Abstract

The geophysical fluid dynamics is defined as fluid dynamics of stratified media in the rotating frame. To simulate both effects in a laboratory, rotation and media stratification, the rotating table and a simple mixing system is needed. Further, for analysis of the fluid motion the measurement techniques are needed. In general for measuring techniques the dye or particle trackers are injected into the media. The analysis of motion can be done with PIV (particle image velocimetry) or LIF (light induced fluorescence) technique.
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1. Introduction

The subject of geophysical fluid dynamics deals with the dynamics of the atmosphere and the ocean. Due to our increasing interest in the environment it is becoming important branch of fluid dynamics. The field is mainly developing by meteorologists and oceanographers. The importance of understanding geophysical fluid is obvious. Living on the Earth is closely connected with atmospheric dynamics. We are exposed to weather changes and it is important to understand its dynamics. The atmosphere is closely connected with the ocean (heat, moisture and momentum exchange).

The two features that distinguish geophysical fluid from other areas of fluid dynamics are the rotation of the Earth and the vertical density stratification of the medium. The ocean have vertical density stratification due to its density being function of pressure, temperature and salinity.

Both stratification and rotation plays important role in the ocean dynamics. The most basic fluid motion, gravity waves, change its motion from 2D to 3D due to Coriolis force. Instead of motion only in the direction of the gravity force, Coriolis force forces particles to rotate in the horizontal plane too. The continuous stratification implies great change in the internal wave propagation. As long as the layered fluid is considered, the Laplace equation describes wave propagation in the media. The waves are propagating only along internal surfaces. In the stratified fluid propagation direction depends on the density gradient, moreover group speed is perpendicular to the phase velocity [1].

In general we are solving the Navier-Stokes equation. Some large scale solutions can be made analytically. For example the Taylor's solution for a rectangular rotating basin [2], but most of the problems are impossible to solve analytically. In the realistic ocean the boundary conditions become very complicated hence Navier Stokes equation cannot be solved.

The alternative is to simulate the rotation of the earth and fluid stratification in a laboratory and analyze its time development. As the geophysical system is simulated one can prepare various experiments, where analytical approach is impossible.

In this seminar I will present laboratory for geophysical fluid dynamics, few measurement techniques and some experiments.
2. Geophysical fluid

Before we start with the experimental techniques we must define the properties of geophysical fluid.

Density of the fluid is in general function of pressure, temperature and salinity. In the ocean, all parameters varies with position. Vertical variations are much more intense than horizontal, so horizontal variations of pressure, salinity and temperature can be neglected.

In oceanography density gradient is usually presented as buoyancy frequency ($N$). The density gradient can be measured by observing particle oscillation. The buoyancy frequency is defined as:

$$\frac{d^2 z}{dt^2} + N^2 z = 0 \quad N^2 = -\frac{g}{\rho_0} \frac{\partial \rho}{\partial z}$$  \hspace{1cm} (2.1)

Equation (2.1) is actually second Newton’s law for the small part of stratified fluid. In case of the linearly stratified fluid the buoyancy frequency is constant.

In the ocean, fluid is typically linearly stratified. Near river outfall sea has often two layer stratification. The river lays fresh water on sea surface (sea water has typical salinity 3.5%). Salty and fresh water does not mix instantly. Fresh water is lighter than salty so it floats on heavier salty water hence there are two layers. Eventually fresh and salty water mix, but region near river outfall is often two layer stratified. Typical examples of two layer fluids are the Norwegian fjords.

Secondly, the Earth’s rotation needs to be considered. The fluid dynamics is described with Navier-Stokes equation. For geophysical fluid dynamics Navier-Stokes equation needs to be rewritten to the rotating coordinate system [1].

$$\rho \frac{D\vec{v}}{Dt} = f \rho - \nabla p + \mu \nabla^2 \vec{v} + (\xi + \frac{\mu}{3})\nabla(\nabla \vec{v})$$  \hspace{1cm} (2.2)

Equation (2.2) considers viscid compressible fluid, where $\rho$ is density, $f$ density of external forces, $\mu$ dynamic viscosity and $\xi$ dilatation viscosity. As the compressibility and the viscosity for water are small they can be neglected. Considering rotating coordinate system and simplification we get:

$$\dot{\vec{v}} + 2\Omega \times \vec{v} + \Omega \times (\Omega \times \vec{r}) + (\vec{v} \nabla)\vec{v} = -\frac{1}{\rho} \nabla p \quad 2\Omega \times \vec{v} = \begin{pmatrix} -fv \\ fu \\ -2\cos\Theta u \end{pmatrix}$$  \hspace{1cm} (2.3)

where $f$ is Coriolis parameter. In equation (2.3) two new terms are introduced. These are Coriolis and centripetal force (second and third term on the left side of equation).

Large scale geophysical problems should be solved using spherical coordinates. If horizontal length scales are much smaller than radius of the Earth, than the curvature of the Earth can be ignored. Small scale (compared to Earth’s radius) problems can be studied by adopting local Cartesian coordinate system on tangent plane (Figure 1). However, Coriolis parameter varies with latitude. This variation is important only for phenomena having very long time scales (several weeks) or very long lengths (at least 1000 km). For our purposes we can
assume \( f \) to be constant. A model using constant \( f \) is called \( f \)-plane model. The variation of \( f \) can be approximated by extending it in Taylor series, so called \( \beta \)-plane model. In this seminar only \( f \)-plane model will be used. \( \beta \)-plane model is used to consider Rossby planetary waves, for example.

3. Simulating geophysical fluid in laboratory

As shown in the previous chapter we have to consider two main effects – rotation and stratification of the medium to simulate geophysical fluid.

As we want to study small scale effects (compared to the Earth’s radius) the \( f \)-plane model and the local Cartesian coordinate system will be used. Therefore, the rotating table with tank on it is needed. We use rotating table in the laboratory for fluid dynamics at Marine Biology Station Piran (RT) (Figure 2). Angular velocity of RT has to be very stable and much higher than Earth’s angular velocity. The system forces in rotating frame must be much higher than system forces caused by the Earth’s rotation. Typical angular velocity used in experiments is \( \omega = 0.3 \, \text{rad/s} \) (earth’s angular velocity is \( \omega = 7.27 \times 10^{-5} \, \text{rad/s} \)).

Tanks with different shapes can be mounted on the RT. On Figure 2 rectangular (87x87 cm) tank is mounted. With changing the shape of the tank the boundary conditions can be changed. Plans for future include even topography and obstacles in the tank, so the realistic boundary conditions could be considered.

3.1. Preparation of linearly stratified fluid

The next step is preparation of linearly stratified fluid. As the layer of water in the tank is shallow, pressure effect on density can be neglected. Moreover, volume of water in the tank is small enough to temper to the surrounding temperature fast, therefore density depends on salinity only - density can be regulated by adding salt.

Principle of preparing stratified fluid is quite simple. Three tanks are needed (Figure 3). The first (P1) is filled with fresh water. The second (P2) is filled with salty water and the third is the target tank where the stratified fluid is prepared (tank on the RT). Water from tank P1 is slowly pumped (with flow rate \( \Phi_1 \)) into P2. P2 is used as water mixer. The water in P2 has to be well mixed through all the time of the experiment. As fresh water pours from P2 to P1, salinity in P2 drops. Water from P2 is pumped in final tank P3 (with flow rate \( \Phi_3 \)). In P3 the floating difusser is used to inject water layer by layer.

Considering continuity equation for P2 we get:
time dependent salinity of water in P2 \( S_2 \) as function of flow rates, initial salinity in P2 and initial volume of water in P2. Both flow rates have to be small and stable.

Equation (3.1) consider time dependence of salinity in tank P2. If flow rates are constant the vertical gradient of salinity in P3 is:

\[
\frac{dC_3}{dz} = -\frac{A}{\Phi_2} \frac{dC_2}{dt}
\]

(3.2)

where \( A \) is plane of P3 tank. From equations (3.1) and (3.2) can be seen that final salinity profile in P3 can be regulated by changing initial parameters (flow rates, initial salinity and initial volume of water in P2).

Typical flow rate used for pumping water from P1 to P2 is \( \Theta = 0.004 \frac{l}{s} \).
This method seems to be quite precise. We managed to prepare linear stratification with $R^2 = 0.9982$. The hardest part is to obtain stable flow rates at such a low flow rate. Flow rate must be small so the injected water in P3 tank does not mix with the layer already in P3. If the table rotation is faster ($\omega > 0.3 rad/s$) than flow rates must be even smaller.

Two layer stratified fluid is much easier to prepare. We fill tank P3 with a layer of salty water and then through floating diffuser inject second layer of lighter water. Typically we use salinity gradient in the two layer stratified fluid about 1.5%.

Typical preparation of linearly stratified layer lasts 5-6 hours. About 70 litres water has to be pumped through the system to get 17 cm thick layer of stratified fluid (Figure 4).
stratification is decayed by temperature and concentration diffusion. However, typical diffusion time is much higher than time needed for experiment to be completed.

\[ P2 \text{- salinity} \]
\[ y = -154.94x + 103.45 \]
\[ R^2 = 0.9971 \]

\[ P3 \text{- salinity} \]
\[ y = -0.1727x + 40.747 \]
\[ R^2 = 0.9866 \]

Figure 4: Measurement of salinity in P2 and P3 tanks

With this setup we can prepare other stratifications. For example, we get exponential profile if flow rates are equal. Consider equation (3.1) with the same flow rates \((\Phi_1 = \Phi_2)\):

\[ S_2(t) = S_{20}e^\frac{\Phi_2}{V_{30}} \]

(3.4)

4. Fluid motion detection

The goal of the experiment is to detect and analyze motion in prepared geophysical fluid.

4.1. Dye injection and camera setup

The idea is to inject a sample of fluid with the dissolved dye into the layer and than observe its time development. With salinity of the sample fluid with the dye we can regulate depth of the dye cloud. In case when linearly stratified fluid is used the depth can be freely chosen. It the two layer stratified fluid the dye is typically injected on internal surface.

The dye must be injected causing minimum disturbance. If disturbance is to strong stratification could be lost and a formation of internal waves could occur.

To reduce disturbance as much as possible the remotely controlled plastic cage is used to contain dye. The dye is injected in the cage very slowly so it does not mix with surrounding water and does not form any internal waves. When sufficient dye is in the cage, the cage is lifted and the dye can move freely.

The dye must be monitored so further analysis is possible after the experiment. Monitoring is performed by two CCD cameras mounted on the frame of RT. The output are two time series of images, one for horizontal analysis and one for vertical analysis.

Image transfer from cameras to stationary computer must be done through buffer due to disturbance, directly transferred images tends to be corrupted. The mobile computer mounted on the RT is used as a buffer. While the experiment is running the images are stored on the mobile computer. One can monitor the experiment by logging on the mobile computer via wireless network. After the experiment is done images are transferred to the stationary computer where further analysis is done.
4.2. Light Induced Fluorescence

General idea of fluid motion analysis is to inject the dye into the sample and then track it. Dye must have specific absorption or emission properties, so it can be distinguished from background. One of possibilities is to use fluorescent dye.

Fluorescein (Na₂C₂₀H₁₂O₅) is commonly used as a fluorescent dye. Two different techniques for concentration measurements can be used. First, the concentration distribution of fluorescein can be reconstructed out of its light attenuation (light attenuation depends on concentration of the fluorescein). The other technique is called Light Induced Fluorescence (LIF). LIF is much more suitable for experiments we are dealing with.

LIF makes use of a light source which induces dissolved dye to fluorescence. The registrating camera is placed perpendicularly above the observed sheet (Figure 6). Because the magnitude of fluorescent signal depends on dye concentration, the two dimensional concentration distribution can be determined. A perfectly parallel light field is preferred, so the whole sheet is uniformly illuminated.

The concentration of fluorescein needed in sample of fluid to clearly distinguish it from background does not affect much on sample density. Fluorescein has an absorption maximum at 490 nm and emission maximum of 514 nm (in water) - it covers wavelength region of 400-700 nm. Ultraviolet lamp is used as a light source and the light emitted from the dye is in the visible area (green), therefore objects illuminated by UV lamp are faintly visible compared to the dye.

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1 Fluorescence is a luminescence that is mostly found as an optical phenomenon in cold bodies, in which the molecular absorption of a photon triggers the emission of a lower-energy photon with a longer wavelength. The energy difference between the absorbed and emitted photons ends up as molecular vibrations or heat.
With use of the LIF technique one can analyze time dependent dye distribution. There is a lot of software written with image recognition algorithms included. However, LIF technique is ideal for visualization of fluid motion.

One of the main type of experiments are vortex simulations. We have already prepared various test experiments. As the cage containing the dye is lifted vortex structures are formed. Interesting area of research is to observe vortex dynamics near obstacles (Figure 7).

Recently we used the LIF technique to visualize injection of waste in the two layer stratified fluid. The waste is supposed to be lighter than lower layer and heavier the upper layer of fluid, so parabolic trajectories of waste towards internal surface are expected (Figure 8). The LIF technique is perfect for that kind of experiment, to create visualization of the flow.

Figure 7: The example of image obtained with top camera using LIF technique. There is vortex dipole in the upper right corner of image and one single vortex near the obstacle. The fluorescein was used as a dye and UV lamp as a light source. The table angular velocity was 0.52 rad/s. The experiment was made in two layer stratified media with density gradient 14 g/dm³. Injected dye had density 1005 g/dm³.
Figure 8: The visualization of the waste disposal into the two layer fluid. The dye (fluorescein) is injected into the two layer fluid. Upper layer has salinity 1.5%, bottom layer 2% and the dye is dissolved in water with salinity 1.2%.

4.3. Particle image velocimetry (PIV)

Another motion detection method, PIV – particle image velocimetry, is much more suitable for measuring time depended velocity vector fields. The main concept is to inject particles, illuminate them and then track bright pixels on captured image. Orgasol powder is injected into the fluid as particle trackers. The orgasol is fine powder with density similar to density of water. As the stratified fluid is used one would expect vertical motion of the orgasol powder. This motion is very slow due to its small size and can be neglected.

Using tracer particles for flow visualization is a well-established technique. It enables to obtain quantitative information about velocities fluids. The idea is to illuminate tracer particles seeded in a fluid with a thin sheet of light. The images of the moving particles in the light sheet can be recorded and processed. We use 50 mW green laser as a light source. A laser beam is split and two cylinders are used to create two laser planes – one parallel and one perpendicular to the fluid layer (Figure 9). With this technique one can observe very thin selected layer of the fluid.

Several techniques, based on the described visualization method, have been developed to measure 2D velocity field in a flow. PIV tracks individual particles in subsequent images whereas PIV determines the average displacement of particles in corresponding image segments between two sequential images. The tracking algorithm functions globally in the following way: first the visualized images are dynamically thresholded to remove background intensity variations. Then the images are processed to obtain coordinates of the particles present in the image. Next, pixel coordinates are remapped from pixel coordinates to physical coordinates and finally particle coordinates of a current frame are matched with particle coordinates from previous frame. A particle in previous frame is matched with the particle in current frame closest to its position in previous frame. The matching algorithm can be improved by not using the real position of the particle in previous frame but instead using an estimation of its position in current frame.
Figure 9: Schematic illustration of the PIV setup

Figure 10: Orgasol particles illuminated with 50 mW green laser. The picture was taken while we were testing new laser.
There are many analysis that can be done using PIV technique, for geophysical fluids velocity vector and vorticity vector fields are the most interesting.

Much more simple technique can be used to measure velocity fields. Instead of sampling short exposure images one may sample long exposure image – streakline photograph, so instead of dots there will be tracks on the image. The track length divided by exposure time gives the velocity of the particle (Figure 12).

However, PIV technique is much more sophisticated and allows various of analysis. The PIV method is also much more accurate and is easier to control.

4.4. **Shadowgraph technique**

Consider finally a simple shadowgraph technique that permits observation of density inhomogeneities in transparent media. The method is based on the refraction. The curvature of the optical path depends on the first derivate of the refractive index ($\frac{\partial n}{\partial z}$). As $\frac{\partial n}{\partial z}$ is constant for linear stratified fluid...
then the deflection of all rays is the same. However, when fluid is disturbed \( \frac{\partial n}{\partial z} \) is not constant anymore. There are local changes, hence rays passing inhomogeneities in the media are deflected more or less than those going through region where \( \frac{\partial n}{\partial z} \) is constant.

The concept of the shadowgraph technique is to uniformly illuminate the fluid and then observe screen on the other side of the tank.

As a result, the disturbances produce intensity patterns on the screen. This method is useful only for qualitative information on the flow field in the linear stratified media. This method is useful for observing internal waves propagation.

Figure 13: Schematic arrangement of a shadowgraph system. S is light source, L lens, \( \rho(z) \) vertical density profile in the tank and \( i(z) \) light intensity on the screen.
5. Conclusion

To summarize, for successful geophysical fluid dynamics experiment we need: rotating table, linearly stratified fluid, dye, monitoring equipment and specific light source (we use laser or UV lamp depending on the tracking technique). As a result of the experiment velocity vector fields are obtained. These velocity vector fields are basis for the further study of the geophysical fluid dynamics.

There are many interesting scenarios to simulate and analyze. The upcoming experiments are simulation of vortex dynamics near obstacles, mixing of fresh and salty water near river outfall, waste disposals into the sea, ... . Interesting experiment is to prepare the cyclone or anticyclone, specially near obstacles.

However, the plan is to construct topography on the bottom of the tank on the RT and then try to simulate as realistic situations as possible.

There is also possible upgrade from standard two dimensional PIV to stereoscopic three dimensional PIV vector field measurement.

Up to date the laboratory described in this seminar is still in preparation phase.

References


