Abstract:
Turbulence in the Earth’s atmosphere limits the performance of ground-based telescopes. The work of adaptive optics is to correct wave-fronts distorted due to atmospheric turbulence. This seminar provides the basics description of the adaptive optics system and its key components. Since adaptive optics is a science still is development, some further developments that will be tested in the following years are also presented in the seminar.
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1. **IMAGING THROUGH THE ATMOSPHERE**

When looking at the stars through the atmosphere the stars are twinkling, which is a long well known phenomenon. Even in the Newton’s days it was known than the point spread function of the telescope obtained by looking at a star was significantly broader than the point spread function which could be observed under laboratory conditions. Newton correctly attributed these effects to “tremors” in the atmosphere:

“*If the theory of making Telescopes could at length by fully brought into Practice, yet would there be certain Bounds beyond which Telescopes could not perform, For the air through which we look upon Stars, is in perpetual Tremor; as may be seen by the tremulous Motion of Shadows cast from high Towers, and by the twinkling of the fix’d stars.*”...

“The only Remedy is a most serene and quiet Air, such as may perhaps be found on the tops of the highest Mountains above the grosser Clouds [1].”

Atmospheric turbulence arises from heating and cooling of the Earth’s surface by the sun. Sunlight warms large land masses during daylight hours, and these warm land masses heat the air. During the night the Earth’s surface gradually cools, and this heat is also coupled into the air. Heating the air in this manner, results in large spatial scale motions. This air eventually becomes turbulent, with the result that the large spatial scale motions break up into progressively smaller scale motions, eventually giving rise to randomly sized and disturbed pockets of air (eddies), each having a characteristic temperature. The index of refraction of air is sensitive to temperature and air density, and hence, the atmosphere exhibits variations in the index of refraction. Air density is greatest at sea level and decays exponentially with height. Optical effects of turbulence therefore generally decrease with altitude, which is the reason for locating astronomical observations on mountain peaks. Plane waves propagating through the atmosphere are no longer planar when they arrive at the surface of the Earth. The study of turbulent air motion is a problem in the field of fluid mechanics. During the 1940’s, Kolmogorov developed a model for how energy is transported from large scale turbulent eddies to small scale turbulent eddies. Kolmogorov’s model provides a spatial power spectrum for the index of refraction fluctuations [1].

The effects of atmospheric turbulence on the image of a point source are depicted in Figure 1. In the absence of a wave-front distortion, the fundamental limit to the performance that can be expected from an ideal telescope is determined by the diffraction of light. The Rayleigh criterion for angular resolution of the telescope states that two point sources are just resolved if the peak of one Airy pattern falls exactly on the first dark ring of the other and is given by:

\[
\theta_{\text{diffraction}} = 1.22 \frac{\lambda}{D}
\]  

![Image of a point source in the absence of atmosphere (a) and with atmospheric distortions for short (b) and long (c) exposure time](image1.png)

Figure 1: Image of a point source in the absence of atmosphere (a) and with atmospheric distortions for short (b) and long (c) exposure time [2].

The wave-front distortion caused by turbulence spreads the energy received from a point source into a diffuse disk, considerably reducing its peak intensity. For very short exposure we will observe distinct spots of essentially telescope diffraction limited size, called speckles that are randomly placed in the focal plane. For long exposures, stellar image is a symmetrical blur with a Gaussian radial brightness dependence called the seeing disk. The twinkling of the stars which results in a blurring of the images of the stars is an effect that astronomers refer to as seeing. Three parameters are often used to characterize seeing:

- Fried parameter \( r_0 \) (atmospheric coherence parameter)

The Fried parameter is a function of turbulence strength, zenith angle and wavelength. The scaling of \( r_0 \) with a wavelength \( \lambda \) and zenith angle \( \theta \) has far reaching practical consequence. Since
it is much easier to achieve diffraction limited performance at longer wavelengths. The seeing also gets worse with increasing zenith angle. For an undistorted plane wave, \( r_o \) is infinity, while for atmospheric distortion it typically varies between 5 cm (poor seeing) and 20 cm (excellent seeing – Mauna Kea or Cerro Paranal) at visible wavelengths. At any given site \( r_o \) varies dramatically from night to night; at any given time it may be a factor 2 better than the median or a factor of 5 worse. The resolution of seeing limited images obtained through an atmosphere with turbulence is characterized by a Fried parameter and is the same as the resolution of diffraction limited images taken with a telescope of diameter \( r_o \). When \( r_o \) is less than the telescope aperture \( D \) the angular resolution of seeing limited images is:

\[
\theta_{\text{seeing}} = 1.22 \left( \frac{\lambda}{r_o} \right) \propto \lambda^{1/5}
\]

Thus, without correction for seeing, the word’s largest telescopes have no better angular resolution than a humble (\( D = 10 \) cm) telescope used at the same site. Of course, the large telescopes can detect much fainter sources than the small ones; resolution isn’t everything [3].

- Characteristic time scale \( \tau_0 \) on which images change significantly due to seeing
  A convenient approximation assumes that the time scale for temporal changes is much longer than the time it takes the wind to blow the turbulence past the telescope aperture. According to this Taylor hypothesis of frozen turbulence, the variations of the turbulence caused by a single layer can therefore be modeled by a frozen pattern that is transported across the aperture by the wind in that layer. If multiple layers contribute to the total turbulence, the time evolution is more complicated, but the temporal behavior of the turbulence can still be characterized by a time constant

\[
\tau_0 = r_o / v,
\]

where \( v \) is the wind speed in the dominant layer. With typical wind speeds of order 20 m/s \( \tau_0 \) is 10 ms for \( r_o = 20 \) cm. The wavelength scaling is the same as for Fried’s parameter. The parameter \( \tau_0 \) is also of great importance for the design of adaptive optics systems. Adaptive optics control loop must have a bandwidths larger than \( 1/\tau_0 \) [3].

- Isoplanatic angle \( \theta_o \)
The light from two stars separated by an angle \( \theta \) on the sky passes through different patches of the atmosphere and therefore experiences different phase variations. The angular anisoplatism limits the field corrected by adaptive optics system. The isoplanatic angle is given by:

\[
\theta_0 = 0.314 (\cos z) \frac{r_o}{H},
\]

where \( H \) is the mean effective turbulence height.

We can see that the isoplanatic angle is affected mostly by high altitude turbulence and depends more strongly on zenith angle than Fried’s parameter. For \( r_o = 20 \) cm and an effective turbulence height of 7 km, eqn. 5 gives \( \theta_0 = 1.8 \) arcsec. For the two stars separated by more than \( \theta_0 \), the short exposure point spread functions are different. In contrast the long exposure point spread functions, which represent averages over many realizations of the atmospheric turbulence, are nearly identical even over angles much larger than \( \theta_0 \) [3].

Understanding the origin of the optical effects of atmospheric turbulence did little to improve the state-of-the-art of astronomy until the modern times. In 1953 Babcock proposed using a deformable optical mirror, driven by a wave-front sensor, to compensate for atmospheric distortions that affected telescope images and his idea did not come to realization until modern fast computers were developed.
2. **ADAPTIVE OPTICS SYSTEM**

Adaptive optics is a system which deals with the control of light in a real-time closed loop fashion. Basic components of adaptive optics consist of a wave-front sensor, corrector and a control computer to perform real-time numerical calculations. Light from the object of interest is captured by the optical system consisting of the telescope and beam train. Part of the light is sampled by the wave-front sensor. Computer calculates the necessary changes to the optical path and sends signals to wave-front corrector where the changes are made. The imager, a camera, collects the light through the compensated optical system. If the light from the object is insufficient for determining the wave-front, supplemental sources such as nearby natural guide stars or artificial laser guide stars are used.

3. **WAVE-FRONT SENSING**

Optical transfer function can be fully characterized by the measurement of wave-front phase $\phi$. It is not possible to directly measure the wave-front phase at optical wavelengths, as today no existing detector responds at temporal frequencies involved. In fact, the available optical detectors measure the intensity of the light. Generally, indirect methods must be used to translate information related to the phase into intensity signals to be processed [5].

Each WFS consists of the following main components:

- **Optical device** which transforms the aberrations into light intensity variations
- **Detector** transforms the light intensity into electrical signal. This signal has an intrinsic noise due to the photon nature of light, but may also contain a contribution of detector noise. Light integration in the detector causes delay in the control loop, which limits the servo bandwidth.
- **Reconstructor** is needed to convert signals into phase aberrations. This computation must be fast enough, which means, practically, that only linear reconstructors are useful.

The most popular techniques of wave-front sensing in adaptive optics are derived from the methods used in optical testing (fabrication and control of telescope mirrors). There are two classes of methods based on either interferometry or geometrical optics concepts. Techniques of the first class use the principle of light beam superposition to form interference fringes coding the phase differences between the two beams, while techniques of the second class use the property that light rays are orthogonal to wave-front. We will discuss two most important methods based on geometrical optics concepts at greater length [6].

3.1 **Shack-Hartmann wave-front sensor**

The principle of Shack – Hartmann (S-H) wave-front sensor is presented in Figure 4.

![Figure 4: Schematic of the adaptive optics systems [2].](image)

![Figure 5: Principle of Shack–Hartmann wave-front sensor for the undistorted and distorted wave-front [6].](image)
A lenslet array (a collection of small identical lenses) is placed in a conjugate pupil plane in order to sample the incoming wave-front. Each lens takes a small part of the aperture, called sub-pupil, and forms an image of the source. If the wave-front is plane, each lens forms an image at its focus and all images are located in a regular grid defined by the lenslet array geometry. As soon as the wave-front is disturbed, to a first approximation each lens receives a tilted wave-front and images become displaced from their nominal positions. Displacements of image centroids in two orthogonal directions \( x, y \) are proportional to the average wave-front slopes in \( x, y \) over the sub-apertures.

Thus, a Shack-Hartmann wave-front sensor measures the wave-front slopes. If \( \Phi \) is the wave-front phase, the angle of arrival \( \alpha \) is estimated as

\[
\alpha_x = \frac{x}{fM} = \frac{\lambda}{2\pi S} \int_{\text{subaperture}} \frac{\partial \Phi(r)}{\partial r_x} d^2r,
\]

where \( f \) is the lenslet focal length, \( M \) the demagnification between the lenslet plane and the telescope entrance plane and \( S \) the area of the sub-aperture. The same equation can be written for the \( y \) slope.

The positions of the images centroids can be measured by charged-coupled device (CCD). A simple estimation is given by:

\[
x = \sum_{i,j} x_{i,j} I_{i,j} \quad \text{and} \quad y = \sum_{i,j} y_{i,j} I_{i,j},
\]

where \( I_{i,j} \) and \((x_{i,j}, y_{i,j})\) are the signal and the position coordinates of the CCD pixel \((i,j)\) [5]. In order to compute the centroids accurately, the individual images must be well sampled, more than 4x4 pixels per sub-aperture. However, each pixel of a CCD detector contributes the readout noise which dominates the photon noise for faintest guide stars. Thus, in some designs there are only 2x2 pixels per sub-aperture. In this case each subaperture has a four quadrant detector (quad cell) [6].

Assuming small displacement and image size smaller than quadrant size, it can be shown that the measured angle of arrival is expressed by:

\[
\alpha_x = \frac{\theta_b}{2} \frac{I_1 + I_2 - I_3 - I_4}{I_1 + I_2 + I_3 + I_4} \quad \text{and} \quad \alpha_y = \frac{\theta_b}{2} \frac{I_2 + I_3 - I_1 - I_4}{I_1 + I_2 + I_3 + I_4},
\]

where \( \theta_b \) is the spot angular size and \( I_1, I_2, I_3, I_4 \) the intensities detected by the four quadrants. To convert position made by the quad-cell into an angle, the image size must be known. When the images are seeing-limited, \( \theta_b = \lambda/r_0 \), the spot size depends on the seeing conditions and its unknown. Therefore, the quad-cell response has to be calibrated, which is achieved by imaging an artificial point source. In addition the response of a quad-cell slope detector is linear only for slopes less than \( \pm \theta_b/2 \), so it may be variable, depending on seeing or object size. This is the price to pay for the increased sensitivity, which is of major importance to astronomers [5, 6].

### 3.2 The Curvature sensor

Instead of measuring wave-front slope it is also possible to measure the wave-front curvature. The principle of this sensor is presented in Fig. 6. The telescope of focal length \( f \) images the source in its focal plane. Two detector arrays are placed out of focus at distance \( l \) from the focal plane. A local wave-front curvature in the pupil produces an excess of illumination in one plane and a lack of illumination in the other.
In the geometrical optics approximation, the difference between the two plane irradiance distributions is a measurement of the wave-front Laplacian and the wave-front radial first derivative at the edge of the beam. The normalized intensity difference is written as:

$$I_1(r) - I_2(-r) = \frac{\lambda f (f - l)}{2\pi l} \left[ \frac{\partial \Phi}{\partial n} \left( \frac{f r}{l} \right) \delta c - \Delta \Phi \left( \frac{f r}{l} \right) \right].$$

The first term in the above equation is the phase gradient at the edge of the aperture (this is written symbolically as a partial derivative over the direction perpendicular to edge multiplied by an "edge function" $\delta_c$). The distance $l$ must be such that the validity of the geometrical optics approximation is ensured. The condition requires that the blur produced at the position of the defocused pupil image must be small compared to the size of the wave-front fluctuations we want to measure, which can be expressed by the following condition:

$$(f - l) \theta_b \leq ld / f$$

The de-focusing is always much less than the focal length $f$, hence the condition of minimum de-focusing is:

$$l \geq \theta_b f^2 / d$$

Larger de-focusing is needed to measure wave-front with higher resolution; the sensitivity of Curvature sensor will be reduced accordingly. This means that it may have problems for sensing high-order aberrations. For point sources and large sub-apertures ($d > r_b$; a case of practical interest) the blur $\theta_b$ is defined by the atmospheric aberrations $\theta_b = \lambda / r_b$. If the adaptive optics system works in the closed loop and the residual aberrations (at the sensing wavelength) become small, the blur is reduced to $\theta_b = \lambda / d$, permitting to reduce de-focusing and to gain the sensitivity. This feature is actually used to a limited extent in the real adaptive optics systems: de-focusing is reduced once the loop is closed. The Curvature sensors that actually work in astronomical adaptive optics systems use the Avalanche Photo-Diodes (APDs) as light detectors. These are single-pixel devices, like photo-multipliers with very little readout noise and small dark current, maximum quantum efficiency is around 60% [5].

## 4. WAVE-FRONT RECONSTRUCTION

The measurements (wave-front sensor data) can be represented by a vector $S$. Length of vector $S$ is twice the number of sub-apertures $N$ for the S-H wave-front sensor (slopes in two directions are measured), and equal $N$ for CS. The unknown (wave-front) is a vector $\Phi$, which can be specified as a phase values on a grid. It is supposed that the relation between the measurements and unknowns is linear, at least to the first approximation. The most general form of a linear relation is given by matrix multiplication

$$S = A\Phi,$$

where the matrix $A$ is called interaction matrix. In real adaptive optics systems the interaction matrix is determined experimentally. All possible signals are applied to a deformable mirror, and the wave-front reaction to these signals is recorded. A reconstruction matrix $B$ performs the inverse operation, retrieving wave-front vector from the measurements:

$$\Phi = BS.$$  

The number of measurements is typically more than the number of unknowns, so the least-squares solution is useful. In the least-squares approach we look for such a phase vector $\Phi$ that would best match the data. The resulting reconstructor is

$$B = (A^T A)^{-1} A^T.$$
In almost all cases the matrix inversion presents problems because the matrix $A^TA$ is singular. The least-squares reconstructor is not the best one. By using a priori information on the signal properties a better reconstruction can be achieved.

5. WAVE-FRONT CORRECTION

Wave-front corrector is any device that corrects wave-front distortions. Such devices introduce an optical phase shift $\phi$ by producing an optical path difference $\delta$. Optical path difference can be obtained by changing index of an optically transmitting material to modify the velocity of propagation (birefringent electro-optical materials as liquid crystal spatial light modulators) or by mechanical movement of the optical surface (deforming mirrors). To date, deformable mirrors are preferably used because they provide short response times, large wavelength independent optical path differences, with a high uniform reflectivity that is insensitive to polarization, properties that are not commonly shared by birefringent materials [5].

5.1 Deformable mirrors

Deformable mirrors physically change shape to correct time-varying wave-front phase. Devices that perform the deformation are called actuators. Either force of displacement actuators can be used to control a deformable mirror. Force actuators such as hydraulic and electromagnetic are used in large active mirrors whereas displacement actuators are used primarily in deformable mirrors less than about 50 cm in diameter. Ferroelectric actuators in the piezoelectric or electrostrictive form are well suited for use as displacement actuators in deformable mirrors because of their small size, excellent stability, and high stiffness [4].

5.1.1 Actuators

Piezoelectric actuators: The direct piezoelectric effect is the creation of an electric charge in a material under an applied stress. The inverse piezoelectric effect is the production of strain-inducing stress as the result of an applied electric field. The most common piezoelectric ceramics used is Pb(Zr,Ti)O₃ or PZT (lead zirconate titanate) which exhibits strong piezoelectric effect with a maximum strain. The key parameter in determining performance is piezoelectric constant $d_{33}$, which relates the change in thickness $\Delta l$ to the applied electric field. In the case of an unloaded piezoelectric disk actuator the relationship is given by:

$$\Delta l = l d_{33} E = d_{33} V.$$  \hspace{1cm} (16)

Values of $d_{33}$ are typically between 0.3 and 0.6 nm/V. To increase the stroke for the same applied voltage the PZT is made of stacks of N disks so the total stroke is multiplied by N. The stroke of a piezoelectric actuator can be made arbitrarily large by increasing N. However, the minimum disk thickness is limited by the maximum electric field (about $2 \cdot 10^6$ V/m) that can be applied to PZT before breakdown occurs.

Displacement is at first approximation linear in voltage but in reality we have a 10-20% hysteresis. This can be compensated to some extent in a closed loop system or with strain gauges. The latest provide independent measure of movement and can reduce hysteresis to below 1%. Typical actuators are 20 mm long and 6 mm in diameter, producing a stroke of 15 μm (unloaded) for an applied voltage of 150 V. A good feature of PZT is its ability to operate over a wide temperature range of -25 to +100°C [4, 5, 7].

Electrostrictive actuators
Electrostriction is a property of all dielectrics, but it can only by observed in non-piezoelectric materials. Deformation of unloaded actuator is proportional to the square of the applied electric field $E$ so that

$$\Delta l = mE^2 = \frac{m V^2}{l},$$

where $m$ is the electrostriction coefficient. The electrostrictive material lead magnesium niobate $\text{Pb(Mg}_{1/3}\text{Nb}_{2/3})\text{O}_3$, commonly referred to as PMN, is a ferroelectric material in which the large electrostrictive strain is due to a huge dielectric constant. PMN actuators have two main drawbacks: nonlinear response and temperature sensitivity. The quadratic relation between deformation and applied voltage is not a serious problem in closed-loop system. It is usually handled by biasing the operating point to half of the peak input voltage. Dielectric constant is a function of temperature and as a consequence displacement and hysteresis are also sensitive to temperature variations. At the optimal working temperature of around 25 °C PMN hysteresis is however less than 1% which is far better compared to PZT [4, 5, 7].

**Figure 9**: Hysteresis (left) and displacement (right) of PZT and PMN as a function of temperature [7].

### 5.1.2 Requirements

The requirements for wave-front correctors used in astronomical adaptive optics are generally specified in the following terms:

- **The number of actuators**, or degrees of freedom, and their spatial arrangement
  
  This is the basic descriptor because it determines the order of compensation that the corrector can produce. When related to the size of the telescope pupil, the actuator spacing $d$ in relation to the value of Fried’s parameter, determines the fitting error of the corrector and is an important parameter. Full wave-front compensation requires an effective actuator spacing of 1 – 1.5 $r_0$.

- **Dynamic range**
  
  The required stroke (total up and down range) is proportional to $\lambda(D/r_0)^{5/6}$. It is practically wavelength independent, and of the order of at least several microns for 10 m telescope and ±10-15 microns for 30 m telescope. Tip and tilt corrections require the largest stroke. If such strokes can not be obtained with deformable mirror separate Tip/Tilt mirror must be used.

- **Response time**
  
  The required response time is of the order of at least a few milliseconds. It increases as the degree of correction decreases. It should be faster than coherence time $\tau_0$.

### 5.1.3 Types of deformable mirrors

Deformable mirrors come in many flavors. They can be constructed as segmented or continuous facesheet deformable mirrors, which can have discrete actuators perpendicular to the surface or continuous actuators such as bimorph deformable mirrors.

#### 5.1.3.1 Discrete-Actuator Deformable mirrors

The structure of a typical deformable mirror using a continuous faceplate and discrete actuators is shown in figure 9. The actuators are mounted on a massive baseplate. The actuators are usually multilayer stacks of piezoelectric or electrostrictive ceramic material. At the top of each actuator is a
coupling that forms the interface between the actuator and the faceplate. The couplings act as flexures to accommodate local tilts in the faceplate. They are carefully sized to control the influence function of the actuators. The faceplate is normally made from a stable, low-expansion material and the same material is usually used for a baseplate, in order to match their thermal properties.

The mechanical design involves compromises among many conflicting factors. The faceplate stiffness should be high to maintain its shape during polishing, but sufficiently low to deflect when pushed or pulled by the actuators. The bending stiffness of the actuators should be high to keep mechanical resonance well above the working bandwidth, but at the same time the faceplate must be allowed to tilt smoothly to interpolate between adjacent actuators. The main factor controlling the shape of the influence function (the shape of the mirror surface when pushed by one actuator) is the actuator bending stiffness to that of the faceplate stiffness. Influence function produced in the two extremes is shown in Figure 10.

The presence of coupling often improves the accuracy with which a random wave-front can be compensated by a deformable mirror, but significant coupling can often be a source of instability. Because of the actuator coupling there is also a reduced inter-segment stroke. The peak deflection of the faceplate will always be less than the free stroke of the actuators, because of the stiffness of the faceplate [4].

5.1.3.2 Segmented mirrors
Segmented mirrors were the first kind of wave-front correctors to be developed and are made of set of identical mirrors distributed over a square of hexagonal array. The simplest type is known as piston-only correctors employing one actuator per segment to correct the average optical phase over each segment. Since they produce wave-front discontinuities they are rarely used. Segments with tree independent actuators can correct tip and tilt as well as absolute phase, resulting in much better fitting of the wave-front. The main advantage is that the cost and difficulty of replacing a single segment is much less than the equivalent maintenance on a continuous surface mirror. With segmented mirrors there is no inter-actuator coupling and as a result segmented mirrors can have higher stroke and denser actuator spacing [4, 5, 8].
The discontinuities (gaps) between segments have an impact on overall performance. Energy is lost through the gaps, and since the gaps cause the mirror to act like grating, energy is diffracted from the central lobe. It is important that the area of the gaps be below 2% [5]. When comparing different types of deformable mirrors we can look at the wave-front fitting error which depends on aberration statistics and mirror type (influence function) and is for Kolmogorov turbulence approximately given by:

$$\sigma^2_{fit} = a_{F} \left[ \frac{d}{r_{0}} \right]^{5/3},$$

where $d$ is a characteristic size of the segment and constant $a_{F}$ depends on a mirror configuration. This constant is approximately 1,26 for piston only, 0,18 for piston plus tip and tilt segmented mirror and 0,28 rad$^2$ for continuous deformable mirror. From that we can estimate the number of actuators and sub apertures needed to get the same fitting error. The number of sub apertures for piston plus tip/tilt versus continuous versus piston only deformable mirror is 1:1.7:10 and for actuators the number is 3:1.7:10. We can see that the continuous deformable mirrors are generally more economical [4].

5.1.3.3 Bimorph

A bimorph mirror consists of two piezoelectric ceramic wafers which are bounded together and oppositely polarized, parallel to their axis. Individual actuators are formed by an array of electrodes which is deposited between the two wafers. Te front and bottom surfaces are grounded. When a voltage is applied to an electrode (actuator) one wafer contracts and the other wafer expands, which produces a local bending. The local curvature being proportional to the voltage, these deformable mirrors are called curvature mirrors and they are particular well suited for integration with a wave-front curvature sensor. This greatly simplifies the wave-front control and essentially eliminates an intermediate wave-front reconstructor. The deformation is produced by forces within the plate itself, which needs only to be simply supported. There is no reaction structure and the actuators do not need individual adjustment. The construction of bimorph mirror is therefore much simpler than that of discrete actuator mirrors and as a consequence bimorph deformable mirrors are a cheaper alternative.

![Bimorph mirror](image)

Bimorph mirrors are able to achieve relatively large strokes with low applied voltages, the stroke being inversely proportional to the square of the thickness. Besides the bending the applied volatage also causes opposite changes of thickness of each wafer. Even if this effect cancels for the entire bimorph, it still produces a displacement at the top and bottom. A dominant bending effect can be achieved by making the size of electrodes at least four times larger than the thickness of the bimorph. The spatial resolution is consequently limited; with the result than practical bimorph mirrors are restricted to fewer than 100 actuators. Hence, bimorph mirrors are best suited for low order compensation systems. Bimorph mirrors cannot assume the shapes of all Zernike polynomials (astigmatism, spherical aberration) without the application of the edge gradients (boundary conditions). This can be achieved by making the active diameter of a bimorph larger than the optical aperture, with additional electrodes to produce the edge gradients. The resonant frequency is typically of the order of several kHz, which is lower than that of a deformable mirror with displacement actuators [5, 8].

5.1.3.4 MEMS (micro-electro-mechanical systems)

New class of deformable mirror, derived from the membrane mirror concept, can be made with thousands of actuators, high bandwidths and fit into a microchip. These devices, called micro-electro-mechanical (MEMS) deformable mirrors, promise to be the first low cost mass produced adaptive optics components. Conventional DMs cost about $1000 per degree of freedom so $1M for 1000 actuators, whereas MEMS currently cost about few tens of $ per degree of freedom.
Components are micro-machined on silicon or metal substrates and the wave-front sensor and reconstructor can be integrated into the same package. The actuation is in the form of electrostatic attraction and repulsion between a thin common electrode membrane that acts as the mirror surface and control electrodes. They have the same basic structure as segmented or continuous faceplates deformable mirrors except that they are often only centimeter in diameter. These devices use very little current to mechanically produce deformations of the optical surface up to 2 microns. The remaining problems of MEMS are insufficient stroke (aiming for 10 microns) and sensitivity to humidity.

Gemini Planet Imager (GPI) which will be deployed in 2010 on the Gemini South telescope will use 4096-actuator Boston Micromachines MEMS deformable mirror. GPI purpose is to directly detect Jupiter-like planets outside of our solar system. The adaptive optics is dedicated specifically to high contrast imaging and is optimized for best performance in the “dark hole” region and minimal systematic errors. The MEMS will have only 3-4 μm stroke, insufficient to fully correct the atmosphere on a 8-m telescope, so the second bimorph mirror will be used to correct for low modes including tip and tilt [8, 9].

5.1.3.5 Deformable secondary mirrors

Because of the requirement to minimize the number of optical surfaces in the beam train to reduce losses due to absorption and scattering deformable mirrors has been incorporated into telescope secondary mirror. Adaptive secondary deformable mirrors have inherently high stroke so there is no need for separate tip/tilt mirror. They are however harder to build (the surface of the mirror is curved – hyperboloid, also actuators are heavier and larger) and harder to handle (they break more easily) [5].

5. GUIDE STARS

5.1 Natural guide stars

Most of the actual astronomical adaptive optics systems use natural guide stars to measure the wave-front. Guide stars must be selected within the isoplanatic angle $\theta_0$ of the target. Typically guide star must lie within about 30 arc seconds of the astronomical target for infrared observations and 10 arc seconds for visible light observations. This imposes a severe restriction on the choice of targets. If a target is selected randomly on the sky the probability to find suitable guide stars are low. For a given distance $\theta$ between the guide star and the target, the residual wave-front error due to anisoplanitism is estimated as:

$$\langle \epsilon_{\text{iso}}^2 \rangle = \left( \frac{\theta}{\theta_0} \right)^2 \propto \lambda^{-2}$$

(19)

On the other hand, the photon noise error is inversely proportional to the photon flux, which is related to the stellar magnitude $m$:

$$\langle \epsilon_{\text{phot}}^2 \rangle \propto \lambda^{-3.6} 10^{-0.4m}$$

(20)

For the infrared wavelengths the limiting magnitude is around 12 and around 16 for the visible wavelengths. It is clear that the sky coverage is a strong function of the imaging wavelength. At longer wavelengths isoplanatic angle is larger and the flux needed to measure wave-front is lower. Both the required photon flux and the isoplanatic angle depend on the turbulence profile. It means that the difficult objects can be observed with adaptive optics only under favorable seeing. Instead of liberating
astronomers from the dependence on seeing, adaptive optics makes this dependence even more critical. The idea is to use artificial laser guide stars, also called lased beacons. The existing two types of lasar guide stars use either the Rayleigh scattering from air molecules or the fluorescence of sodium atoms in the mesosphere [6].

Figure 16: Rayleigh laser for William Herschel Telescope (left) and sodium laser at Keck observatory (right).

5.2 Rayleigh laser guide star
The beam of a pulsed laser is focused at altitudes between 10 and 20 km above ground, the return signal is obtained as back scattered light by the air density fluctuations. This scattering is more efficient at short wavelengths, which explains the blue color of the clear sky and the increased extinction of blue stellar light. The return beam is therefore bluer than the outgoing beam. Our laser will not just give us a nice point source and a small spot in the atmosphere. We get scatter from the laser all along its path. So in addition the beam is pulsed and the return is gated with a fast shutter, so that the altitude is defined by the light travel time to and from the layer. There is a tradeoff between the rate of the pulses and the altitude that the laser is gated to. The laser cannot be used for atmospheric fluctuations faster than the light travel time there and back. The Rayleigh laser guide star is based on the 200 W copper vapor laser emitting the green light [6, 10].

5.3 Sodium laser guide star
The sodium layer at an altitude of about 90 km and with thickness of about 10 km surrounds the Earth. It is most likely formed by micro meteor ablation. Parameters of the layer (total number of atoms, mean altitude, profile) change seasonally, but also on the time scales of days, hours and even minutes. On the average, there are some $10^{13}$ sodium atoms per square meter. The sodium atoms can be excited by the laser beam tuned to the D2 line (589 nm) and radiate at the same wavelength. Because of finite amount of sodium atoms there is a saturation limit of about $1.9 \times 10^8$ photons/sec. The 25 W sodium lasers can produce guide star of magnitude 9 in the visible band [6, 10].

5.4 Problems with laser guide stars
On the one hand laser guide stars improve sky coverage and permit the observations otherwise impossible. But on the other hand they cause additional sources of error (cone effect, tilt) which result in a reduced performance compared to natural guide stars adaptive optics.

- **Tilt problem**
The laser beam passes through the atmosphere twice, once on the way up and once on the way down. The lowest order terms in the wave-front correction, called tip and tilt, cancel out and measurement of the laser guide star does not sense them. Solutions are to use a natural guide star for tip and tilt only. This can be done in infrared where the isoplanatic angle is larger and the good sky coverage can be achieved. The other solution is to use a polychromatic laser guide star. Sodium can be excited to higher levels to emit other lines. Using the wave-fronts at more than one wavelength the atmospheric tilt can be calculated.
The cone effect / focal anisoplanatism

The laser guide stars are at infinite altitude $H$, and therefore the light from the guide star does not pass through the same atmosphere as the beam from the astronomical target. There are three distinctive effects: the turbulence above $H$ is not sensed by laser guide star, the outer proportions of the stellar wave-front are not sensed and laser and stellar wave-front are scaled differently. While the laser wave-front is corrected by the adaptive optics, the stellar wave-front has a residual error for a telescope with diameter $D$ due to the cone effect:

$$\left\langle \varepsilon_{\text{cone}}^2 \right\rangle = \left( \frac{D}{d_0} \right)^{5/3}$$

(21)

We have introduced a new parameter characterizing the cone effect $d_0$, which is a function of a isoplanatic angle and altitude of a guide star. For a Sodium guide star it is typically 5 times larger than for a Rayleigh guide star. The error is worse for larger telescopes and for lower altitudes. We can increase the altitude for the Rayleigh laser to get smaller cone effect, but the higher we go the less air there is, so the strength of scattering is lower. The Sodium laser guide star has a clear advantage with the respect to the cone effect. But the current sodium lasers are 10 times more expensive than Rayleigh lasers of equal power.

Other problems

Powerful laser beams directed into the sky are dangerous for aircraft pilots and for satellites (blinding and damage to optical equipment). They also increase local light pollution at the observatory, requiring coordination between the telescopes. In addition laser guide stars can not operate under spectroscopic sky conditions (light cirrus clouds) because of the strong beam scattering [6].

6. CONCLUSION

As we have learned performance of ground-based telescopes is limited due to turbulence in the Earth’s atmosphere. Adaptive optics is a fairly simple system that significantly improves distorted pictures and even better improvement is expected for some further developments of the original adaptive optics concept such as multi-conjugate adaptive optics. Multi-conjugate adaptive optics (MCAO) tries to overcome the cone effect by sensing and correcting for the whole atmospheric volume probed by the observed field of view. The process of implementing MCAO correction consists of three main steps. The first one is to measure the deformation of the wave-front due to atmospheric turbulence along different directions in the field of view.
This is performed by with several wave-front sensors looking at different guide stars. The greater the number of guide stars, the better the knowledge of the wave-front distortion in the sky field of interest. The second step is called atmospheric tomography and consists of reconstructing the vertical distribution of the atmospheric turbulence at different locations of the field, in order to obtain a three-dimensional mapping of the turbulence above the telescope. The third step is to apply the wave-front correction to the whole field and not only in a specified direction. This is achievable by using several deformable mirrors which are optically conjugated to different altitudes in the atmosphere above the telescope. Wave-front sensing can be either star oriented (as many wave-front sensors as guide stars) or layer oriented which allows sensing all the guide stars simultaneously and uses as many wave-front sensors as deformable mirrors [11].

Multi-conjugate adaptive optics on the Gemini-S (Cerro Pachon – Chile) telescope is planed to begin in 2010. The main system parameters are summarized in the table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>DM conjugate ranges</td>
<td>0, 4.5 and 9 km</td>
</tr>
<tr>
<td>Number of guide stars</td>
<td>3 NGS and 3 LGS</td>
</tr>
<tr>
<td>Guide Star geometry</td>
<td>(0,0) and (+/-42.5, +/-42.5) arcsecs (LGS)</td>
</tr>
<tr>
<td>WFS Orders</td>
<td>16 by 16 (LGS); Tip-tilt (NGS)</td>
</tr>
<tr>
<td>LGS Laser Power</td>
<td>Equivalent to 125 PDEs/cm2/s at WFS</td>
</tr>
<tr>
<td>NGS magnitudes</td>
<td>3 times 19</td>
</tr>
<tr>
<td>Control bandwidths</td>
<td>33Hz (LGS); 0-90Hz (NGS)</td>
</tr>
</tbody>
</table>

Table 1: System parameters for multi-conjugate adaptive optics for Gemini South telescope [12].

Images with multi-conjugate adaptive optics will be basically diffraction-limited, in term of FWHM, over the full 1 square arcmin field of view [12].

References