the values of the electrical and magnetic field as well as the permittivity ($\varepsilon$) and permeability ($\mu$) tensors. By choosing a different set of permittivity and permeability tensors, one can therefore simulate the behavior in a different coordinate system, like a coordinate system with a hole, where a part of space is invisible for electromagnetic waves.

Since the idea is interesting and relatively simple, there have also been attempts to build such a cloak [3]. The permittivity and permeability tensors needed to make the cloak are anisotropic and too complex to appear in nature. However, the development of metamaterials has made it possible to construct materials whose permittivity and permeability change independently and arbitrarily. The main interest in developing metamaterials in the first place was making structures that would have a negative index of refraction. But metamaterials can also be made anisotropic and their electromagnetic properties can be changed over a large range of values, so they have also proven useful in constructing invisibility devices.

2 Theory

Let us now look at the theory behind making an invisibility cloak. The equations we need are of course Maxwell equations in a space without sources or currents. Therefore we have:

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$
$$\nabla \times \vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Where the permittivity ($\varepsilon$) and permeability ($\mu$) are generally tensors. Since these equations are invariant under coordinate transformations, their form does not change in a new coordinate system and we get:

$$\nabla' \times \vec{E}' = -\mu'_0 \frac{\partial \vec{H}'}{\partial t}$$
$$\nabla' \times \vec{H}' = \varepsilon'_0 \frac{\partial \vec{E}'}{\partial t}$$

Where the $\nabla'$ of course means derivation in the new coordinates. The other quantities transform as:

$$\vec{E}' = (A^T)^{-1} \vec{E} \; , \; \vec{H}' = (A^T)^{-1} \vec{H}$$
$$\mu' = \frac{1}{\text{det} A} A \mu A^T \; , \; \varepsilon' = \frac{1}{\text{det} A} A \varepsilon A^T$$

Where the matrix $A$ and its inverse have elements:

$$A_{ij} = \frac{\partial x'_i}{\partial x_j} \; , \; A^{-1}_{ij} = \frac{\partial x_i}{\partial x'_j}$$
The easiest way to prove the above transformed equations is to insert the new values and show that the transformed equation holds. Since both the Maxwell equations have the same form, it is enough that we show it for one. Let us take the Fourier transform of the first equation:

\[ \nabla' \times \vec{E}' = -i\omega' \mu_0 \vec{H}' \]

If we now insert the transformed quantities we get:

\[ -i\omega' \mu_0 \vec{H}' = -\frac{1}{\det A} A \mu A^T (A^T)^{-1} \vec{H} = \frac{1}{\det A} A \nabla \times \vec{E} \]

In index notation, where \( \epsilon \) is the Levi-Civita tensor, the above equation reads:

\[
-i\omega' \mu_0 (\mu' \vec{H}')_i = \frac{1}{\det A} \frac{\partial x'_l}{\partial x_j} \epsilon_{jkl} \frac{\partial E_l}{\partial x_k} = \frac{1}{\det A} \frac{\partial x'_l}{\partial x_j} \epsilon_{jkl} \frac{\partial x'_m}{\partial x_k} \frac{\partial E_l}{\partial x'_m} \]

We need to compare the above equation with:

\[
(\nabla' \times \vec{E}')_i = (\nabla' \times (A^T)^{-1} \vec{E})_i = \epsilon_{ijk} \frac{\partial}{\partial x'_j} \left( \frac{\partial x_l}{\partial x'_k} E_l \right) \]

The first term in the last equation vanishes during summation, because the Levi-Civita tensor is antisymmetric and we can swap the dummy indices \( j \) and \( k \).

The terms in Eq.(1) and Eq.(2) will be identical if:

\[
\epsilon_{jkl} \frac{\partial x'_l}{\partial x_j} \frac{\partial x'_n}{\partial x_k} = (\det A) \epsilon_{ink} \frac{\partial x_l}{\partial x'_k} \]

Multiplying both sides by matrix \( A \) (multiplying by \( \frac{\partial x'_l}{\partial x_i} \) and summing over \( l \) on the right side) we get:

\[
\epsilon_{jkl} \frac{\partial x'_l}{\partial x_j} \frac{\partial x'_n}{\partial x_k} \frac{\partial x'_p}{\partial x'_l} = (\det A) \epsilon_{inp} \]

Both sides of the equation are antisymmetric in indices \( i, n \) and \( p \). The right side is antisymmetric because the Levi-Civita tensor is antisymmetric. On the left side we notice, that swapping indices \( i \) and \( n \) is the same as swapping indices \( j \) and \( k \) (and similar for all other pairs). Therefore the left side is also antisymmetric in \( i, n \) and \( p \). Because of this it suffices to show that the equation holds for one set of \( i, n, p \). If we take \( i = 1, n = 2 \) and \( p = 3 \), we get the known formula for the determinant:

\[
\det A = \epsilon_{jkl} \frac{\partial x'_l}{\partial x_j} \frac{\partial x'_n}{\partial x_k} \frac{\partial x'_3}{\partial x'_l} = \epsilon_{jkl} A_{1j} A_{2k} A_{3l} 
\]

We have shown that permittivity and permeability really transform as:

\[
\varepsilon' = \frac{1}{\det A} A \varepsilon A^T, \quad \mu' = \frac{1}{\det A} A \mu A^T \]

With:

\[
A_{ij} = \frac{\partial x'_i}{\partial x_j} \]

There are two ways to interpret the electromagnetic behavior in the transformed space. The obvious way is that both sides of equation (3) represent the same material properties, but in
different coordinate systems. The quantities change because of the topology of the transformation.

The alternative interpretation is that both sides of the equation (3) represent different material properties, but both in the same space. We can therefore imagine a space with a property we would like to implement (a hole for example) and then construct the coordinate transformation that would implement the desired behavior in the flat, Cartesian space. Using equation (3) we can calculate the material properties that would give us this behavior.

2.1 Spherical cloak

If we wish to make an invisibility cloak we have to imagine a space with a hole. The simplest cloak is a spherical one and the transformation is a spherically symmetric transformation that compresses the space in a volume of radius \( b \) into a shell of inner radius \( a \) and outer radius \( b \). The transformation outside of radius \( b \) is the identity transformation. It is important to always apply the transformation only to a limited region if we wish to make a cloak of finite dimensions.

A position vector in the original (Cartesian) space has components \( x^i \) and in the transformed space it has coordinates \( x'^i \). The magnitude is the same in both spaces:

\[
r^2 = x^i x^j \delta_{ij} = x'^i x'^j g_{ij}^t
\]

Where \( g_{ij}^t \) is the metric of the transformed space. But in our interpretation we will take the components \( x'^i \) to be the components in the Cartesian space. Also the magnitude (\( r' \)) is calculated in the Cartesian space:

\[
r'^2 = x'^i x'^j \delta_{ij}^t
\]

Our transformation maps points from a radius \( r \) to a radius \( r' \) according to the function:

\[
r' = \frac{b - a}{b} r + a
\]

We notice that when \( r = 0 \) then \( r' = a \) and also when \( r = b \) then \( r' = b \).

To have a better idea of the coordinate systems and position vectors, Fig.1 (A) shows a position vector and a ray in a flat Cartesian space (A), in a transformed space (B) and in a flat space with a material that simulates the behavior in a transformed space (C).

Since our transformation is radially symmetric it must hold:

\[
\frac{x'^i}{r'} = \frac{x^i}{r} \delta^t_i
\]

If we insert the relation between \( r \) and \( r' \) (Eq.(5)) we get:

\[
x'^i = \frac{b - a}{b} x^i \delta^t_i + a \frac{x^i}{r} \delta^t_i
\]

Now that we have this relation we can calculate the transformation matrix \( A^t_{ij} \). With derivation we get:

\[
A^t_{ij} = \frac{\partial x'^i}{\partial x_j} = \frac{b - a}{b} \delta^t_j + \frac{a}{r} \delta^t_j - \frac{a}{r^3} x^k \delta_{kj} \delta^t_i
\]

The components of the above expression are:

\[
A^t_{ij} = \begin{pmatrix}
\frac{r'}{r} - \frac{ax^2}{r^3} & -\frac{axy}{r^3} & -\frac{axz}{r^3} \\
-\frac{ayx}{r^3} & \frac{r'}{r} - \frac{ay^2}{r^3} & -\frac{ayz}{r^3} \\
-\frac{azx}{r^3} & -\frac{azy}{r^3} & \frac{r'}{r} - \frac{a^2}{r^3}
\end{pmatrix}
\]
Figure 1: The blue line shows the path of the same ray in flat space (A), transformed space (B) and flat space with a material simulating the transformed space (C). Shown is also a position vector $\vec{x}$ in flat (A) and transformed (B) systems and a vector $\vec{x}'$ where transformed components are interpreted in flat space (C). [2]

To calculate the determinant of this matrix we can rotate into a coordinate system where:

$$x^i = (r, 0, 0)$$

Here all of the off-diagonal elements vanish and we get for the determinant:

$$\det(A^j_i) = \frac{r' - a}{r} \left( \frac{r'}{r} \right)^2$$

To get the desired material properties for a spherical invisibility cloak we use Eq.(3). If our original space is free space than permittivity and permeability are both equal to 1. After multiplying the matrices we get:

$$\varepsilon^{i'j'} = \mu^{i'j'} = \frac{b}{b - a} \left( \delta^{i'j'} - \frac{2ar' - a^2}{r^4} x^{i'} y^{j'} \right)$$

Again, using the proper rotation, the off-diagonal elements vanish and we get for the material properties:

$$\varepsilon_r = \mu_r = \frac{b}{b - a} \left( \frac{r' - a}{r'} \right)^2$$

$$\varepsilon_\theta = \mu_\theta = \frac{b}{b - a}$$
With the calculated material properties one can now calculate the light propagation in a space with the spherical cloak. This can be done in two ways. Since we have the material properties, we can solve the Maxwell equations. But the result must be equal if we just take the behavior in flat space and transform it with the same transformation used for the space transformation. The ray propagation in a spherical cloak is shown in Fig. 2; we can see how the light bends around the inner, ‘invisible’, shell.

Figure 2: Ray behavior in a spherical cloak. The medium lies between the two spheres. [2]

### 2.2 Cylindrical cloak

Let us now analyze a cylindrical cloak. The reason for this is that it is much easier to build a cylindrical cloak than a spherical one. The transformation is the same as that of the spherical case, only that it’s now applied only to the two dimensions normal to the cylinder’s axis. For this reason we define two operators, one that projects to the z-axis and the other one that projects to the x-y plane:

\[
Z^{ij} = \delta_3^i \delta_3^j
\]

\[
T^{ij} = \delta_1^i \delta_1^j + \delta_2^i \delta_2^j
\]

Since the transformation in the x-y plane is the same as in the spherical case and the transformation in the z direction remains the identity, the transformation matrix remains:

\[
A_j^{ij} = \frac{r^j}{r} T_j^{ij} - \frac{a r^k r^h \delta_1^i \delta_2^j}{r^3} + Z_j^{ij}
\]

(9)

Where \( r \) is now the distance from the z-axis. The matrix written out in components is:

\[
A_j^{ij} = \begin{pmatrix}
\frac{r^j}{r} - \frac{a x^2}{r^3} & \frac{-a x v}{r^3} & 0 \\
\frac{-a x v}{r^3} & \frac{r^j}{r} - \frac{a y^2}{r^3} & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

(10)
Again, by rotating into a system where the off-diagonal elements vanish, we can get the determinant:

$$\det(A'_{ij}) = \frac{r' - a}{r}$$

By using Eq. (3) and considering that our original space has both permittivity and permeability equal to 1, we get the formula for the material properties in the transformed space:

$$\varepsilon = \mu = \frac{r}{r - a} - \frac{2ar - a^2}{r^2(r - a)} \mathbf{r} \otimes \mathbf{r} + \left( \frac{b}{b-a} \right)^2 \frac{r - a}{r} Z$$

(11)

With the rotation into a system where the off-diagonal elements vanish we get the components for the material properties:

$$\varepsilon = \mu = \frac{r - a}{r}$$

(12)

$$\varepsilon = \mu = \frac{r}{r - a}$$

(13)

$$\varepsilon = \mu = \left( \frac{b}{b-a} \right)^2 \frac{r - a}{r}$$

(14)

3 Experiments

Soon after the first proposal of the invisibility cloak was published [1], there were attempts to build it. The first one was in the same year [3]. The theory gives us the material properties needed, but the practical application opens questions about the materials to use and the effects of the approximations. Although one can calculate the needed material properties for any shape of cloak, they are anisotropic and can have large ranges of values. Therefore artificial materials are needed to satisfy them, but even they cannot completely implement the desired properties and experimenters are forced to use simplifications. Computer simulations are a useful tool for analyzing the effects of such approximations.

3.1 Reduced material properties

Although the equations for the material properties for the cylindrical cloak Eq. (12), (13) and (14) seem simple, they still represent six different parameters that are a function of radius ($r$). That makes the material very anisotropic and too complicated to realise. To simplify the required properties the experiment was performed where the electric field was polarized along the z (cylinder) axis. In this case only the components $E_z$, $H_r$ and $H_\phi$ are nonzero and therefore only $\varepsilon_z$, $\mu_r$ and $\mu_\phi$ are relevant.

Further, the Maxwell equations inside the material now reduce to (written in component form):

$$\left( \nabla \times \bar{H} \right)_z = \frac{1}{r} \left( \frac{\partial (rH_\phi)}{\partial r} - \frac{\partial H_r}{\partial \phi} \right) = \frac{\partial D_z}{\partial t}$$

$$\left( \nabla \times \bar{E} \right)_\phi = \frac{\partial E_z}{\partial r} = -\frac{\partial D_r}{\partial \phi} = -\frac{\partial (\varepsilon_\phi H_\phi)}{\partial t}$$

$$\left( \nabla \times \bar{E} \right)_r = \frac{1}{r} \frac{\partial E_z}{\partial \phi} = \frac{1}{r} \frac{\partial D_\phi}{\partial \phi} = -\frac{\partial (\varepsilon_r H_r)}{\partial t}$$

If $\varepsilon_z$ is not a function of $r$ then the above equations depend on only two material parameters ($\varepsilon_\phi \mu_\phi$ and $\varepsilon_r \mu_r$) instead of three. Now we can choose one of the original parameters arbitrarily to achieve some favorable conditions. A good choice is to select $\mu_\phi = 1$, which gives us $\varepsilon_z = \left( \frac{b}{b-a} \right)^2$ and $\mu_r = \left( \frac{r-a}{r} \right)^2$. This makes only the $\mu_r$ spatially inhomogeneous. We will refer to these properties as reduced material properties:
\[ \mu_\phi = 1 \] (15)
\[ \mu_r = \left( \frac{r - a}{r} \right)^2 \] (16)
\[ \varepsilon_z = \left( \frac{b}{b - a} \right)^2 \] (17)

Using these simplifications instead of the exact functions does not change the light propagation inside the material, but it does change the behavior on the interface with free space. This cloak now has a nonzero reflectance.

### 3.2 Cloak design and metamaterials

The material specification in Eq.(15) - (17) is simple enough to be realizable, but it is still anisotropic and special materials are needed. As can be seen, the parameters \( \mu_\phi \) and \( \varepsilon_z \) are constant and \( \mu_r \) varies radially. The cloak described in the experiment in [3] was built from split-ring resonators (SRRs).

![Figure 3: SRR design. Parameters \( a_\theta \) (l plus the spacing between SRRs), l and w are kept constant, but changing the parameters r and s changes the effective values for \( \varepsilon_z \) and \( \mu_r \). [3]](image)

A split-ring resonator looks like a ring or a square with a small segment removed, which leaves a gap. Usually two concentric rings are present with the gap placed in opposite directions, but in our case the SRRs had only one ring (Fig.3). To construct a metamaterial multiple SRRs are placed next to each other on a plane and then more planes are put together in layers to get a 3D structure. For wavelengths larger than the size of the SRRs the electromagnetic response of the structure can be described with an effective \( \mu_{eff} \) and \( \varepsilon_{eff} \) that is a function of frequency. Changing the shape of the SRRs, one can therefore obtain the desired value for the \( \mu_{eff} \) at a certain frequency. The behavior of \( \mu_{eff} \) depends on the shape and properties of individual SRRs, but the shape of the function is similar for all split rings. Each individual ring acts as a
small LC-circuit and the incident light coupling to its resonance produces the response that is described by $\mu_{\text{eff}}$. The frequency dependency of a SRR array has the following shape:

$$\mu_{\text{eff}}(\omega) = 1 - \frac{\omega^2_{mp}}{\omega^2 - \omega^2_0 + i\Gamma\omega}$$

Fig. 4 shows the real and imaginary parts of $\mu_{\text{eff}}$ of a typical SRR medium near its resonance. As seen, the real part covers a wide range of values near the resonance, including negative values in a small bandwidth. This is the source for making structures with negative index of refraction, but for making an invisibility cloak, we only wish to make an anisotropic material. This is easily done, since the structure with SRRs all aligned in the same direction only affects the magnetic fields that penetrate the plane of the rings. The above described metamaterial is therefore already anisotropic. To make an isotropic material one would have to position the SRRs in the structure in all three dimensions.

![Figure 4: Shape of the functions of real and imaginary parts of permeability for a typical SRR medium near resonance. [4]](image)

The invisibility cloak was built from 10 concentric shells with the axes of the SRRs positioned along the radial direction (Fig.5), so they only affected the radial component of $\mu$ as needed. The values for the inner ($a$) and outer ($b$) radius of the cloak were $a = 27.1\ mm$ and $b = 58.9\ mm$ and the optimal frequency used was 8.5 GHz (microwaves). The wavelength to use had to be big compared to the distance between the shells of the cloak, but not too big compared to the size of the cloak itself. Also the electromagnetic properties of the SRRs are frequency dependant, so the cloak was only designed to operate in a small frequency interval. Further, the SRRs also have a small imaginary component of the material parameters, so the cloak has a non zero absorption.

3.3 Results and simulations

The cloak has been tested and the electric field around the cloak has been measured and compared with the computer simulations. The object to be shielded was a Cu cylinder that exactly fit the inner radius of the cloak. Fig.6 shows the results, including the simulation of the cloak with the exact material parameters (A) and measurements with the bare Cu cylinder without the cloak (C). The measurements are in agreement with the simulation for the cloak with the
Figure 5: Design of the invisibility cloak with 10 shells and the diagram of the SRRs of the inner and outer shells. Included are also the plots for the values of $\mu^\phi$ (constant value 1), $\varepsilon^z$ (constant value of 3.423) and $10 \mu_r$ in the region from $a = 27.1$ mm to $b = 58.9$ mm. [3]

reduced material parameters, especially the light propagation inside the cloak. Also apparent is the reduction of backward (reflection) and forward (shadow) scatter of the cloaked cylinder compared to the bare one.

The approximations one needs to consider when making a cloak can be divided into three parts: non zero absorption, need for stepwise approximation of the cloak and using reduced properties instead of exact. When making a cloak all three effects combine, but with simulations one can observe each effect individually (Fig.7).

In the ideal case (Fig.7 (A)) the cloaking is perfect. There is no shadow and no reflection, so the plane wave outside the cloak is the same as if no object was present.

The presence of absorption is apparent in case on Fig.7 (B). Here the loss tangent (defined as $\tan \delta \frac{Im\varepsilon}{Re\varepsilon}$ and similar for $\mu$) is 0.1 for the electric and magnetic loss. The shadow behind the cloak is clearly visible and expected, but, as in the ideal case, there is no backscatter, so the plane wave on the left of the cloak is again unaltered. With proper materials one can achieve the loss tangent much smaller than 0.1.

Plot in Fig.7 (C) shows the case when the ideal cloak is approximated by eight layers. Here we see that the plane wave on all sides of the cloak is perturbed, so we have forward as well as backward scattering. But the effects are not too big and most of the light is still bent around the cloak. Similar also is the behavior of the cloak with reduced parameters (Fig.7 (D)), which also has a non zero reflectance as expected, because of the simplified properties.
Figure 6: Simulations and measurements of the electric field and power flow (black lines) around the cloak. A: simulation of the exact material parameters; B: simulation of the reduced material parameters, approximating the cloak with 10 shells and using absorption loss; C: measurements of the bare cylinder without the cloak; D: measurements of the cylinder with the cloak. [3]

Figure 7: Simulations of the electric field around the cloak. A: exact material parameters; B: exact parameters with a loss tangent of 0.1; C: eight-layer stepwise approximation of exact parameters; D: reduced material parameters. [5]
3.3.1 Cloak at optical frequencies

In the previously described experiment the cloak was designed for microwave frequencies. The main goal of the research is to make a cloak that would work with visible light, but the above mentioned design is not appropriate for this, because SRRs cannot be made small enough compared to the wavelength. Also the magnetic properties of SRRs depend on the frequency of the light, so the cloak only operates in a small bandwidth.

To try to overcome these problems other cylindrical cloak designs have been proposed: [6], [7]. A useful simplification is to try to avoid magnetic properties of the material with the use of light where magnetic field (instead of electric) is polarized along the z-axis. Now the relevant material parameters of the cylindrical cloak are $\mu_z$, $\varepsilon_\phi$ and $\varepsilon_r$ and, since we can choose one arbitrarily, we decide to make $\mu_z = 1$. This removes the need for magnetic response of the material. The other two parameters also change, so the new properties are now (compare with previous Eq.(15), (16) and (17)):

$$\mu_z = 1 \quad (18)$$

$$\varepsilon_\phi = \left(\frac{b}{b-a}\right)^2 \quad (19)$$

$$\varepsilon_r = \left(\frac{b}{b-a}\right)^2 \left(\frac{r-a}{r}\right)^2 \quad (20)$$

But still, even making a cloak with these properties is a demanding task and there have been no attempts yet. The material is still anisotropic and, since the parameters are just a different set of reduced parameters, the cloak would still have nonzero reflectance. Recently, there has been a lot of interest in making a ‘carpet cloak’ (also called ground cloak) [8]. Here the cloak is placed on a mirror with a bump. If the cloak works properly, the light should reflect from the mirror as if it was flat, therefore making the area under the bump invisible (Fig.8).

![Figure 8: Diagram of the carpet cloak. The reflection from a mirror with a bump is distorted (b). But if a cloak is put on the bump, the light should reflect the same as from a flat mirror, therefore making the bump and what is under it invisible (c). [9]](image)

The benefits of such a cloak are that it does not require materials with a wide range of permittivity and permeability. The values for the cylindrical cloak can go to infinity (Eq.(13)) or zero (Eq.(12)) near the inner radius, which is difficult to realise in practice. Another important advantage of the carpet cloak is that choosing a suitable coordinate transformation the anisotropy of the materials can be minimized to a small enough value to be neglected. The cloak can be therefore made from isotropic dielectrics instead of metamaterials. Unlike with the cylindrical cloak, when we started with the transformation and calculated the material parameters, here the process is reversed. One imagines the shape of the cloak and then calculates the transformation that minimizes the anisotropy of the material. Now the
values for the material parameters have to be calculated numerically. Values of the parameter $n^2$ defined as:

$$n^2 = \frac{\varepsilon}{\varepsilon_{\text{background}}}$$

for two different transformations are plotted in Fig.9. The parameter $\varepsilon_{\text{background}}$ is the permittivity of the background (1 in free space). If the anisotropy is small enough to be neglected (the case in Fig.9 (b)), then the dielectric profile of the medium is described by the above equation and its magnetic permeability is equal to 1. Here the task was to find the transformation that would map a square into a square with a ‘bump’ and give the smallest amount of anisotropy in the medium. But as we can imagine, the shape of the cloak does not have to be a square.

Figure 9: The color map of $n^2$ for the carpet cloak for two different transformations. a: the area above the bump is just compressed so the vertical lines remain parallel. b: the grid lines remain orthogonal to each other. This transformation also minimizes anisotropy. [8]

There have also been experiments to build a carpet cloak [9], [10]. Both used similar techniques and dimensions, but one used a triangular shape of the cloak and the other used a square shape. The cloak was built from either Si with holes or SiO$_2$ with small Si posts and the desired distribution of the index of refraction was achieved by varying the density of holes or Si posts. The background index was 1.58 and 1.65 respectively, so the range of the index used in the cloak was easier to build. Both cloaks were small with the area around 100µm$^2$ and the area of the bump around 1µm$^2$. Fig. 10 shows the diagram and a photo of the cloak made from Si with holes.

Both cloaks were tested using infrared light with wavelength of 1400 nm - 1800 nm and the distribution of reflected light was in both cases similar to the case with a flat reflective surface, so the bump was successfully hidden. Fig. 11 shows the measurements and simulations of the reflection of a Gaussian beam from a flat mirror, a mirror with a bump and the same mirror with the bump covered with a cloak. The experiment shows little difference between the
Figure 10: The design of the carpet cloak. a: region C2 is the background with uniform index of 1.58. Region C1 is the actual cloak with a gradient index. b: image of the cloak. The width and depth of the bump are 3.8 \( \mu m \) and 400 nm. [9]

reflection from a cloaked bump and a flat mirror as well as good agreement of simulations with the measurements.

Figure 11: Cloaking at a wavelength of 1540 nm. Reflection of a Gaussian beam. The measurements are in the left column and the simulation is in the right column. a: flat surface without the cloak. b: surface with a bump and without the cloak. c: bump with a cloak on top. [9]

4 Conclusion

As shown, the invisibility is indeed physically possible. The idea for electromagnetic invisibility comes from the fact that Maxwell equations have the same form in all coordinate systems. Since
the same is true for all other laws of physics, it leads us to believe that a similar cloaking device is possible in other wave systems as well. Indeed, there has been a proposition for an acoustic cloaking shell [11].

The biggest obstacle in making an invisibility cloak in practice is satisfying the complex material properties needed in such a cloak and realizing them on a scale that could be used with visible light. The problem still remains unsolved, but interest shown in the topic could mean that invisibility will soon become reality.

References


