Effect of Packing on Multiplication Factor in Pebble-Bed Reactor

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ABSTRACT

Random packings of monodisperse hard spheres have long been of interest as models for the arrangement of atoms in simple liquids and glasses. These results can be used for studying the pebble bed nuclear reactor core. It is experimentally shown that there exists a range of random packing densities with an upper limit of 0.64 and a lower limit of 0.60. The pebble-bed reactor (PBR) core consists of a large number of randomly packed spherical fuel elements. The effect of fuel element packing density variations on multiplication factor of a typical PBR is studied using WIMS code. It is observed that multiplication factor increases with packing fraction.
1 INTRODUCTION

The pebble-bed reactor (PBR) is an advanced high temperature gas-cooled nuclear reactor intended for producing process heat at high temperature (e.g., pyrolytic hydrogen production). The PBR core is a cylindrical vat filled with a large number (up to 500,000) of randomly packed spherical fuel elements (pebbles) and cooled by helium flowing in the free spaces between the pebbles [1-3]. Each pebble measures 6 cm in diameter and has two radial material zones: the inner zone made of mixture of enriched uranium dioxide and graphite and the outer zone made of pure graphite. The thickness of the outer zone is approximately 5 mm. During operation, pebbles are continuously added at the top of the core. The fuel elements move downward in the reactor core and discharge at the core bottom. After that fuel elements pass the burn-up measurement device one by one. The fuel elements, which have reached the burn-up target, will be re-circulated into the reactor core, and the spent fuel elements exceeding the burn-up target will be discharged and transported into the spent fuel storage tank.

The pebbles are small enough (compared to the size of the core) to be treated as a granular medium. Therefore we can expect in PBR to exhibit phenomena typical for granular matter. In this work we shall describe only the effects which affect the PBR multiplication factor and should therefore be considered when designing PBR. The most important effect is variation of the packing density.

In cylindrical vats, the effective packing density of hard spherical objects is lower than theoretical and even varies with relative position in the vat, e.g., near the wall [4]. Moreover, the packing density depends on the way the vat is filled. If the vat is shaken after being randomly filled, the packing density will increase. If this happens in a pebble-bed reactor (e.g., as a consequence of an earthquake or operation-induced vibrations) its multiplication factor will change due to change in the neutron leakage governed primarily by the effective core material density.

The packing density is usually described by the parameter ‘packing fraction’ defined as the ratio between the hard volume of the objects (in our case spheres) and the total volume they fill. The purpose of this work is to calculate the multiplication factor of the pebble-bed reactor core at various packing fractions. The packing fraction is varied within the expected interval for normal operation as well as in the interval typical for accident conditions.

Figure 1: Schematic picture of PBR fuel element [3]
2 SPHERE PACKING

Packing problems, such as how densely objects can fill a given volume, are among the most ancient and persistent problems in mathematics and science. It was hypothesized by J. Kepler in 1611 that face-centred cubic lattice and hexagonal close packed lattice are the densest possible (have the greatest packing fraction), and this assertion is known as Kepler’s conjecture (J. Kepler: “The packing will be the tightest possible, so that in no other arrangement could more pellets be stuffed into the same container.”). In 1900 the famous mathematician D. Hilbert presented a list of 23 mathematical problems. The 18th of these, Kepler’s conjecture, can be phrased: “How can one arrange most densely in space an infinite number of equal solids of given form, e.g., spheres with given radii..., that is, how can one so fit them together that the ratio of the filled to the unfilled space may be as great as possible?” The Kepler's conjecture is intuitively obvious, but the proof remained elusive until it was accomplished by T. Hales in 1998 [4].

Random packings of uniform hard spheres have been studied by biologists, material scientists, engineers and physicists to understand the structure of living cells, liquids, granular media, glasses and amorphous solids to mention but a few examples. The first person to undertake the experiments with the ball bearings in the bag was an Irish crystallographer and biophysicist, J. D. Bernal in the 1950’s [5]. The random packing of balls became known as Bernal packing. The packing fraction of the Bernal packing was found to be roughly 0.64; or slightly less if the balls were not shaken.

The second important person who thoroughly studied stable [6] random arrangements was G. D. Scott [7] who poured thousands of spherical balls into rigid containers. When the container was gently shaken to optimise the packing, the packing fraction was found to be 0.636. One thing became clear quite early: no amount of shaking the container would cause the balls to come together in an elegant ordered structure. When the container was not shaken, the loosest random packing was found to have the packing fraction of 0.60. He found that there existed a range of random-packing densities, with an upper limit of 0.64 and lower limit of 0.60. The upper limit is often referred to as random close packing (RCP) and the lower limit as random loose packing (RLP).

The prevailing notion of random close packing is that it is the maximum density that a large, random collection of spheres can attain and that this density is a universal quantity. It is now generally agreed that random-close packing represents the densest packing of uniform spheres having a random structure and that it has a density of 0.6366 ± 0.0004[8].

No clear definition exists for the RLP, except for the implication that it represents the loosest possible, random packing that is mechanically stable. The packing fraction value of 0.60 was obtained by rolling spheres gently into a container without shaking. Lower values for the packing fraction can be obtained only by eliminating gravity [4]. The lowest densities which can be experimentally obtained are around ~0.56 [9].

The various theoretical and experimental packing fractions for rigid spheres are summarised in Table 1.

<table>
<thead>
<tr>
<th>Packing</th>
<th>Packing fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loosest possible [10]</td>
<td>0.06</td>
</tr>
<tr>
<td>Simple cubic lattice</td>
<td>0.52</td>
</tr>
<tr>
<td>Loosest random in lack of gravity [9]</td>
<td>0.56</td>
</tr>
<tr>
<td>Loosest random [4]</td>
<td>0.60</td>
</tr>
<tr>
<td>Dense random packing [8]</td>
<td>0.64</td>
</tr>
<tr>
<td>Face-centred cubic and Hexagonal close packed lattice</td>
<td>0.74</td>
</tr>
</tbody>
</table>
In order to observe the effects of packing densification we designed a simple experimental device, presented in Figure 2. We poured approximately 120,000 ball bearings (4 mm in diameter) in a Plexiglas® cylinder 10 cm in diameter and 100 cm tall. After shaking the decrease of the height of the column of ball bearings was observed. The packing fraction increase was estimated to be about 2.5%, which is in a good agreement with other experiments [4]. A close-up photograph of the ordering of ball bearings after shaking is shown in Figure 3.

![Figure 2: The experimental set-up](image1)

![Figure 3: Ordering of ball bearings after shaking](image2)

### 3 MULTIPLICATIN FACTOR IN PEBBLE BED REACTOR

The primary objective in the design and operation of a nuclear reactor is the utilization of the energy released by a controlled chain reaction of nuclear fission events maintained within the reactor core. Every fission reaction releases 2-3 neutrons and ~200 MeV of energy. A given neutron will be “born” in a fission event and will then usually scatter about the reactor until it meets its eventual “death” in either an absorption reaction or by leaking out of the reactor. Certain numbers of these neutrons will be absorbed by fissile nuclei (such as $^{235}$U and $^{239}$Pu) and induce further fission, thereby leading to the birth of a new generation of fission neutrons and so on. Such process is called a fission chain reaction. It can be described by the multiplication factor $k$ defined as

$$k \equiv \frac{\text{Number of neutrons in one generation}}{\text{Number of neutrons in preceding generation}} \quad (3.1)$$

Note that if $k = 1$, the number of neutrons in any two consecutive fission generations will be the same, and hence the chain reaction will be time-independent. We refer to a system
characterized by $k = 1$ as being critical. If $k < 1$, the system is said to be subcritical, and for $k > 1$ we refer to such system as being supercritical.

The above definition of the multiplication factor $k$ is for $k \approx 1$ equivalent to the following

$$k = \frac{\text{Rate of neutron production in reactor}}{\text{Rate of neutron losses (absorption plus leakage) in reactor}} \quad (3.2)$$

The neutrons are produced in fission reactions. The rate of neutron production is proportional to the fission macroscopic cross section [11] and the average number of neutrons released per fission. Similarly is the rate of neutron losses due to absorption proportional to absorption macroscopic cross section. The ratio of the fission and absorption reaction rates is the multiplication factor of an infinite medium ($k$-infinity) without neutron leakage. To determine the multiplication factor of a realistic, finite reactor ($k$), the neutron leakage must be taken into account. The infinite medium multiplication factor $k$-infinity is multiplied by the probability that the neutron will remain in the reactor. The non-leakage probability depends on the size and geometry of the reactor. For simple geometry (cylinder, cube) it can be calculated analytically [12] that the effective multiplication factor formula takes the following form

$$k = \frac{\langle n\Sigma_f \rangle}{\langle \Sigma_a \rangle} \cdot \frac{1}{1 + \langle D \rangle B^2}, \quad (3.3)$$

where the first term denotes $k$-infinity and the second term denotes the neutron non-leakage probability. $D$ is known as the diffusion constant. It is inversely proportional to total macroscopic cross section and consequently to the atomic number density. $B^2$ is the leakage operator eigenvalue and depends on the geometry of the reactor. For a homogeneous cylinder

$$B^2 = \left( \frac{\pi}{H} \right)^2 + \left( \frac{2.405}{R} \right)^2, \quad (3.4)$$

where $H$ and $R$ are active core height and radius, respectively. We see for a typical PBR with $R = 2$ m and $H = 9$ m that the second term in Eq. (3.4) is much bigger than the first. Consequently, changes in $H$ due to packing fraction changes at fixed core radius will not change $B^2$ significantly.

Neutron energies are in a fission reactor in the range from $10^{-3}$ to $10^7$ eV. Since neutron cross sections depend significantly on energy and also on the density and type of the material in the reactor, cross sections have to be averaged over energy and over volume of the reactor as indicated by brackets in Eq. (3.3). Such calculation is normally performed by special computer programs.

4 MODEL AND CALCULATION

The multiplication factor calculation was performed for a homogenous reactor core. A homogenous core is a good approximation because the neutron diffusion length ($\sim$ 10 cm) is much larger than heterogenities in the core ($\sim$ 1 mm). The homogenisation was performed for spherical 'unit cell' of the core, consisting of three regions:
1. inner fissile region, mixture of UO$_2$ and graphite
2. outer graphite shell, pure graphite
3. inter-pebble space, filled with helium

Volumes and densities of the materials were varied according to the problem under investigation. Changing of packing fraction in our model means changing of region 3 outer radius while other dimensions remain fixed. In other words changing of packing fraction means changing the core effective density.

WIMS computer code in 69 energy groups homogenous medium approximation is used for the calculation of the multiplication factor [13]. The WIMS program averages the macroscopic cross sections and the diffusion coefficient on the basis of microscopic cross sections, which are tabulated in the libraries of cross sections and on the basis of the material composition of the reactor, which is provided in the input. The leakage from the reactor was modelled by taking into account Eqs. (3.3) and (3.4).

5 DEPENDENCE OF $K$ ON PACKING FRACTION

Calculated $k$ as a function of the packing fraction is presented in Figure 4. The entire range of packing fractions was considered, including values expected at normal conditions (around 0.6) as well as at hypothetical and accident conditions. Packing fractions under 0.52 are attainable only in special conditions, such as occurrence of large voids in the core. Packing fractions above 0.74 correspond to (partly) crushed fuel pebbles [14]. The markers in the graph represent the calculated values of $k$. The lines in the graph are the smoothed connection lines between the calculated values.

It is observed that the $k$ increases with packing fraction over entire range. As the infinite medium multiplication factor $k$-infinity does not change with density (packing fraction) in homogeneous system, the effect must be explained by the reduced leakage of neutrons, described by $DB^2$ term. $B^2$ is increasing with increasing packing fraction meaning the decreasing of the volume of the core. The effect is opposite to the trend observed in Figure 4 but small, as expected from Eq. (3.4). It is out weighted by the effect in diffusion coefficient $D$, which is inversely proportional to the density, i.e. to the packing fraction, as can be seen from Figure 5.

![Figure 4: K-effective versus packing fraction and fuel temperature](image)

Figure 4: K-effective versus packing fraction and fuel temperature
We can see from Figure 4 that PBR features a strong negative temperature effect. We also note that $k$ is close to one at operating temperatures ($\sim 1100^\circ C$) and expected packing fraction ($\sim 0.60$). This is in agreement with Ref. [15] and confirms our calculation model. It can be also noted from the result in Figure 4 that in accidents, involving crushing of the core, the $k$ could increase leading to reactivity excursion.

![Figure 5: Effective diffusion coefficient calculated in WIMS as a function of packing fraction](image)

**Figure 5:** Effective diffusion coefficient calculated in WIMS as a function of packing fraction

## 6 CONCLUSION

It was shown that packing fraction of the pebbles in the PBR's core has a strong influence on reactivity. In general, multiplication factor increases when packing density is increased. The packing fraction could increase for up to 0.04 (from 0.60 to 0.64) as a consequence of earthquake or other core vibration. Our results supported also by the references show that this could result in the core multiplication factor increase of 3\% [16]. Such increase of the multiplication factor might lead to power excursion, if it happened in a very short period of time (a second or less) in which control rods could not be inserted into the core. However the power excursion may be prevented by the built in control mechanisms (such as control rods) if the increasing of $k$ lasted for a minute or more.
7 REFERENCES

[6] Packing of spheres is mechanically stable, when it is in static equilibrium under an existing set of externally applied forces
[11] Macroscopic cross section is defined as the product of the microscopic cross section and the atomic number density
[14] If fuel elements crushed, the crushed parts would fill the inter pebble space leading to increase of packing fraction.
[16] In Nuclear power plant Krško, during normal operation, k is 1.00000 ± 0.00001